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ABSTRACTS OF TALKS GIVEN AT THE 2ND INTERNATIONAL CONFERENCE ON STOCHASTIC METHODS*

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The Second International Conference on Stochastic Methods (ICSM-2) was held May 25–31, 2017 in the village of Abrau-Durso (at the Moryak Hotel) on the Black Sea coast, the same place where the first conference (ICSM-1) took place in 2016. As in 2016, the organizers of this conference were the Steklov Mathematical Institute of RAS (Department of Theory of Probability and Mathematical Statistics), Moscow State University (Department of Probability Theory), and the Don State Technical University (Department of Higher Mathematics), the main university of Rostov-on-Don. The chairman of this conference was A. N. Shiryaev, academician of the Russian Academy of Sciences, who also headed the Organizing Committee and the Program Committee.

The conference committees were as follows. **Organizing Committee:** I. V. Pavlov (Deputy Chairman), E. V. Burnaev, M. V. Zhiltukhin, V. V. Shamraev, and S. Ya. Shatskikh; **Program Committee:** A. A. Gushchin (Deputy Chairman), Yu. E. Gliklikh, and D. B. Rokhlin. Organizational issues were solved at the conference by the **Local Organizing Committee** consisting of I. V. Pavlov (Chairman), V. V. Shamraeva, S. I. Uglich, and N. P. Krasiy.

In addition to scientists from Russia, scientists from the UK, Sweden, Bulgaria, France, the USA, and Uzbekistan took part in the conference. Thirteen lectures and 41 talks were given. The themes of the lectures were as follows: *A. N. Shiryaev* (*jointly with E. A. Feinberg*), On forward and backward Kolmogorov equations of general jump Markov processes; *A. B. Piunovskiy*, On the strategies in controlled jump Markov processes; *A. A. Gushchin*, The joint law of terminal values of a nonnegative submartingale and its compensator; *Yu. E. Gliklikh*, Stochastic equations and inclusions with mean derivatives and their applications; *D. B. Rokhlin*, Central limit theorem under uncertainty and the problem of prediction with expert strategies; *F. S. Nasyrov*, Representation of solutions of wave equations as mathematical expectations; *V. G. Zadorozhnii*, On moment functions of a solution of differential equations which are multiplicatively perturbed by random noise; *A. A. Lykov and V. A. Malyshev*, How statistical is nonequilibrium statistical physics?; *I. V. Pavlov*, Interpolating martingale measures and Haar extensions of financial markets; *S. Ya. Shatskikh* (*jointly with L. E. Melkumova*), Maximum-likelihood method in de Finetti's theorem; *O. E. Kudryavtsev*, Numerical methods for liquidity estimation in models admitting jumps; *Ya. I. Belopol'skaya*, Probabilistic representations of the solution to Cauchy problem for parabolic system with cross-diffusion; *N. V. Smorodina*, Representation of solutions to initial-boundary value problems by mean values of functionals of processes reflecting from the boundary.

The joint and sectional sessions were chaired by A. A. Gushchin, S. Ya. Shatskikh, F. S. Nasyrov, E. L. Presman, V. G. Zadorozhnii, S. M. Sitnik, I. V. Tsvetkova, V. V. Shamraeva, A. V. Nikitina, Yu. E. Gliklikh, O. E. Kudryavstev, V. V. Rodochenko, and L. E. Melkumova.

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ICSM-2 was superior to ICSM-1 in terms of the number of speakers, the scientific level of the talks (among the participants of ICSM-2 were one academician of the Russian Academy of Sciences, 22 doctors of science, 20 candidates of science, and seven postgraduates), and in terms of organizational aspects of arrival and departure of the participants. The abstracts of the talks underwent peer review and are published below.

The Organizing Committee arranged an excursion to Dinomorskoe village, where at the "Raduga" sports and fitness center of the Don State Technical University a number of lectures and talks were given.

Financial support from the Don State Technical University (B. Ch. Meskhi, rector) and the Russian Foundation for Basic Research (grant 17-01-20079-g) contributed immeasurably to the successful implementation of the conference.

The Third International Conference on Stochastic Methods will be held in June of 2018 at the "Raduga" sports and fitness center (Divnomorskoe).

A. N. Shiryaev, I. V. Pavlov, T. B. Tolozova, V. V. Shamraeva

V. I. Arkin, A. D. Slastnikov (Moscow, Russia)**. Threshold strategies for optimal stopping problems for Itô diffusion processes.** [1]

Let $X_t,\ t\geqslant 0,\ X_0=x$ be a one-dimensional regular diffusion Itô process with values in the interval I and with boundary points l, r, which may or may not belong to the interval. Consider the optimal stopping problem $\mathbf{E}^x g(X_\tau)e^{-\rho\tau}\chi_{\{\tau<\infty\}}\to\max$, where the maximum is taken over some class of Markov moments τ (not necessarily a.s. finite).

If one knows the structure of the solution (for example, the first exit time from the process beyond some level), then one can search for the specific form of the optimal stopping time in a more narrow class of Markov moments (which is much simpler) and then prove its optimality among all Markov moments. The general problem here is as follows: what are conditions for the stopping time, which is optimal in some class of moments, to be optimal also among all Markov moments?

In the present paper, this problem is searched for in the classes of stopping times defined by one or two thresholds: $\tau_p=\inf\{t\geqslant 0\colon X_t\geqslant p\}$ and $\tau_{(a,b)}=\inf\{t\geqslant 0\colon X_t\notin(a,b)\}$, respectively.

Let τ_{p^*} be the optimal stopping time in the class $\mathscr{M}_1=\{\tau_p,\, l<p<r\}$ for all x, $l<x<r$. Then a necessary and sufficient condition for τ_{p^*} to be an optimal stopping time over all Markov moments (for all x, $l<x<r$) is that the benefit function g is ρ-excessive for the process $X_{t\wedge\tau_{(p^*,r)}}$, $t\geqslant 0$, with the initial state x, $p^*<x<r$.

A similar result also holds for the time $\tau_{(a^*,b^*)}$, which is optimal in the class of Markov moments $\mathscr{M}_2=\{\tau_{(a,b)},\, l<a<b<r\}$.

We also put forward optimality conditions for a stopping time in the class $\mathscr{M}_1$ and give conditions for a function to be ρ-excessive.

A. S. Asylgareev (Ufa, Russia)**. Pathwise comparison theorems for stochastic differential equations and their applications.**

Consider two stochastic differential equations with the Stratonovich integral with respect to the Wiener process:

$$d\xi_t^{(k)}=\sigma_k(t,\xi_t^{(k)})*dW_t+b_k(t,\xi_t^{(k)})\,dt,\quad \xi_t^{(k)}|_{t=t_0}=\xi_0^{(k)},\qquad k=1,2. \tag{1}$$

[1]Supported by the Russian Science Foundation (grant 15-06-03723).

The aim of the present study, which continues [1], is to prove comparison theorems for the stochastic differential equations (1). Our approach is based on the fact that the solutions to equations (1) can be written as $\xi_t^{(k)} = \varphi_k(t, W_t + C_k(t))$, where $\varphi_k(t,u)$ are deterministic functions and $C_k(t)$ are solutions of an ordinary differential equation with a random right-hand side (see [2]). Our main result is as follows.

THEOREM 1. *We assume that the following conditions are satisfied for all* $t \geqslant 0$: (a) $\varphi_2\big(t, \int_{\xi_0^{(1)}}^{u} (\sigma_1(t,\psi))^{-1}\, d\psi\big) \geqslant u$ *for all* $u \in \mathbf{R}$; (b) $\sigma_2(t,u) > 0$ *for all* $u \in \mathbf{R}$; (c) $C_2(t) \geqslant C_1(t)$ *almost surely. Then* $\xi_t^{(2)} \geqslant \xi_t^{(1)}$ *for all* $t \geqslant 0$ *with probability* 1.

The comparison theorems proved in our study are used for investigating pathwise stability of equations of the form (1).

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I. V. Atlasov (Moscow, Russia)**. Operation of two parallel devices such that at least one of them should work.**

This paper extends one problem from Gnedenko's book [1]. Consider the operation of a system composed of two interchangeable devices. These devices operate in sequence, break, or undergo repair. The system halts if the repair time of one device exceeds the operation time of the other, that is, when there is such a time interval when one device is being repaired and the other is already broken. In the book [1], the change time of one (broken) device by the other (repaired) was considered immaterial; a characteristic function of the system operation time was considered, and ways to increase the mean operation time of the system were put forward. This study was continued in [2] and [3].

