

M. A. Leshchinskaya, E. A. Pchelintsev (TSU, Tomsk, Russia). **Adaptive function estimation in semimartingale regression**¹.

Let the observed process $(y_t)_{0 \leq t \leq 1}$ satisfies the stochastic differential equation on the probability space $(\Omega, \mathcal{F}, \mathbf{P})$: $dy_t = S(t)dt + \varepsilon d\xi_t$, $0 \leq t \leq 1$, where $\varepsilon > 0$ is a noise intensity, ξ_t is a square-integrable semimartingale with a distribution from some class \mathcal{Q}_ε . The function S belongs to the Sobolev class $\Theta = \left\{ S \in \mathcal{L}_2[0, 1] : \sum_{j=1}^{\infty} e^{2\kappa j^\alpha} \theta_j^2 \leq \mathbf{r} \right\}$ with $\alpha \in (0, 1)$, $\kappa > 0$, $r > 0$ and $(\theta_j)_{j \in \mathbb{N}}$ are the Fourier coefficients of the function S with respect to the orthonormal basis $(\varphi_j)_{j \geq 1}$.

The problem is to estimate S from observations $(y_t)_{0 \leq t \leq 1}$. The quality of the estimate \widehat{S} is measured by the robust quadratic risk $\mathcal{R}(\widehat{S}, S) = \sup_{Q \in \mathcal{Q}_\varepsilon} \mathbf{E}_Q \|\widehat{S} - S\|^2$.

For the case of known smoothness parameters, a weighted projection estimate is constructed

$$\widehat{S}_\gamma(t) = \sum_{j=1}^n \gamma_j \widehat{\theta}_j \varphi_j(t), \quad \gamma_j = 1 - e^{-\kappa(n^\alpha - j^\alpha)}, \quad \widehat{\theta}_j = \int_0^1 \varphi_j(t) dy_t,$$

$$n = \max \left\{ l \geq 1 : e^{\kappa l^\alpha} \sum_{j=1}^l e^{\kappa j^\alpha} - \sum_{j=1}^l e^{2\kappa j^\alpha} \leq \varepsilon^{-2} \mathbf{r} \right\},$$

for which the robust risk asymptotics is found, i.e. $\mathcal{R}(\widehat{S}_\gamma, S) \sim \kappa^{-1/\alpha} \varepsilon^2 |\ln \varepsilon|^{1/\alpha}$ for $\varepsilon \rightarrow 0$ [1]. In the adaptive setting, when α, κ, r are unknown, a model selection procedure S^* is proposed based on sharp oracle inequalities.

Теорема. The model selection procedure S^* is asymptotically efficient, i.e.

$$\lim_{\varepsilon \rightarrow 0} \frac{\sup_{S \in \Theta} \mathcal{R}(S^*, S)}{\varepsilon^2 |\ln \varepsilon|^{1/\alpha}} = \kappa^{-1/\alpha}.$$

REFERENCES

[1] Pchelintsev E., Pergamenschikov S.M., Povzun M.A. Efficient estimation methods for non-Gaussian regression models in continuous time // Annals of the Institute of Statistical Mathematics. 2022. Vol. 74, № 1. P. 113–142.

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