

Tight Asymptotic of Probability of Singularity of $n \times n$ Random Matrix with Uniform Distributed ± 1 Entries

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Abstract

We prove the conjecture: probability that P_n of Bernulli ± 1 square matrix is singular has tight asymptotic $4\binom{n}{2}2^{-n}$. We also prove precise asymptotic $P_n - 4\binom{n}{2}2^{-n} \sim 16\binom{n}{4}\left(\frac{3}{8}\right)^n$.

There is a number of works devoted to determination of tight asymptotic of probability P_n that random square matrix with independent uniformly distributed ± 1 entries is singular.

The the general conjecture stated

Conjecture 1 *The following asymptotic equality is valid*

$$P_n \sim 4\binom{n}{2}2^{-n}. \quad (1)$$

In this article we prove this conjecture.

Theorem 1 *Asymptotic (1) for P_n is true.*

Using the same arguments as in the proof of Conjecture 1, we prove

Conjecture 2 *The following asymptotic equality is valid*

$$P_n - 4\binom{n}{2}2^{-n} \sim 16\binom{n}{4}\left(\frac{3}{8}\right)^n. \quad (2)$$