Stationary Distribution of a Multi-Class Queueing Network with Unreliable Nodes on Random Graphs. E.Yu. Kalimulina (MSU, IITP RAS, Moscow, Russia)

The classical open queueing networks of Jackson, BCMP and Kelly type admit a product-form stationary distribution under a stochastic routing matrix that does not depend on the state of individual nodes. When nodes may fail and routing adapts to the current working subnetwork, both the correct problem statement and the description of the stationary regime require a separate analysis. The present work proposes such a setting in the multi-class case on a random geometric graph and describes its stationary regime in closed form.

Let η be a homogeneous Poisson point process on $\mathbb{R}^2$ and let $G^{(R)}$ be the finite subgraph of the Delaunay triangulation $\mathrm{Del}(\eta)$ induced by a window $W \subset \mathbb{R}^2$. Each node $v \in V_R$ carries an independent Bernoulli flag $\xi_v \in \{0,1\}$ (working / failed). A fixed compact *service region* $\Lambda_0 \subset \mathbb{R}^2$ partitions the nodes into PS servers of rate μ inside Λ_0 and IS (infinite-server) transit nodes outside; external Poisson streams of rate γ_s arrive only at PS nodes $s \in \Lambda$. On its first service each customer is assigned a destination $w \in \Lambda$ by a tag matrix R^0, and is then routed step by step along the *current* shortest path $\sigma(s, w;\, G(\xi))$ of the working subgraph $G(\xi) := G^{(R)}[\{v : \xi_v = 1\}]$; if w becomes unreachable in $G(\xi)$, the customer is lost.

Fix $\xi \in \{0,1\}^{V_R}$ and let $\Lambda_\xi := \{u \in \Lambda : \xi_u = 1\}$, $T_\xi := \{z \in V_R \setminus \Lambda : \xi_z = 1\}$. Let P^ξ be the class-resolved routing matrix induced by tag-shortest-path routing on $G(\xi)$, let $(a^\xi_c)_c$ be the unique non-negative solution of the multi-class traffic equations for P^ξ, and define the load

$$\rho^\xi_u \;:=\; \frac{1}{\mu} \sum_{\substack{c \in \mathcal{K}_\xi:\\ \mathrm{loc}(c)=u}} a^\xi_c, \qquad u \in \Lambda_\xi.$$

Theorem 1. *Assume $\rho^\xi_u < 1$ for every $u \in \Lambda_\xi$. Then the reduced multi-class process on $\Lambda_\xi \cup T_\xi$ is positive recurrent, and its unique stationary distribution has Kelly-type product form: $N_u \sim \mathrm{Geom}(\rho^\xi_u)$ independent over $u \in \Lambda_\xi$; $m_{z,w} \sim \mathrm{Pois}(a^\xi_{(z,w)}/\mu)$ independent over $z \in T_\xi$; conditionally on N_u, the class composition $(n_{u,c})_c$ is multinomial with weights proportional to $a^\xi_{(u,c)}$. Moreover, the detour load on $u \in \Lambda_\xi$ admits the* exact *representation*

$$\rho^\xi_{u,\mathrm{tag}} \;=\; \frac{1}{\mu} \sum_{s \in \Lambda_\xi} \gamma_s \sum_{\substack{w \in \Lambda_\xi\\ w \neq s}} r^0_{sw}\, \mathbf{1}\big\{u \in \sigma(s, w;\, G^{(R)}(\xi)) \setminus \{s, w\}\big\}.$$