The Density of Cross-Persistence Diagrams and its Applications

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Topological Data Analysis (TDA) provides tools for studying geometric and topological properties of data using methods from algebraic topology. Persistence diagrams describe the birth and death of topological features across scales and are widely used in machine learning and data analysis. Recently introduced cross-persistence diagrams extend this framework to pairs of point clouds and allow one to capture interactions between different manifolds.

In this work, we study probabilistic properties of cross-persistence diagrams and prove the existence of their density. This result establishes a theoretical foundation for statistical methods based on cross-persistence, including density estimation and machine learning approaches for comparing point clouds sampled from different manifolds.

Theorem 1. *Let $n, k \geq 1$. Assume that M and N are real analytic compact d-dimensional connected submanifolds, possibly with boundaries, and let $\mathbf{X} \in M^n$ and $\mathbf{Y} \in N^k$ be random variables with densities with respect to the Hausdorff measures $\mathcal{H}_{dn}$ and $\mathcal{H}_{dk}$, respectively.*

Then, for the Vietoris–Rips cross-persistence filtration $\mathcal{K}$, the expected measure $\mathbb{E}[D_s[\mathcal{K}(\mathbf{X},\mathbf{Y})]]$ admits a density with respect to the Lebesgue measure on Δ for all $s \geq 1$. Furthermore, $\mathbb{E}[D_0[\mathcal{K}(\mathbf{X},\mathbf{Y})]]$ admits a density with respect to the Lebesgue measure on the vertical line $\{0\} \times [0, \infty)$.

The obtained result allows one to apply classical probabilistic and statistical methods to cross-persistence diagrams and provides a basis for machine learning algorithms that analyze topological interactions between point clouds.