In the present study, we consider a system consisting of two elements operating in parallel. The system halts if both devices are under repair. The change time of one (broken) device with the other (repaired) is considered important. We build the characteristic function of the continuous operation time of the system. We find the average operation time of the system and consider ways to increase it. Next, we consider the characteristic function in the case when the operation times and repair times of each of the devices are distributed exponentially and consider recommendations of how, by varying parameters of these distributions, to increase the mean operation time of the system. For this case, we also consider the structure of the distribution function of the operation time of the system.

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Ya. I. Belopol'skaya (St. Petersburg, Russia)**. Probabilistic representations of the solution to Cauchy problem for a parabolic system with cross-diffusion.** [2]

We propose an approach (see [1], [2], [3]) that enables one to obtain probabilistic representations of generalized solutions to the Cauchy problem for systems of nonlinear parabolic equations appearing in various problems in physics, biology, chemistry, and other areas:

$$\partial_t u_q = \operatorname{div}\left(\sum_{m=1}^{d_1} F^{qm}(x,u)\nabla u^m\right) + \sum_{m=1}^{d_1} a^{qm}(x,u,\nabla u)\nabla u_m + \sum_{m=1}^{d_1} c_{mq}u_m,$$
$$u_q(0)(x) = u_{q0}(x), \qquad q = 1,\dots,d_1, \quad x \in \mathbf{R}^d.$$

Our approach is demonstrated for an example of the Cauchy problem for the simplest system

$$\partial_t u_m = \Delta(u_m[u_1+u_2]) + c_m(u)u_m, \quad u_m(0,x) = u_{m0}(x) > 0, \qquad m = 1,2, \tag{1}$$

of quasilinear parabolic equations with cross-diffusion, which is a particular case of the Shigesada, Kawasaki, Teramoto model [4] that describes the evolution of densities of concentration of competing populations.

Let $M_u(x) = \sqrt{u_1(t,x)+u_2(t,x)}$, $c_m(u) = a_m - b_m u_1 - c_m u_2$ and $\widetilde{w}(\theta) = w(t-\theta) - w(t)$. Consider the system of stochastic differential equations

$$d\widehat{\xi}(t) = [M_u\partial_x M_u](\widehat{\xi}(\theta))\,d\theta - M_u(\widehat{\xi}(\theta))\,d\widetilde{w}(\theta), \qquad \widehat{\xi}(0) = x, \tag{2}$$
$$d\eta_m(\theta) = \widetilde{c}_u^{\,m}(\xi(\theta))\eta_m(\theta)\,d\theta + C_m^u(\xi(\theta))\eta_m(\theta)\,dw(\theta), \qquad \eta_m(0) = 1, \tag{3}$$

where $C_u^m(\xi(\theta)) = -\nabla M_u(\xi(\theta))$, $\widetilde{c}_u^{\,m}(\xi(\theta)) = c_u^m(\xi(\theta)) - \langle \nabla M_u(\xi(\theta)), \nabla M_u(\xi(\theta))\rangle$.

We set $\phi_{0,\theta}(y) = \xi_{0,y}(\theta)$, $\psi_{0,\theta}^m(x) = \widehat{\xi}_{0,x}(\theta) = \xi_{0,y}(t-\theta)$, $\widehat{\eta}_m(\theta) = \eta_m(t-\theta)$ and note that $\psi_{0,\theta}^m \circ \phi_{0,\theta}^m(y) = y$, $\eta_m(t) = U_m(0,t)$, $\widehat{\eta}_m(t) = \widehat{U}_m(0,t)$, $m = 1,2$, and $\widehat{U}_m(0,t)U_m(0,t) = 1$.

Theorem 1. *Let u_1, u_2 be bounded strictly positive differentiable functions satisfying* (1) *in the generalized sense. Then there exist random processes $\widehat{\xi}_m(t)$, $\eta_m(t)$, $m = 1,2$, satisfying* (2), (3) *such that the functions u_1, u_2 admit a probabilistic representation of the form*

$$u_m(t) = \mathbf{E}\big[\widehat{\eta}_m(t)u_{0k} \circ \psi_{0,t}\big], \qquad m = 1,2. \tag{4}$$

Theorem 2. *There exists a closed system of stochastic relations that includes* (2)–(4) *and is associated with system* (1).

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[2]Supported by the Russian Science Foundation (grant 15-01-01453).

A. N. Beskopyl'nyi, N. I. Beskopyl'naya (Rostov-on-Don, Russia). **Stochastic models of mechanical properties of materials.**

The stochastic nature of the microstructure of metals is responsible for inhomogeneity in the development of plastic strain and fracture both for elastic [1] and brittle materials [2], [3]. To find the form of distribution $F(x) = \mathbf{P}(\Theta \leqslant x)$ of the random variable Θ (the ultimate strength of metal), we consider an approach based on special transformation of random variables [4]. The stress–strain curve for uniaxial tension is adequately approximated by the power-like function $\sigma = \sigma_0 + A_0\varepsilon^m$ on the interval $\sigma \in [\sigma_0; \sigma_B]$. Consider the new variable $u = u(\sigma)$, which is responsible for accumulation of plastic strain: $u = \int_{\sigma_0}^{\sigma_B} \varepsilon'(z)\,dz = B_0(\sigma_B - \sigma_0)^{\gamma+1}$. Now the distribution law for the ultimate strength reads as $G(x) = 1 - \exp[-B_0(x - \sigma_0)^{\gamma+1}]$. This is a three-parameter Weibull law with shift parameter, which plays the leading role in assessing the strength of metal and subsequent strength analysis. We consider also various options of approximation of the strain–stress dependence, and corresponding distribution laws are obtained. Experimental verification of the results obtained show that the above approach provides the best approximation in terms of several statistical criteria.

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V. A. Bovkun (Ekaterinburg, Russia). **On models leading to an infinite-dimensional stochastic Cauchy problems.** [3]

Consider the heat propagation problem in a one-dimensional rod of length l with due account of random thermal actions on the lateral surface and with isolated end-points. Let $u(x,t)$ be the rod temperature in section $x \in [0;l]$ at time $t \geqslant 0$, and let $u(x,0) = f(x)$ be the initial temperature distribution in the rod. When subject to random thermal actions, the rod gains heat γ or $-\gamma$ per unit length during unit time interval with probability λ.

Taking into account the description of the physical model, the variation of heat quantity in the section can be represented as a sum of two components: the deterministic and the stochastic. In [1] it was shown that the stochastic component can be described using an $L^2[0;l]$-valued cylindric Wiener process $\{W(t),\, t \geqslant 0\}$ (see, for example, [2]). As a corollary, the following result holds.

Theorem 1. *A stochastic Cauchy problem for the process of heat propagation in the rod is described as follows*:

$$c\rho S\big(u(t,x) - f(x)\big) = \alpha S \int_0^t u_{xx}(s,x)\,ds + \gamma\sqrt{2\lambda}W(t), \quad t \in [0;T], \quad x \in [0;l];$$

[3] This research was carried out with the financial support of the Programme of the President of the Russian Federation for the Support of Leading Scientific Schools (grant NSh-9356.2016.1) and of the Russian Academic Excellence Project (agreement 02.A03.21.0006 of August 27, 2013, between the Ministry of Education and Science of the Russian Federation and Ural Federal University).

here c is the specific heat capacity of the rod, ρ is the rod density, α is the heat-conduction coefficient, and S is the area of the section at x.

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M. N. Bogacheva, L. B. Zelentsov (Rostov-on-Don, Russia)**. State prognosis of investment construction projects based on stochastic modeling.**

In the process of implementation of an investment construction project (ICP) z, the behavior of the system is described by the sequence of interrelated identically distributed time series

$$\widetilde{Q}_z(t_i) = \widetilde{Q}_z^1(t_i) \to \widetilde{Q}_z^2(t_i) \to \cdots \to \widetilde{Q}_z^n(t_i), \qquad \widetilde{Q}_z(t_i) = f(t_i)\{T_z, S_z, H_z\},$$

where $\widetilde{Q}_z(t_i)$ is the state vector of the ICP z at the discrete planning horizon t_i; $f(t_i)\{T_z, S_z, H_z\}$ are the state characteristics of the ICP z at the discrete planning horizon t_i; T_z, S_z, and H_z are, respectively, the duration, net cost, and reliability of the project.

