Platonova M. V. (St. Petersburg, Russia). **On a probabilistic approach to the Cauchy problem solution for an evolution equation with a higher even-order differentiation operator.**

Consider the Cauchy problem for an evolution equation with a higher even-order differential operator on its right-hand side $\frac{\partial u}{\partial t} = \mathcal{L}u$, $u(0,x) = \varphi(x)$, where $\mathcal{L} = (-1)^{\frac{m+2}{2}} \frac{c^m(x)}{m!} \frac{d^m}{dx^m}$. It is well known that for $m = 2$ and $c(x) \in C_b^2(\mathbf{R})$ such that $c(x) \neq 0$ for all $x \in \mathbf{R}$, the solution of the Cauchy problem can be represented as $u(t,x) = \mathbf{E}\,\varphi(\xi_x(t))$, where $\xi_x(t)$ is a diffusion process satisfying the stochastic differential equation $d\xi_x(t) = c(\xi_x(t))\,dw(t)$. For $m \geqslant 4$, such a representation for the solution is impossible. Nevertheless, in this case one can construct a probabilistic approximation of the solution of the Cauchy problem.

Let $\xi_1, \xi_2, \ldots$ be i.i.d. bounded random variables with a symmetric distribution $\mathcal{P}$ and $\mathbf{E}\,\xi_1^m = 1$.

Denote by $\eta_n(t)$ a Poisson process with parameter n, independent of $\{\xi_j\}_{j=1}^{\infty}$, and by $t_1, t_2, \ldots, t_j$ the jump times of $\eta_n(t)$ up to time t. Define the process $\zeta_n^x(t)$ as follows: $\zeta_n^x(t) = x$ for $t \in [0, t_1)$ and $\zeta_n^x(t) = \zeta_n^x(t_j-) + \sigma c(\zeta_n^x(t_j-)) \frac{\xi_j}{n^{1/m}}$ for $t \in [t_j, t)$.

Let $\sigma = 1$ for $m = 4l+2$, and $\sigma = e^{\frac{i\pi}{m}}$ for $m = 4l$. For $M > 0$, denote by P_M the projection in $L_2(\mathbf{R})$ onto the Paley–Wiener space $PW_M(\mathbf{R})$. Define the operator semigroup $P_{n,M}^t \varphi(x) = \mathbf{E}\,[\mathcal{R}\,\varphi](\zeta_n^x(t))$, where $\varphi \in PW_M(\mathbf{R})$ and $\mathcal{R}$ is a regularization operator. The generator of this semigroup is the operator $P_M \mathcal{L}_n P_M$, where

$$\mathcal{L}_n \psi(x) = n \int_{-\infty}^{+\infty} \Big(\psi\big(x + \tfrac{\sigma c(x) y}{n^{1/m}}\big) - \psi(x) - \sum_{j=1}^{m-1} \psi^{(j)}(x) \tfrac{\sigma^j c^j(x) y^j}{j!\, n^{j/m}} \Big)\, d\mathcal{P}(y).$$

Theorem. *Let $c(x) \in C_b^{m(k-1)}(\mathbf{R})$, $0 < c_1 \leqslant c(x) \leqslant c_2 < \infty$, $M = M(n) = n^{\frac{2}{m(m+2)}}$ and $\varphi \in W_2^{mk}(\mathbf{R})$, $k \geqslant 2$. Then as $n \to \infty$*

$$\big\| P_{n,M}^t P_M \varphi(\cdot) - u(t,\cdot) \big\|_{L_2(\mathbf{R})} \to 0.$$

This work was supported by the Ministry of Science and Higher Education of the Russian Federation (agreement 075-15-2025-344 dated 29/04/2025 for Saint Petersburg Leonhard Euler International Mathematical Institute at PDMI RAS).