V. I. Afanasyev. Excursions and meanders of a discrete bridge and a Brownian bridge[1]

An integer random walk $\{S_i,\ i \geq 0\}$ with zero drift and finite variance σ^2 is considered, provided that $S_n = 0$. Set $\beta(m) = \max\{i \leq m : S_i = 0\}$, $\gamma(m) = \min\{i > m : S_i = 0\}$ for $m \in \{0, 1, \ldots, n-1\}$. Let $s \in (0,1)$ and $n_s = \lfloor ns \rfloor$. For random vectors $(n_s - \beta(n_s), \gamma(n_s) - n_s)$ and $(n_s - \beta(n_s), S_{n_s})$, local limit (as $n \to \infty$) theorems are proved, and the following result is derived from them. Let the symbol $\stackrel{D}{\to}$ denote convergence in distribution. Set

$$f_s(x,y) = \frac{1}{2\pi} \frac{1}{\sqrt{(s-x)(1-s-y)}\,(x+y)^{3/2}},\ x \in (0,s),\ y \in (0,1-s);$$

$$g_s(x,y) = \frac{1}{2\pi} \frac{|y| \exp\left(-\frac{y^2}{2}\left(\frac{1}{1-s} + \frac{1}{x}\right)\right)}{x^{3/2}\sqrt{(1-s)(s-x)}},\ x \in (0,s),\ y \in \mathbf{R}.$$

Theorem. *If* $n \to \infty$, *then*

$$\left\{ \frac{n_s - \beta(n_s)}{n}, \frac{\gamma(n_s) - n_s}{n} \,\middle|\, S_n = 0 \right\} \stackrel{D}{\to} (\zeta_1, \zeta_2),$$

$$\left\{ \frac{n_s - \beta(n_s)}{n}, \frac{S_{n_s}}{\sigma\sqrt{n}} \,\middle|\, S_n = 0 \right\} \stackrel{D}{\to} (\zeta_1, Y),$$

where (ζ_1, ζ_2) *and* (ζ_1, Y) *are random vectors with distribution densities* $f_s(\cdot,\cdot)$ *and* $g_s(\cdot,\cdot)$ respectively.

The theorem allows us to study excursions and meanders of a Brownian bridge.

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