As a local criterion of optimality of the management efficiency of a construction project, we take the aggregate time loss level over a certain planning period. To enhance the forecast precision, we propose to use the method of adaptive regression modeling [1] and the pseudogradient method for renewal of coefficients of ICP models, which together enable one to obtain models with high degree of approximation and forecast to ensure timely adoption of management decisions.

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E. V. Burnaev (Skolkovo Institute of Science and Technology; Institute for Information Transmission Problems of the Russian Academy of Sciences (Kharkevich Institute), Moscow)**, G. K. Golubev** (Aix-Marseille Université, Institute for Information Transmission Problems of the Russian Academy of Sciences (Kharkevich Institute), Moscow)**. On one problem of multichannel signal detection.** [4]

We consider a statistical problem of detecting a signal with unknown energy in a multichannel system, observed in a Gaussian noise. We assume that the signal can appear in the kth channel with a known small prior probability π_k. Using noisy observations from all channels, we would like to detect whether the signal is presented in one of the channels or we observe pure noise. We describe and compare statistical properties of the maximum posterior probability test and optimal Bayes test. In particular, for these tests we obtain limiting distributions of test statistics and define sets of their undetectable signals.

It seems that one of the first mathematical works about Bayesian signal detection for multichannel systems was [1], where a statistical model composed of n Rayleigh

[4]Supported by the Russian Science Foundation (grant 14-50-00150).

channels was studied. The problem of Bayesian signal detection with known entropy in Gaussian channels was considered in [2]. In this paper it was assumed that a signal can appear in one of n channels with equal prior probabilities.

In the present paper, we investigate the situation where prior probabilities of signals observed in different channels are different and the energy of a signal is unknown and is a nuisance parameter. Since the statistical problem of signal detection in a multichannel system is high-dimensional, its solution, as opposed to low-dimensional problems, depends significantly on the available prior information about detectable signals, and so the results provided in our paper differ significantly from those of the papers listed above.

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V. A. Vasiliev, T. V. Dogadova (Tomsk, Russia). **Adaptive optimal prediction of multivariate diffusion processes.** [5]

Consider a prediction problem of the multivariate diffusion-type process

$$dX(t) = \Lambda X(t)\,dt + dW(t)$$

with unknown dynamics matrix Λ. The forecast $\widehat{X}(t)$ for $X(t)$ is built from observations $(X(s))_{s\leqslant t-u}$, $u > 0$, on the basis of truncated estimates of the matrix Λ. The truncated estimation method, which was proposed in [1] for discrete-time systems, is suitable for evaluation of relation-type functionals from dependent samples of a fixed volume with given accuracy in the sense of the metric L_{2m}, $m \geqslant 1$.

We prove the optimality of the adaptive prediction procedure with risk function of the form

$$R_T = \frac{A}{T}\mathbf{E}\mathrm{e}_T^2 + T,$$

which is structurally similar to that considered in [2] for multivariate discrete-time autoregression. Here, the parameter A represents the cost of aggregate prediction error and

$$\mathrm{e}_T^2 = \frac{1}{T}\int_u^T \|\mathrm{e}(s)\|^2\,ds, \qquad \mathrm{e}(s) = \widehat{X}(s) - X(s).$$

Such a choice of risk function enables one to optimize the volume of observations depending on the requirements for the prediction quality by applying the quality criterion $R_T \to \min_T$.

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G. A. Vlaskov, A. M. Mozhaev (Rostov-on-Don, Russia)**. Modeling of stochastically convecting ionosphere.**

Traditional models of distribution of electron concentration N_e in the F-region of polar ionosphere are based on two determining factors: ionization q and the large-scale electrical field of magnetospheric convection generating the transfer $\vec{v}$ of ionospheric plasma [1]. The continuity equation $\partial N_e/\partial t + \vec{v}\,\nabla N_e = q - \beta N_e$ is solved, where β is the recombination coefficient, and q is the ion formation function. These models are deterministic. However, as measurements show, the electrical field undergoes substantial fluctuations, especially in the auroral zone. This means that the functions involved in the equation are random (depending on the coordinates). Hence N_e is a random function of coordinate and time (that is, a random field). Using the Lagrange approach and taking into account that the ionospheric plasma is "frozen into" the geomagnetic field, we assume that the random component of the motion of tubes of force is Brownian. Consideration of simplified problems subject to analytic solution shows that the presence of stochastic fluctuations in the convection should result in blurring of the mean values N_e [2]. The existence of solutions for equations of such type was proved in [3, section 14.4]. At present this can hardly be solved analytically, and so the numerical methods (and in particular, the Monte Carlo method) turn out to be the most efficient. We found numerically the distribution of electronic density in some characteristic zones of the upper polar ionosphere and obtained histograms, charts of expectation, and variance for N_e.

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T. A. Volosatova, A. G. Danekyants (Rostov-on-Don, Russia)**. Modeling of quasilinear complex systems: The case of three probabilistic priorities with unit sum.** [6]

This report continues [1], which was concerned with the model of an economic system with three priorities in a case when the objective function reproduces competing demands. We consider an optimization problem with the objective function

$$F = \mathbf{E}[F_1^{\alpha_1} F_2^{\alpha_2} F_3^{\alpha_3}], \qquad F_i(x) = \left(\sum_{k=1}^{n} a_k^i x_k + b_i\right) I\left\{\sum_{k=1}^{n} a_k^i x_i + b_i > 0\right\},$$

where $I\{A\}$ is the indicator function of a set A, and α_i are the random variables (priorities), $\mathbf{P}(\alpha_i > 0) > 0$, $\mathbf{P}(\alpha_i < 1) > 0$, $i = 1, 2, 3$, and $\alpha_1 + \alpha_2 + \alpha_3 = 1$. We study only models in which there exist points of local and global maxima of the function F. We let S denote the set of stationary points of the function $F(x)$. If we assume that $S \neq \varnothing$, then the system of vectors $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$ is linearly dependent. Suppose that in this system there is a pair of linearly independent vectors $\vec{a}_1$ and $\vec{a}_2$. A necessary condition for a global maximum is the existence of numbers $c_1 < 0$, $c_2 < 0$ such that $\vec{a}_3 = c_1\vec{a}_1 + c_2\vec{a}_2$. In this report, we consider an extremal problem for

[6] Supported by the Russian Foundation for Basic Research (grant 16-01-00184-a).

the function $F(s,t) = \mathbf{E}[(s+b_1)^{\alpha_1}(t+b_2)^{\alpha_2}(c_1 s + c_2 t + b_3)^{\alpha_3}]$, provided this relation is satisfied.

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Yu. E. Gliklikh (Voronezh, Russia)**. Stochastic equations and inclusions with mean derivatives and their applications.** [7]

In this report we survey results on equations and inclusions with mean derivatives obtained by the Voronezh school after the publication of the book [1].

The concept of the mean derivative was introduced by E. Nelson in the 1960s for the purpose of his so-called stochastic mechanics (a variant of quantum mechanics; see [2], [3], [4]). Later it was shown that equations with mean derivatives are also natural in various branches of mathematical physics, economics, etc.

In [5] (see also [1]) on the basis of minor modification of Nelson's ideas, in addition to Nelson's right-, left-derivatives, as well as symmetric and antisymmetric derivatives, we introduced the mean derivative (which we called quadratic), which in principle made it possible to find a process from its mean derivative, that is, jointly from one of the classical Nelson's derivatives and the quadratic derivative (by default, in the Nelson's approach the quadratic derivative always agrees with the unit operator multiplied by a constant number, and hence he has never introduced it). In particular, many equations with mean derivatives appearing in mathematical physics, economics, etc. were proven to be solvable (see examples in [1]).

Differential inclusions with mean derivatives appear naturally in applications in the same way as ordinary differential inclusions originate from ordinary differential equations. For example, in equations with mean derivatives with control and feedback at each point (t,x) of the extended phase space, one should consider all values of the right-hand side for all possible control values. So, the right-hand side can be looked upon as a set-valued mapping, the equation becoming an inclusion. We study inclusion obtained in this way and consider an optimal control problem.

