Martynov Gennady. (IPPI RAS). Goodness-of-fit test for the Weibull distribution family.

In the analysis of stochastic systems, it is common to select the distribution of the input random influence from a number of standard distributions. However, if it is possible to obtain observations of the random variable present in the system, it is useful to apply a goodness-of-fit test to verify is these observations conconsistent with the previously chosen distribution.

If the hypothesis is accepted, then a conclusion can be made about the possibility of using this distribution in the analysis of the quality of the system functionality with the used sample size. Otherwise, the value of the test statistic is a measure of the deviation of the distribution from the unknown hypothetical one, by which production losses are estimated if the hypothetical distribution is used. An important part of such research is obtaining the limiting distribution of the statistic of the chosen goodness-of-fit test.

The main part of the report considers a five-parameter family of mixture of two Weibull distributions. The Cramer-von Mises-Smirnov omega-square test was chosen as the test. Formulas for the covariance functions of the corresponding limiting Gaussian processes are obtained. Exact significance levels are calculated for both the described family of distributions and for subfamilies with a smaller number of unknown mixture parameters.

Theorem 1. *The three parametric family of the mixture of two Weibull distribution has a form* $F(x, \lambda_1, \lambda_2, k) = 0.5W(x, \lambda_1, k) + 0.5W(x, \lambda_2, k)$. *Let the estimations of the unknown parameters be:* $\hat{\theta} = (\hat{\lambda_1},\ \hat{\lambda_2},\ \hat{k})'$. *The corresponding empirical process weakly converges to Gaussian process with the covariance function* $C(t, \hat{\theta} = \min(t, \tau) - t\tau - q^\top(t; \hat{\theta}) I^{-1}(\hat{\theta}) q(\tau; \hat{\theta})$, $q(s; \hat{\theta}) = ((\partial/\partial\hat{\theta}) F(x; \hat{\theta}))|_{x=F^{-1}(s;\hat{\theta})}$, *and* $I(\hat{\theta})$ *is the Fisher information matrix.*