On convergence of control problems in the theory of financial markets with transaction costs

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Control problem. Fix a finite horizon T and parameters $\mathbf{M} = (\mathbb{B}, Y, S, K)$, where $\mathbb{B} = (\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$, $S = (S^1, \dots, S^d) > 0$ is the d-dimensional price process, Y is the m-dimensional information process, and K is a constant closed convex cone with $\operatorname{int} K^* \neq \varnothing$. A strategy B is an $\mathbb{F}$-adapted càdlàg K-decreasing finite-variation process. For $x \in K$ put

$$\widehat{V}^i_t = x^i + \int_{[0,t]} (S^i_u)^{-1}\, dB^i_u, \qquad V^i_t = S^i_t \widehat{V}^i_t.$$

The strategy is admissible if $V^{x,B}_t \in K$ for all $t \in [0,T]$; the set of such strategies is denoted by $\mathcal{A}(x, \mathbf{M})$. We use the utility function

$$U(v,s) = \frac{1}{1-\gamma}\ell(v)^{1-\gamma}, \quad 0 < \gamma < 1, \qquad \ell(v) = \sup\{\lambda \in \mathbb{R} :\ v - \lambda e_1 \in K\}.$$

The control problem is to compute

$$u(x, \mathbf{M}) = \sup_{B \in \mathcal{A}(x,\mathbf{M})} \mathbb{E}\, U(V^{x,B}_T, S).$$

Main result. Let $\mathbf{M}^n = (\mathbb{B}^n, Y^n, S^n, K)$ and $\mathbf{M}$ have the same cone K, let $x \in \operatorname{int} K$, and suppose that the laws $\mathcal{L}(Y^n, S^n \,|\, \mathbb{P}^n)$ converge weakly to $\mathcal{L}(Y, S \,|\, \mathbb{P})$ in the Skorokhod J_1 topology on $D([0,T]; \mathbb{R}^{m+d})$. Assume that the families

$$\{U(V^{x,B^n}_T, S^n) :\ n \geq 1,\ B^n \in \mathcal{A}(x, \mathbf{M}^n)\}, \qquad \{U(V^{x,B}_T, S) :\ B \in \mathcal{A}(x, \mathbf{M})\}$$

are uniformly integrable. There is also a common $\varepsilon > 0$ and price systems Z^n, Z such that Z^n_t/S^n_t, $Z_t/S_t \in \varepsilon\text{-}\operatorname{int} K^*$; moreover, $\mathbb{P}^n$ is contiguous with respect to $\mathbb{Q}^n = Z^{n,1}_T \mathbb{P}^n$. Then

$$u(x, \mathbf{M}^n) \longrightarrow u(x, \mathbf{M}).$$

If B^n are asymptotically optimal, then there is an optimal strategy $B \in \mathcal{A}(x, \mathbf{M})$, and along a subsequence $\mathcal{L}(Y^n, S^n, B^n) \Rightarrow \mathcal{L}(Y, S, B)$: in the J_1 topology on (Y, S) and in the Meyer–Zheng topology on B.