Of special importance are equations and inclusions with current velocities (symmetric mean derivatives), because these derivatives are natural physical analogues of the standard velocity in physics. Such equations present the greatest challenge.

The main emphasis in this talk is given to the following questions: solvability of equations and inclusions with mean derivatives (in particular, with current velocities); existence of optimal solutions (in particular, for equations with control of geometrical Brownian motion type and with current velocities); stochastic Leont'ev-type equations describing certain radio and electrical units with noises; second-order equations appearing in mathematical physics, etc. A brief introduction to the theory of mean derivatives is also given.

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A. A. Gushchin (Steklov Mathematical Institute of Russian Academy of Sciences, Moscow, Russia)**. The joint law of terminal values of a nonnegative submartingale and its compensator** [1]**.** [8]

We characterize a set $\mathbb{W}$ of possible joint laws of terminal values of a nonnegative submartingale X of class (D), starting at 0, and a predictable increasing process (compensator) from its Doob–Meyer decomposition. The set of possible values remains the same under certain additional constraints on X, for example, under the condition that X is an increasing process or a squared martingale. Special attention is paid to extremal (in a certain sense) elements of the set $\mathbb{W}$ and to the corresponding processes.

Namely, let μ be a probability measure on $\mathbf{R}_+$ with finite mean. We let $Q(u)$, $u \in (0,1)$, denote the lower quantile function for μ, that is,

$$Q(u) := \inf\{x\colon \mu([0,x]) \geqslant u\},$$

and define

$$Q^*(u) := \int_0^u \frac{Q(t)}{1-t}\, dt.$$

Then $Q^*(u)$, $u \in (0,1)$, is also a lower quantile function of a probability measure on $\mathbf{R}_+$ with the same mean as that for μ. We denote this measure by μ^*.

Let $X = (X_t)_{t\geqslant 0}$, $X_0 = 0$, be a nonnegative submartingale of class (D) on a stochastic basis $(\Omega, \mathscr{F}, (\mathscr{F}_t)_{t\geqslant 0}, \mathbf{P})$ with compensator $A = (A_t)_{t\geqslant 0}$. We assert that

(1) if $\operatorname{Law}(X_\infty) = \mu$, then

$$\mathbf{E} f(A_\infty) \leqslant \int_0^1 f(Q^*(u))\, du \quad \text{for any convex function } f; \tag{1}$$

(2) if a measure μ is given, then there exists a stochastic basis and a nonnegative submartingale X on it with $\operatorname{Law}(X_\infty) = \mu$ satisfying the above assumptions and such that $\operatorname{Law}(A_\infty) = \mu^*$; that is, we have equality in (1) for any convex f;

(3) if X satisfies the above assumptions, $\operatorname{Law}(X_\infty) = \mu$ and $\operatorname{Law}(A_\infty) = \mu^*$, then

$$\operatorname{Law}(X_\infty, A_\infty) = \operatorname{Law}(Q(U), Q^*(U)),$$

where U is a random variable uniformly distributed on $(0,1)$.

In addition, we give a complete characterization of all such submartingales. In particular, a necessary condition is that X lie in the class (Σ) [2].

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D. S. Donchev (Sofia, Bulgaria)**. Probability densities of a Wiener process exit through one-sided boundaries.**

In earlier studies (see [1]) we succeeded in characterizing the probability densities of a Wiener process exit through any smooth boundaries in terms of a solution of a parabolic partial second-order equation. It turns out that by using appropriate substitutions and the Laplace transform, this equation can be reduced to a first-order equation, which is explicitly solvable only in three cases: parabolic boundaries, as well as boundaries involving the square root and rational functions. As an example, we consider a boundary that has not been studied so far.

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A. D. Zhivaikina, A. A. Peresetskii (Moscow, Russia)**. Credit ratings of Russian banks and revocation of bank licenses in 2012–2016.**

We consider 11 credit ratings of Russian banks assigned either by international or Russian rating agencies in 2012–2016. Econometric models of these ratings are constructed from open data: financial indicators of banks and macroeconomic indices. Based on historical data on revocation of bank licenses, we construct econometric models for the probability license revocation separately for each specific formulation of revocation. These models enable us to analyze to what degree the Central Bank employs ratings and to what degree rating agencies consider the possibility of license revocation in a short-term outlook.

M. V. Zhitlukhin (Steklov Mathematical Institute of Russian Academy of Sciences, Moscow, Russia)**. On new inequalities for the maximum of a fractal Brownian motion** [1]**.** [9]

For a fractal Brownian motion B^H with exponent $H \in (0, 1/2)$, we put forward new upper and lower estimates for the difference of the expectation of the maximum of B^H_t on the interval $t \in [0, 1]$ and the maximum of $B^H_{t_i}$ over a finite point set $t_i = i/n$, $0 \leqslant i \leqslant n$. These results are used to improve the available estimates for the expectation of the maximum of B^H and to derive an upper estimate for the Pikands constant. It is shown how the new estimates can be used to estimate the expectations of a fractal Brownian motion.

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Zadorozhnii V. G. (Voronezh, Russia)**. On moment functions of a solution of differential equations multiplicatively perturbed by random noise.**

We consider differential equations $dx/dt = \varepsilon(t,\omega)Ax + f(t,\omega)$, $x(0) = x_0$, which are multiplicatively perturbed by a random noise. Here t is time, X is a finite-dimensional space with the inner product $\langle \cdot\,, \cdot \rangle$, $A\colon X \to X$ is a linear operator, $x\colon \mathbf{R} \to X$ is the sought-after function, ε is a random process, f is a vector random process, and $x_0 \in X$ is a random vector. The characteristic functional of the processes ε, f is assumed to be known (see [1]): $\psi(u,v) = \mathbf{E}\exp\bigl\{i\int_T[\varepsilon(s,\omega)u(s) +$

[9]Supported by the Russian Science Foundation (grant 14-21-00162).

$\langle f(t,\omega), v(s)\rangle]\,ds\}$, where $T \subset \mathbf{R}$ is the interval where the problem is studied, and $\mathbf{E}$ is the expectation with respect to the distribution function of the processes ε, f.

The search problem of moment function of solutions is reduced to nonrandom differential equations involving traditional and variational derivatives.

We consider the auxiliary mapping

$$y(t,u,v) = \mathbf{E}\biggl(x(t)\exp\biggl\{i\int_T \bigl[\varepsilon(s,\omega)u(s) + \langle f(s,\omega), v(s)\rangle\bigr]\,ds\biggr\}\biggr).$$

Note that $y(t,0,0) = \mathbf{E}x(t)$.

Under the additional assumption that x_0 is independent of the random processes ε, f, we have the problem for $y(t,u,v)$

$$\frac{\partial y(t,u,v)}{\partial t} = -iA\frac{\delta_p y(t,u,v)}{\delta u(t)} - i\frac{\delta_p \psi(u,v)}{\delta v(t)}$$

with the initial condition

$$y(t_0,u,v) = \mathbf{E}(x_0)\psi(u,v).$$

Here $\delta_p y(t,u,v)/\delta u(t)$ is the partial variational derivative [1]. The solution to the resulting nonrandom Cauchy problem reads as

$$y(t,u,v) = \psi\bigl(uE - iA\chi(t_0,t),v\bigr)\mathbf{E}x_0 - i\int_{t_0}^t \frac{\delta_p\psi(uE - iA\chi(s,t),v)}{\delta v(s)}\,ds.$$

Here $\chi(s,t,\tau)$ is the function of τ, which agrees with $\operatorname{sign}(\tau - s)$ when τ lies in the interval $(\min\{s,t\},\max\{s,t\})$, and vanishes for other values of τ; E is the identity operator.

