**Reversible canonical measure for a zero-range process with independent Markov node failures. E.Yu. Kalimulina, A.A. Esin (IITP RAS, MSU, Moscow, Russia)**

We consider a conservative particle transport model on a network with independent Markov node failures: an edge is open only when both its endpoints are active.

Let $G_0 = (V, E)$ be a finite connected undirected graph, $|V| = N \geq 2$, with vertex degrees $k_i$, $i \in V$. The rates of the zero-range process (ZRP) are given by a function $u\colon \mathbb{Z}_+ \to [0, \infty)$ with $u(0) = 0$, $u(n) > 0$ for $n \geq 1$; set $p(0) := 1$, $p(m) := \prod_{n=1}^{m} u(n)^{-1}$ for $m \geq 1$. At each vertex an independent two-state Markov chain $I_v(t) \in \{0,1\}$ is given with rates $\alpha > 0$ $(1 \to 0)$ and $\beta > 0$ $(0 \to 1)$; set $a := \beta/(\alpha+\beta)$, and let $\mu_a := \bigotimes_{v\in V} \mathrm{Bernoulli}(a)$, which is reversible since $a\alpha = (1-a)\beta$. The random subgraph $G(t) := (V, \{\{i,j\} \in E : I_i(t)I_j(t) = 1\})$ is uniquely determined by $I(t)$. The joint Markov process $(\eta(t), I(t))$ on $\mathbb{Z}_+^V \times \{0,1\}^V$ has generator

$$\mathcal{L}^{\mathrm{tot}} f(\eta, I) = \sum_{i\in V} \sum_{j\sim i} I_i I_j \, \frac{u(\eta_i)}{k_i} \big[ f(\eta^{i\to j}, I) - f(\eta, I) \big] + \mathcal{L}_I f,$$

where $\eta^{i\to j}$ denotes the configuration after a particle has moved from $i$ to $j$ (the term vanishes if $\eta_i = 0$ since $u(0) = 0$); $\mathcal{L}_I$ is the generator of $|V|$ independent two-state chains.

**Theorem 1.** *Let $\{q_i\}_{i\in V}$ be positive weights; let $\{A_{ij}(I)\}_{i,j\in V}$ be symmetric conductances [3]: $A_{ij}(I) = A_{ji}(I) \geq 0$, $A_{ij} \equiv 0$ if $\{i,j\} \notin E$; let $\mathcal{L}_I$ be the generator of an irreducible Markov chain on $\{0,1\}^V$ with reversible measure $\mu$, independent of $\eta$. Then for the generator*

$$\widetilde{\mathcal{L}} f(\eta, I) = \sum_{i,j\in V} A_{ij}(I) \, \frac{u(\eta_i)}{q_i} \big[ f(\eta^{i\to j}, I) - f(\eta, I) \big] + \mathcal{L}_I f$$

*and any $M \in \mathbb{Z}_+$ the canonical measure*

$$\pi_{N,M}(\eta, I) = \frac{1}{Z_{N,M}} \, \mu(I) \prod_{i\in V} p(\eta_i) \, q_i^{\eta_i} \cdot \mathbf{1}\Big\{ \sum_{i\in V} \eta_i = M \Big\}$$

*is reversible with respect to $\widetilde{\mathcal{L}}$ and unique on the sector $\{\sum_i \eta_i = M\}$ provided the joint chain is irreducible. In the principal example $q_i = k_i$ (the unnormalised invariant measure of the simple random walk on $G_0$), $A_{ij}(I) = I_i I_j \, \mathbf{1}_{\{i\sim j\}}$ and $\mu = \mu_a$; the joint chain is irreducible whenever $G_0$ is connected, $N \geq 2$ and $\alpha, \beta > 0$, and the reversible measure takes the explicit form*

$$\pi_{N,M}(\eta, I) \propto \prod_{i\in V} p(\eta_i) \, k_i^{\eta_i} \cdot \prod_{i\in V} a^{I_i} (1-a)^{1-I_i} \cdot \mathbf{1}\Big\{ \sum_i \eta_i = M \Big\}.$$

*Consequences for the $\eta$-marginal.* (i) In the principal example the $\eta$-marginal of $\pi_{N,M}$ coincides with the classical ZRP canonical measure [1], $\pi^{\eta}_{N,M}(\eta) \propto \prod_i p(\eta_i) \, k_i^{\eta_i} \mathbf{1}\{\sum \eta_i = M\}$, for all $\alpha, \beta > 0$ and without any limiting procedure in the activation rates; in particular, for any sequence of base graphs $\{G_N\}$ on which the failure-free ZRP admits a thermodynamic limit with critical density and condensation [2], symmetric independent node activity does not shift the static phase diagram nor relocate the condensate. (ii) In the regime $\alpha + \beta \to \infty$ with $a$ fixed, standard averaging over the fast environment yields the generator $a^2 \mathcal{L}^{\mathrm{ZRP}}_{\eta}$ for the $\eta$-projection: the dynamics is the failure-free ZRP slowed down by a factor $a^{-2}$. Within this reversible product-form class, a non-trivial shift of $\rho_c$ is possible only through a modified single-site invariant measure, $\eta$-dependent failures, or a non-reversible edge dynamics with a different stationary structure.

## References


1. *Andjel E. D.* Invariant measures for the zero range process // Ann. Probab. 1982. V. 10. No. 3. P. 525–547.
2. *Evans M. R., Hanney T.* Nonequilibrium statistical mechanics of the zero-range process and related models // J. Phys. A. 2005. V. 38. P. R195–R240.
3. *Doyle P. G., Snell J. L.* Random Walks and Electric Networks. Washington, DC: Math. Assoc. America, 1984. (Carus Math. Monogr., V. 22.)