Nikiforov N., Pchelintsev E., Pergamenshchikov S. (TSU, Tomsk, Russia).
On efficient estimation of functions from the Sobolev class with exponential coefficients.[1]

We consider the problem of estimating a function $S \in \mathcal{L}_2[a,b]$ in additive regression

$$y_k = S(t_k) + \xi_k, \quad t_k = a + (2k-1)(b-a)/(2n)\,, \quad 1 \le k \le n,$$

where $(\xi_k)_{1\le k\le n}$ is heteroscedastic weak white noise with distribution $\mathbf{p} \in \mathcal{P}_n$ such that $\max_{1\le k\le n} \int_{\mathbb{R}^n} x_k^2\,\mathbf{p}(dx) \le \varsigma_{*,n}$. We assume that the Fourier coefficients of the function S w.r.t. the orthonormal basis $(\psi_j)_{j\ge 1}$ in $\mathcal{L}_2[a,b]$ on the sieve $\mathcal{T}_n = \{t_1, ..., t_n\}$ belong to some ellipsoid in $\mathbb{R}^n$, i.e. the function S belongs to the Sobolev class $\Theta = \big\{S = \sum_{j=1}^n z_j\psi_j \,:\, \sum_{l=1}^n e^{2\kappa l^\alpha}\, z_l^2 \leqslant r\big\}$, $\kappa > 0$, $0 < \alpha < 1$, $r > 0$. The quality of any estimate $\widehat{S}_n$ under observations $y = (y_k)_{1\le k\le n}$ is measured by the robust quadratic risk

$$\mathcal{R}_n(\widehat{S}_n, S) = \sup_{\mathbf{p}\in\mathcal{P}_n} \frac{b-a}{n} \sum_{k=1}^n \mathbf{E}_{\mathbf{p},S}(\widehat{S}_n(t_k) - S(t_k))^2\,.$$

Here $\mathbf{E}_{\mathbf{p},S}$ is expectation w.r.t. the distribution of y. Let us determine the weighted LSE [1]. We put $n_* = [(2\kappa)^{-1/\alpha}(\ln n)^{1/\alpha}/(b-a)] + 1$ and

$$\widehat{S}_n(t) = \sum_{j=1}^{n_*} \left(1 - e^{-\kappa(n_*^\alpha - l^\alpha)}\right) \widehat{\theta}_j \psi_j(t)\,, \quad t \in \mathcal{T}_n\,. \tag{1}$$

Theorem. For any $0 < \alpha < 1$, $\kappa > 0$ and $r > 0$ the estimate (1) is asymptotically robust efficient, i.e. $\lim_{n\to\infty} n(\ln n)^{-1/\alpha}\varsigma_{*,n}^{-1} \sup_{S\in\Theta} \mathcal{R}_n(\widehat{S}_n, S) = (2\kappa)^{-1/\alpha}$.

References

[1] Nikiforov N., Pchelintsev E., Pergamenshchikov S. Robust superefficient estimation method for nonparametric regression models // Annals of the Institute of Statistical Mathematics. 2026. In press. P. 1–24.

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