In particular, for $u=0$, $v=0$ we get

$$\mathbf{E}x(t) = \psi\bigl(-iA\chi(0,t),0\bigr)\mathbf{E}x_0 - i\int_0^t \frac{\delta\psi(-iA\chi(s,t),0)}{\delta v(s)}\,ds.$$

If random processes ε, f are independent and are given by the characteristic functionals $\varphi_\varepsilon(u)$, $\varphi_f(v)$, then the formula for $y(t,u,v)$ is more transparent:

$$y(t,u,v) = \varphi_\varepsilon\bigl(uE - iA\chi(t_0,t)\bigr)\mathbf{E}x_0 + \int_{t_0}^t \varphi_\varepsilon(uE - iA\chi(s,t))\mathbf{E}f(s)\,ds.$$

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A. A. Zamyatin (Moscow, Russia)**, Iasnogorodski R.** (Paris, France)**. Regular quantum and random walks.**

We consider the Hilbert space $l_2(\mathbf{N}^2)$ over the field of complex numbers, where $\mathbf{N} = \{1,2,\dots\}$ take an orthonormal basis $e_{m,n}$, $m,n \geqslant 1$, and define the bounded linear self-adjoint operator (Hamiltonian) $H = H_0 + V\colon l_2(\mathbf{N}^2) \to l_2(\mathbf{N}^2)$, which acts on the basis vectors $e_{m,n}$ as follows:

$$H_0 e_{m,n} = -\lambda(e_{m+1,n} + e_{m-1,n} + e_{m,n+1} + e_{m,n-1}), \qquad n, m \geqslant 2,$$
$$H_0 e_{1,n} = -\lambda(e_{2,n} + e_{1,n+1} + e_{1,n-1}), \qquad n \geqslant 2,$$
$$H_0 e_{m,1} = -\lambda(e_{m+1,1} + e_{m-1,1} + e_{m,2}), \qquad m \geqslant 2,$$
$$H_0 e_{1,1} = -\lambda(e_{2,1} + e_{1,2}),$$
$$V e_{m,n} = \mu_1 \delta(m-1) e_{m,n} + \mu_1 \delta(n-1) e_{m,n} + \mu \delta(m-1)\delta(n-1) e_{m,n},$$

where $\lambda, \mu, \mu_1 \in \mathbf{R}$.

A quantum walk is defined as a dynamics of the form $f(t) = \exp(-iHt)f(0)$, where $f(t) = \sum_{m,n=1}^{\infty} f_{m,n}(t) e_{m,n}$. The Hamiltonian H defines the system consisting of three particles on the one-dimensional lattice $\mathbf{Z}_+$ such that one of them is fixed at the point 0, while the other two particles are free: they interact with the fixed particle when at least one of these two free particles hits the point 1. A quantum walk is regular in the sense that the free particle never reaches the point 0, where the fixed particle is located. The wave function $f(t)$ defines the state of free particles: the particles are at the points m, n at time t with probability $p_{m,n}(t) = |f_{m,n}(t)|^2$. If an eigenvalue of the operator H is taken as an initial state, then these probabilities are independent of time.

The talk is concerned with the discrete spectrum of H. We find an explicit form of its eigenvectors. The problem was solved by the method proposed earlier for finding a stationary distribution of an ergodic random walk in a quadrant of the plane.

N. P. Krasiy (Rostov-on-Don, Russia)**. Optimization of quasilinear models with several independent priorities.** [10]

We continue a study of the model proposed in [1]. Our purpose is to single out conditions for existence of local and global maxima of the objective function $F(x) = \prod_{j=1}^{3} \mathbf{E} F_j^{\alpha_j}$, where $\alpha_j \in [0;1]$ are random variables (priorities) and

$$F_j(x) = \biggl(\sum_{i=1}^{n} a_{ij} x_i + b_j\biggr) I\biggl\{\sum_{i=1}^{n} a_{ij} x_i + b_j > 0\biggr\},$$
$$a_{ij} \in \mathbf{R},\ b_j \in \mathbf{R}, \quad x = (x_1, \dots, x_n) \in \mathbf{R}^n.$$

The values of priorities of competing structures are set by a referee who is interested in the most efficient performance of the system on a large scale. It is shown that the linear dependence of the vectors $\overline{a}^{(j)} = (a_{1j}, \dots, a_{nj})$, $j = 1, 2, 3$, provides a necessary condition for existence of a stationary point of the function $F(x)$. Two cases are possible: (1) the vectors lie on distinct lines, and (2) all vectors are collinear. In the first case, we show that if there exists a local extremum point (t_1^*, t_2^*) of the objective function $F(t_1, t_2) = \mathbf{E}(t_1 + b_1)^{\alpha_1} \mathbf{E}(t_2 + b_2)^{\alpha_2} \mathbf{E}(-c_1 t_1 - c_2 t_2 + b_3)^{\alpha_3}$, where $c_1 > 0$ and $c_2 > 0$ are some constants, then all intersection points of the hyperplanes $t_1^* = \sum_{i=1}^{n} a_{i1} x_i$ and $t_2^* = \sum_{i=1}^{n} a_{i2} x_i$ are points of local maximums of the function $F(x)$. In the model corresponding to the second setting, there is no finite maximum if the vectors $\overline{a}^{(j)}$, $j = 1, 2, 3$, have the same direction. If among these vectors there are some with opposite directions, then there exists a unique point t^* such that all points of the hyperplane $t^* = \sum_{i=1}^{n} a_{i3} x_i$ are points of global maximum of the function $F(x)$.

[10] Supported by the Russian Foundation for Basic Research (grant 16-01-00184-a).

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O. E. Kudryavtsev (Rostov-on-Don, Russia). **Numerical methods for liquidity estimation in models admitting jumps.** [11]

The liquidity risk is of key significance for financial markets. We define the liquidity risk as the inability to close a position at an appropriate price. Especially, its role grows at sharp movements of financial asset prices, which are observed on most trading platforms. Practitioners use Lévy processes for modeling jumps in prices at financial markets. The liquidity risk has two components related to the value of immediate closing position and the value of waiting for a possibility to complete a transaction. Analyzing the second component of illiquidity, Longstaff [1] considered an investor with a single security in a portfolio who is restricted in ability to sell his/her asset.

Given the absence of trading restrictions, the investor could sell the asset by the maximal price reached during the given period. The expected difference between the maximal price for the period and the price at the end of the period gives an upper bound for the value of illiquidity and can be interpreted as a European floating strike lookback put with an initial asset price $S = e^x$:

$$V(T,x) = \mathbf{E}\big[e^{-rT}(e^{\overline{X}_T} - e^{X_T}) \,\big|\, X_0 = x\big],$$

where X_t is a Levy process starting at x, $\overline{X}_T$ is a supremum process, T is the time to option expiration, and r is the riskless rate.

A repurchase agreement (*repo*) can serve as an example of such trading restrictions such that a participant in a financial market actually takes a loan using the assets as collateral by means of selling them with an obligation to repurchase the securities at the end of the loan term at a pre-agreed price. Since the ownership of the assets is transferred from the seller to the buyer during the period of the agreement, the securities become illiquid for the seller. Repo operations, by their character, are short-term and have a term of less than 1 year. At the Moscow Exchange, agreements indicated can last for 1, 7, or 14 days.

Assessing a liquidity risk before a repo transaction, an investor should dynamically monitor the risk, taking into account variation of the price of the asset used as collateral. If at time moment T_1 the maximal price of the asset reaches a value $H = e^h$ under current price $S = e^x$, then the value illiquidity can be estimated via the price of a seasoned European floating strike lookback put:

$$V(T_1, T_2; x, h) = \mathbf{E}_{T_1}\big[e^{-r(T_2-T_1)}(e^{\overline{X}_{T_2}} - e^{X_{T_2}}) \,\big|\, X_{T_1} = x,\, \overline{X}_{T_1} = h\big].$$

Setting $T = T_2 - T_1$, we reduce solving the problem to computing the function

$$\begin{aligned} V(T,x) &= \mathbf{E}^x\big[e^{-rT}(e^{\max\{\overline{X}_T, h\}} - e^{X_T}) \,\big|\, X_0 = x\big] \\ &= \mathbf{E}^x\big[e^{-rT}(e^{\overline{X}_T} - e^{X_T}) \,\big|\, X_0 = x\big] + \mathbf{E}^x\big[e^{-rT}(H - e^{\overline{X}_T})\mathbf{1}_{\{\overline{X}_T < h\}} \,\big|\, X_0 = x\big]. \end{aligned}$$

The expectations obtained can be efficiently computed by using the Wiener–Hopf method and approximate factorization formulas obtained in [2].

[11] Supported by the Russian Fund for Basic Research (grant 18-01-00910).

