Ronzhin A.F. (Lebedev Institute of Precision Mechanics and Computer Engineering, Russian Academy of Sciences, Moscow, Russia) **Accelerated assessment of average uptime**.

Independent devices whose uptime (operating time to failure) is are random variables are being tested $X_1, X_2, \ldots, X_N$. Rearrange the values of earnings per failure in ascending order $X_{1,N} \leq X_{2,N}, \ldots, X_{N,N}$. Let's construct a sequence of random variables

$$\begin{aligned} Y_{1,N} &= N \times X_{1,N}, Y_{2,N} = (N-1) \times (X_{2,N} - X_{1,N}), \ldots, \\ Y_{m,N} &= (N-m+1) \times (X_{m,N} - X_{m,N-1}), \ldots, \\ Y_{N,N} &= 1 \times (X_{N,N} - X_{N,N-1}). \end{aligned}$$

As a first assessment parameter $T = MX_1$, consider statistics $T_{n,N} = n^{-1} \times (Y_{1,N} + Y_{2,N} + \ldots + Y_{n,N}), n \leq N$.

Let's divide $N = m \times n$ the devices into n groups of m devices each. Let be the time of the first failure in the i-th group. Along with the statistics $T_{n,N}$, consider the second statistic as an estimate of parameter T

$$T_n^{(m)} = m \times n^{-1} \times (Z_1^{(m)} + \ldots + Z_n^{(m)}).$$

Theorem. Let the uptime of N independent devices have an exponential distribution with a parameter T. Then, for any integers $m, n \leq N$, the statistics $T_{n,N}$ and $T_n^{(m)}$ are unbiased and consistent with estimates of the parameter T, their mathematical expectation and variance are T and T^2/n, respectively. Random variables $n \times T_{n,N}$ and $n \times T_n^{(m)}$ have a gamma distribution with parameters (n, T^{-1}). For mathematical expectation of the test time for constructing statistics $T_{n,N}$ and $T_n^{(m)}$ the relations are valid

$$\begin{aligned} \tau_{n,N} &= T \times (1/(N-n+1) + 1/(N-n+2) + \ldots + 1/(N)) \\ \tau_n^{(m)} &= m^{-1} \times T \times (1 + 1/2 + \ldots + 1/n), \\ \tau_{n,n} &= \tau_n^{(1)}, \tau_{n,n\times m} \leq \tau_n^{(m)}, (n/N) \leq T^{-1} \times \tau_{n,N} \leq (n/(N-n+1)), N \geq n. \end{aligned}$$

Remark. The inequalities are proved in [1]

$$m^{-1} \times T \leq \tau_{n,n\times m} \leq (m-1)^{-1} \times T.$$

References

[1] Tsitsiashvili G.S. Ergodicity of statistical estimates of the intensity of the Poisson flow. Far Eastern Mathematical J, 2019, V. 19, № 22 – P.256-260.