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On random assemblies with restrictions on the component sizes.

Let U_n be a set of assemblies having weight n that are decomposed into indecomposable components of weights $k \le n$. Suppose that for every $k \in N$ there are m_k indecomposable components. Let A be a set having a density $\varrho > 0$ in the set of natural numbers. With $U_n(A)$ we denote a subset of assemblies of U_n, having the component volumes belonging to the set A.

Theorem 1 *Suppose that $m_k = (k-1)!\exp(\lambda k)\theta_k$ for some real λ, and $\theta_k \to \theta \in (0,\infty)$ as $k \to \infty$. Also assume that $|k : k \le n, k \in A, m-k \in A|/n \to \varrho^2$ for some constant $C \in [1,\infty)$ uniformly by $m \in [n, Cn]$. Then the next asymptotic relation holds:*

$$|U_n(A)| \sim \frac{\sqrt{2\pi}}{\Gamma(\varrho\theta)} e^{-\varrho\theta\gamma} n^n e^{n(\lambda-1)} n^{\varrho\theta-1/2} e^{c_n(A)} (L(n))^\theta,$$

as $n \to \infty$, where γ is the Euler's constant and

$$c_n(A) = \sum_{k\in A,\ k\le n} \frac{\theta_k - \theta}{k},\ L(n) = \exp\left(\sum_{k\in A,\ k\le n} \frac{1}{k} - \varrho \ln n\right).$$

In addition, for a certain $a > 0$ and $r = o(n)$ as $n \to \infty$ the next estimate is true:

$$d_{TV}((\zeta_{kn}, k \in A,\ k \le r), (\xi_k, k \in A,\ k \le r)) = O(1)\left(\frac{r}{n}\right)^a.$$

Here, ζ_{kn} is the number of components of size $k \in A$ of a random assembly uniformly distributed on $U_n(A)$ and $\xi_k, k = 1, 2\ldots$ is a sequence of independent Poisson random variables with parameters θ_k/k, and $d_{TV}(\cdot)$ is the total variation distance.

Since the set A has an asymptotic density, it is easy to deduce that the sequence $L(n)$ slowly varies at infinity as well as the sequence $\exp(c_n(A))$.