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D. I. Lisovskii (Steklov Mathematical Institute of Russian Academy of Sciences, Moscow, Russia)**. On a distribution of the first hits of a random boundary by a Brownian motion.** [12]

We are concerned with a model of Brownian motion with a *change point*; namely, consider the process $X_t = x + \mu(t-\theta)^+ + \sigma B_t$, where $\mu, x \in \mathbf{R}$, $\sigma > 0$, $(B_t)_{t\geqslant 0}$ is a standard Wiener process, and $\theta \sim \operatorname{Exp}(\lambda)$ is an exponentially distributed random variable, which is interpreted as an unobservable moment of occurrence of a *change point*. It is assumed that a given Brownian motion $(B_t)_{t\geqslant 0}$ and a random variable θ are independent. For the process X_t, we define in the natural way the first hits of a given level:

$$\tau_a^\theta = \inf\{t \geqslant 0\colon X_t \geqslant a\}, \quad \sigma_b^\theta = \{t \geqslant 0\colon X_t \leqslant b\};$$

here $a > x$, $b < x$; with the help of some new notation, these instances can be rewritten in an equivalent way as

$$\tau_{\widetilde{a}}^\theta = \inf\{t \geqslant 0\colon B_t \geqslant \widetilde{a} - \widetilde{\mu}(t-\theta)^+\}, \quad \sigma_{\widetilde{b}}^\theta = \{t \geqslant 0\colon B_t \leqslant \widetilde{b} - \widetilde{\mu}(t-\theta)^+\};$$

here $\widetilde{a} > 0$, $\widetilde{b} < 0$, and $\widetilde{\mu} \in \mathbf{R}$. They can be interpreted as the first hits of a *random boundary* of a special form by the Brownian motion B_t. We are also concerned with the stopping time $\gamma_{a,b}^\theta = \tau_a^\theta \wedge \sigma_b^\theta$, which is the first exit time by the process X_t from the interval $[b, a]$. We note that the above random variables τ_a^θ, σ_b^θ, and $\gamma_{a,b}^\theta$ extend the well-known first hit times of given levels by a Brownian motion (see [1] and [2]). In this talk we put forward new Laplace transforms, and explicit expressions for densities, and expectations of the above stopping times.

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A. A. Lykov, V. A. Malyshev (Moscow, Russia)**. How statistical is non-equilibrium statistical physics?**

Consider a system of N particles on the line with interaction

$$U = \sum_{1\leqslant i<j\leqslant N} V(|x_j - x_i|),$$

where the potential of interaction between particles reads as

$$V(x) = \frac{\omega^2}{2}\begin{cases} \phi(x), & 0 < x \leqslant a - a_1,\\ (x-a)^2, & a - a_1 < x \leqslant a + a_1,\\ \text{const}, & x > a + a_1 \end{cases}$$

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for an arbitrary smooth function $\phi(x)$ and $a = 1/N$, $a_1 = r/N > 0$, $r < 1$, $\omega = \omega' N > 0$. The initial conditions are as follows:

$$x_{k+1}(0) - x_k(0) = \frac{1}{N} X\left(\frac{k}{N}\right) > 0, \qquad \dot{x}_{k+1}(0) - \dot{x}_k(0) = \frac{1}{N} V\left(\frac{k}{N}\right),$$
$$x_1(0) = 0, \quad \dot{x}_1(0) = v, \quad X(0) = X(1) = 1, \quad V(0) = V(1) = 0$$

for some $v \in \mathbf{R}$ and $X, V \in C^4([0,1])$.

THEOREM. *Let $t \geqslant 0$ and $k = 1, \ldots, N-1$. Then*

$$\frac{1-\gamma}{N} \leqslant x_{k+1}(t) - x_k(t) \leqslant \frac{1+\gamma}{N},$$

where

$$\gamma = 2\alpha + \frac{\beta N}{\omega} = 2\alpha + \frac{\beta}{\omega'}, \qquad \alpha = \int_0^1 |X''(y)|\, dy, \qquad \beta = \int_0^1 |V''(y)|\, dy,$$

which means that there is no collision between particles.

Making $N \to \infty$, we get a regular continuum system of particles [1] with paths $y(t,x)$, velocities $u(t,y)$, and initial conditions $y(0,x) = x$, $u(0,y) = v(x)$.

From the no-collision condition, we derive a system of three equations [2]: the continuity equation, the Euler equation, and the state equation (a relation between density ρ and pressure p):

$$\rho_t + (u\rho)_y = 0, \quad u_t + uu_y = -\frac{1}{\rho} p_y, \quad p = -\frac{(\omega')^2}{\rho} + (\omega')^2.$$

In addition, we survey three "stochastic" methods of derivation based on the stochastic dynamics, kinetic Boltzmann equations, or the chain of BBGKY equations for correction functions. We discuss heuristic and unproved moments in these methods of derivation.

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G. V. Martynov (Institute for Information Transmission Problems of the Russian Academy of Sciences (Kharkevich Institute), Moscow, Russia)**. Goodness test for the Gaussianity conjecture of a random process** [1]**.** [13]

Let $(X, \mathscr{B}, \mu)$ be a probability space, where X is a real separable Hilbert space, and let $\mathscr{B}$ be a Borel σ-algebra on X. We test the conjecture that the measure μ is equivalent to a Gaussian measure with zero expectation and a trace class covariation operator K. As alternatives, we consider all possible measures, including Gaussian ones. Let X_i, $i = 1, \ldots, n$, be observations of X, and let $(X_{i1}, X_{i2}, \ldots)$, $i = 1, \ldots, n$, be their decompositions in the basis corresponding to the operator K. Let α_i, $i = 1, 2, \ldots$, be the characteristic numbers of the operator K. Consider $T_{ij} = G(\alpha_j X_{ij})$,

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$i = 1, \dots, n$, $j = 1, 2, \dots$, where G is the function of the standard one-dimensional normal distribution. We replace each observation X_i by the vector $T_i = (T_{i1}, T_{i2}, \dots)$ lying in $[0,1]^\infty$ and having with H_0 a uniform distribution on $[0,1]^\infty$. Let the distribution function on $[0,1]^\infty$ be defined by $F(t) = \prod_{i=1}^\infty t_i^{r_i}$, where $t = (t_1, t_2, \dots)$ and $r_1, r_2, \dots$ is a sequence decreasing from 1 to 0. In a similar way, we define the empiric distribution function $F_n(t) = n^{-1} \sum_{i=1}^n \prod_{j=1}^\infty I\{T_{ij} < t_j^{r_i}\}$. A conjecture can be tested with the help of the Cramér–Mises statistics $\omega_n^2 = n \int_{[0,1]^\infty} (F_n(t) - F(t))^2 \, dt$, which can be evaluated by the Monte Carlo method. Using Prokhorov's theorem, it is proved that the empirical process $\sqrt{n}\,(F_n(t) - F(t))$, $t \in [0,1]^\infty$, is weakly convergent in $L_2([0,1]^\infty)$ under some conditions on $\{r_i\}$ to a Gaussian process with the covariation function

$$C(s,t) = \prod_{i=1}^\infty \min(s_i^{r_i}, t_i^{r_i}) - \prod_{i=1}^\infty s_i^{r_i} t_i^{r_i}, \qquad s, t \in [0,1]^\infty.$$

We propose a method for evaluating precise eigenvalues and functions for $C(s,t)$. We calculate a table of limit distribution of the statistics ω_n^2 for the infinite-dimensional Hilbert setting and with $r_i = i^{-3(1-i^{-1/2})}$. So, $\mathbf{P}\{\omega_n^2 \leqslant 0.9\} = 0.16450$. The limit distribution is independent of K, but it depends on the choice of $\{r_i\}$ and on the dimension of the Hilbert space. The above method can be applied, for example, for testing that a random process is Wiener or for testing the uniformity of distribution in a unit cube of large dimension.

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L. E. Melkumova (Samara, Russia). **Comparison regression analysis results for the Newton–Kantorovich, Ridge, and LASSO methods.** [14]

The problem of evaluating the coefficients B of a linear regression from the initial data $\mathbf{W}$, $\mathbf{V}$ using the Newton–Kantorovich, Ridge, and LASSO methods (see [1]) can be formulated as follows:

$$\begin{aligned} &1) \quad \|\mathbf{V} - \mathbf{W}B\|^2 \longmapsto \min, \\ &2) \quad \|\mathbf{V} - \mathbf{W}B\|^2 + \lambda \|B\|_2 \longmapsto \min, \\ &3) \quad \|\mathbf{V} - \mathbf{W}B\|^2 + \lambda \|B\|_1 \longmapsto \min. \end{aligned}$$

Wine Quality (UCI Machine Learning Repository) data were analyzed for red wine (1599 observations) and white wine (4898 observations); see [2]. Eleven wine product properties are taken as predictors, and qualitative assessments (from 0 to 10) are taken as responses.

For the analysis of multicollinearity, we evaluate the variance increase factors of predictors VIF_j, $j = 1, \dots, 11$. The maximal values of VIF are as follows: 7.125 for red wine, and 28.233 for white wine. A cross-validation method is used to find optimal values of regularization parameters λ for Ridge and LASSO, which are used for construction of regression models. The data for each kind of wine are randomly

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partitioned into the learning and control samples. For all kinds of regression, models are constructed from learning samples, and errors (the residual sum of squares) are calculated from control samples. Using the LASSO method allows us to reduce the number of predictors for both white and red wines. It is found that the RSS quantity for control samples for MNK is larger than for the Ridge and LASSO methods.

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A. A. Muromskaya (Moscow, Russia)**. Optimal reinsurance for a company making combination insurance contracts.**

Consider operations of an insurance company which makes insurance contracts covering directly $k \geqslant 2$ risks. We assume that each of such risks can be given by the company for a reinsurance of arbitrary type and at each time $t \geqslant 0$ the insurance company has the ability to choose parameters d_t^i of reinsurance of the ith risk in accordance with the capital value $X_t^{\overline{d}}$. The process $\overline{d}_t = (d_t^1, \dots, d_t^k)$, where $d_t^i = d^i(X_t^{\overline{d}})$ are measurable functions of the company capital, define the reinsurance strategy. The principal task of the company is to search for an optimal strategy of reinsurance that maximizes the nonruin probability. In accordance with this task, we obtain the Hamilton–Jacobi–Bellman equation and prove the existence and uniqueness of the solution to this equation. We also prove the existence of an optimal reinsurance strategy such that the nonruin probability of the company is maximal. Our results extend and continue the studies on the search of optimal reinsurance strategies in models with a fixed type of reinsurance contract and one risk within one insurance contract (see [1] and [2]). Numerical examples are given to illustrate the above theoretical results in the case of independent risks and in the case of dependent risks, and joint distribution for these risks is constructed using a copula.

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F. S. Nasyrov (Ufa, Russia)**. Representation of solutions of wave equations as mathematical expectations.**

We show that solutions to both the Cauchy problem for oscillation of an unbounded string and the first, second, and third boundary-value problems for the oscillation equation of a bounded string can be represented as mathematical expectations. Additionally, as distinct from [1], which employed fairly involved machinery for this purpose (in particular, a generalized random process which was a limit of some sequence of random walks was constructed), we show that the solutions can be represented as mathematical expectations of deterministic functions of a random variable with uniform distribution on the interval.

Some results can be extended to the case of wave equations of dimensions $n = 2, 3$.

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I. V. Pavlov (Rostov-on-Don, Russia)**. Interpolating martingale measures and Haar extensions of financial markets.** [15]

In this talk we give a survey of recent results on finding interpolating martingale measures, as well as new results obtained by the author and his research team. To avoid technicalities, we consider only one-step processes that assume at the final time only a countable number of values (possibly repeated). The initial values of these processes are always constant.

We let $Z = (Z_n, \mathscr{F}_n)_{n=0}^1$ denote a one-step process, where $\mathscr{F}_0 = \{\Omega, \varnothing\}$, $\mathscr{F}_1$ is generated by the partition of Ω into a countable number of atoms B_k^i ($k \in \mathbf{N} = \{1, 2, \dots\}$, $1 \leqslant i < m_k + 1$, $1 \leqslant m_k \leqslant \infty$), $Z_0 = a$, $Z_1(B_k^i) = b_k$ (b_k are distinct real numbers, $b_k \neq a$ for any $k \in \mathbf{N}$). We assume that $\inf_k b_k < a < \sup_k b_k$. The set of martingale measures P on $\{\Omega, \mathscr{F}_1\}$ such that $p_k^i := P(B_k^i) > 0$ and $b_l \neq \sum_J b_k p_k^i / \sum_J p_k^i$, for all $l (1 \leqslant l < \infty)$ and all subsets $J \subset \{(k,i), 1 \leqslant k < \infty, 1 \leqslant i < m_k + 1\}$ with finite complement J^c is denoted by $\mathscr{P}$; this set is called the set of special interpolating martingale measures.

We first assume that the σ-algebra $\mathscr{F}_1$ is finite. This is equivalent to the inequalities $r < \infty$ and $m_k < \infty$ for all $0 \leqslant k \leqslant r$. It is easily seen that a necessary condition that $\mathscr{P}$ be nonempty is that $m_1 = \cdots = m_r = 1$ and that a be distinct from any of the numbers $b_1, \dots, b_r$. Under these conditions $\mathscr{P} \neq \varnothing$ (see [1], [2]).

We now assume that the σ-algebra $\mathscr{F}_1$ is infinite, but $r < \infty$. It can be assumed without loss of generality that $b_1 < \cdots < b_r$. Clearly, if there exists a unique index k, $0 \leqslant k \leqslant r$, such that $m_k = \infty$, then $\mathscr{P} = \varnothing$. We assume that there exist at least two indexes k and k' such that $m_k = \infty$ and $m_{k'} = \infty$. Then (1) if $r = 2$ or $r = 3$, then $\mathscr{P} \neq \varnothing$ (see [3]); (2) if $r = 4$, $m_k = \infty$ ($k = 1, 2, 3, 4$), and $b_1 < a < b_2$ or $b_3 < a < b_4$, then $\mathscr{P} \neq \varnothing$ (see [4]); (3) if $r \geqslant 4$ and if $b_1, \dots, b_r$ are rational, then $\mathscr{P} \neq \varnothing$ (see [3]); (4) if $b_1 < a < b_2 < b_3 < b_4 < b_5 < \cdots$ and if $b_k - b_{k-1} \geqslant b_{k-1}$, for any $k \geqslant 2$, then $\mathscr{P} \neq \varnothing$.

In this talk, for $r = 4$, we give other sufficient conditions for the set $\mathscr{P}$ to be nonempty. The following theorem is also new.

Theorem. *If among the numbers $a, b_1, b_2, \dots$ only one number is irrational* (*and the other numbers are rational*), *then* $\mathscr{P} \neq \varnothing$.

It is worth noting that Shamraeva recently proved that under condition (4) there exist martingale measures satisfying much more stringent interpolating condition than the membership to the set $\mathscr{P}$.

In this talk, we give a detailed description of the extension scheme for completing incomplete arbitrage-free finance markets with the use of martingale measures from $\mathscr{P}$.

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A. Yu. Perevaryukha (St. Petersburg, Russia)**. Modeling the phenomenon of collapse of the exploited fish population with stochastic uncertainty.** [16]

We propose a dynamic model with a stochastic component for a specific scenario of collapse: a special case of the rapid degradation of the exploited population of large fish. This scenario has practical importance, because prior to the collapse of bioresources their state was estimated as favorable for fishing. [1]. The stochastic supplement in the model is necessary to describe the unexpected possibility of recovering a small population, as in the case of a whitefish *Coregonus clupeaformis* in Lake Ontario. We applied a discrete–continuous model of "stock⇔recruitment" type based on the differential equations of loss of fish populations with a trajectory in iterative form $x_{n+1} = \psi(x_n)$, where $\psi(x) = N(T)$ is a solution of the Cauchy problem at the interval of juvenile vulnerability $t \in [0, T]$ with initial conditions $N(0) = \lambda x_{n-1}$, depending on the state of the stock, and λ is the average fertility of fish. Earlier, the author of [2] described the effect of the threshold state of the number of fish U_1 in the form of a repeller singular point: $\lim_{n\to\infty} \psi^n(x_0) = U_0$, $U_0 < \varepsilon$, for all $x_0 < U_1$. The attractor for a state $x_0 > U_1$ of a population is a cycle without a cascade of bifurcations: $p = 2^I$, $I \to \infty$. We propose a hypothesis that successful reproduction of a small group of fish is probabilistic; then there is a range of values, rather than a single value, of the state of a small group $U_1 \in \Omega_x$, where the reproductive process of the fish is caused by random influences. Let the probability of an event of replenishment $\psi(x_0) > x_0$, $x_0 \in \Omega$, and the probability gradually decreases with further depletion $x_n \to 0 + \epsilon$. In the computing model there is a trigger functional: $\Theta(N(0)) = [1 + \exp(-\kappa N(0)^2)]$, $\lim_{N(0)\to\infty} \Theta(N(0)) = 1$, which is supplemented by a random variable γ with an exponential distribution law: $\widetilde{\Theta} = \Theta(N(0)) \times \gamma$. So we successfully solve the problem of local perturbation in the equation for the reduction of the number of the first stage of development of young fish:

$$\dot{N} = -N(t)\big(\alpha w(t)N(t) + \beta\widetilde{\Theta}N(t)\big), \qquad t \in [0,\tau], \quad \tau < T.$$

We take into account the speed of dimensional development of the individuals of a generation: $\dot{w} = v(N^{-2/3}(t))$, $w(0) = \widehat{w}$. For the senior development stage the delayed regulation is introduced in the form of the right-hand side of f_2:

$$\dot{N} = f_2(N(t-\varsigma), w(\tau)), \qquad t \in [\tau, T], \quad \varsigma < \tau.$$

The method of inclusion in the model $\widetilde{\Theta}$ allows us to describe the stochastic behavior of the trajectory in the narrow range of the state of the stock. $x_n \in U_1 \pm \Omega(\gamma)$. The new model in computational experiments demonstrates a slow recovery of the population while maintaining the number of reproductively isolated groups of large fish.

[16] Supported by the Russian Foundation for Basic Research (grant 17-07-00125), developed by SPIIRAS.

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E. A. Pechersky (Moscow, Russia)**. Markov dynamics of configurations of two types of particles** [1]**,** [2]**.** [17]

We are concerned with the Markov dynamics of a Jackson network with two types of queries forming queues at service nodes. Service nodes form d-dimensional torus $\mathbf{T}^d$. One type of query is called *standard*, and the other type of query is called *special*. Standard queries (irrespective of the special ones) form a conventional Jackson network. In the system, there is a finite number of special queries that are conserved in the course of dynamics. Their dynamics depends on the configuration of standard queries. So, the state space of the Markov process describing the system dynamics is the set of configurations $\mathscr{X} = \mathbf{N}^{\mathbf{T}^d} \times \mathbf{N}^{\mathbf{T}^d}$. A state is a pair $(\underline{n}, \underline{y}) \in \mathscr{X}$, where $\underline{n} = (n_i,\, i \in \mathbf{T}^d)$, $\underline{y} = (y_i,\, i \in \mathbf{T}^d)$ is the number of standard and special orders, respectively. Let L be the total number of special orders, when $L = \sum_{i\in\mathbf{T}^d} y_i$. The dynamics of standard orders depends on the intensities λ_i, $i \in \mathbf{T}^d$, of arrival of standard queries at each node i, the intensities μ_i of escaping the system from the node i, and the jump intensities β_{ij} of standard queries. The dynamics of special queries depends on the jump intensities τ_{ij}. The following theorem holds under some symmetry conditions and constraints on the dynamics of special queries, which depend on the configuration of standard queries.

Theorem. *There exist constants $\gamma_i > 0$, $i \in \mathbf{T}^d$, such that the standard distribution π of the Markov dynamic of requests on $\mathscr{X}$ is as follows*:

$$\pi(\underline{n}, \underline{y}) = \prod_i \left(\frac{\lambda_i}{\mu_i}\right)^{n_i} \gamma_i^{y_i}.$$

Under these conditions there is no detailed balance. However, there are relations, which we call a balance of three states.

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A. B. Piunovskiy (Liverpool, UK)**. On the strategies in controlled jump Markov processes.** [18]

In the theory of controlled jump processes, there are two main areas: semi-Markov decision processes (SMDP) and continuous-time Markov decision processes (CTMDP).

Suppose $\mathbf{X}$ and $\mathbf{A}$ are the standard Borel spaces of states and actions; $q_x(a)$ is the jump rate from the state $x \in \mathbf{X}$ under action $a \in \mathbf{A}$, and $c(x, a)$ is the corresponding cost rate. Let $x \in \mathbf{X}$ be the initial state.

[17] Joint work with G. Schütz and A. A. Yambartsev.
[18] The author is thankful to IMA (UK) for support.

In the framework of SMDP, a decision maker can choose action $a \in \mathbf{A}$; e.g., in the form of the feedback control $a = \varphi(x)$, this action remaining constant on the interval $(0, \Theta_1]$, where the state x is constant. The distribution of Θ_1 is exponential, $\mathbf{P}(\Theta_1 \leqslant t \mid x) = 1 - e^{-q_x(\varphi(x))t}$, and the cost rate is $c(x, \varphi(x))$.

In the more general case, the decision maker chooses a standard Borel space $\boldsymbol{\Xi}$, simulates the random element $\xi \in \boldsymbol{\Xi}$ according to one or another stochastic kernel $p(d\xi|x)$, and applies the time-dependent actions $A(t) = \varphi(x, \xi, t)$. If $\boldsymbol{\Xi} = \mathbf{A}$ and $\varphi(x, a, t) = a$, then this is just the randomized version of the feedback control described above. Such control strategies, as defined by $\{\boldsymbol{\Xi}, p, \varphi\}$, are *realizable* in the sense that there exists a random process $A(u, \widetilde{\omega})$ such that

$$\forall\, q_x(a) \quad \mathbf{P}(\Theta_1 \leqslant t \mid x) = \int_{\widetilde{\Omega}} \left[1 - \exp\left\{- \int_{(0,t]} q_x(A(u, \widetilde{\omega}))du\right\}\right] \widetilde{\mathbf{P}}(d\widetilde{\omega}),$$

and for any $c(x, a)$ the actual expected cost rate equals

$$\int_{\widetilde{\Omega}} c(x, A(t, \widetilde{\omega}))\, \widetilde{\mathbf{P}}(d\widetilde{\omega}).$$

Such control strategies are called ξ-strategies. Some versions thereof were studied in the 1960–1970s by Howard, Bather, Yushkevich, Feinberg, Federgruen, Varaiya, and others.

In the framework of CTMDP, the decision maker chooses the stochastic kernel $\pi(da|x, t)$, resulting in the expression

$$\mathbf{P}(\Theta_1 \leqslant t \mid x) = 1 - \exp\left\{- \int_{(0,t]} \int_{\mathbf{A}} q_x(a)\, \pi(da|x, u)\, du\right\},$$

and the actual expected cost rate is $\int_{\mathbf{A}} c(x, a)\, \pi(da|x, t)$.

Such control strategies are called π-strategies. They were studied, starting from the 1980s, by Kitaev, Feinberg, Guo, Prieto-Rumeau, Hernández-Lerma, and others. π-strategies are *not realizable*, unless the kernel π is degenerate.

The first issue here is that ξ-strategies and π-strategies have (almost) no overlap. Because of that, SMDP and CTMDP were developed in parallel. In the current talk, the general class of π-ξ-strategies is proposed, which makes possible the unified description of all the models of controlled jump Markov processes.

Further, one has to understand which classes of strategies are sufficient for solving optimization problems, and which classes are realizable. It appears that a convenient class of realizable and sufficient strategies is formed by the so-called Poisson-related strategies.

One can find details in [1], [2].

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E. L. Presman (Moscow, Russia). **Stock control model with price depending on a continuous-time finite-state Markov process.** [19]

There is a vendor which needs to consume some intermediate product (goods) at a constant rate. If the product price depends on the state of a continuous-time Markov chain (the model proposed by Sonin), then it is expedient to arrange a warehouse and purchase goods both continuously and discretely. The storage cost is proportional to quantity of goods at the warehouse. The problem is to manage the warehouse activity in order to minimize cost of purchase and storage (discounted or limited per time unit).

Sonin and Hill [1] considered expenses which are limiting per unit time and assumed that, for each state, there is a threshold such that if the quantity of goods at the warehouse exceeds this threshold, then no procurement is required; otherwise a one time buy should be arranged to reach this threshold and then performed procurements at unit rate in order that the stock is equal to the threshold value until the next jump of the Markov process. In [1], for the case of two states and some subcases of three states, optimal threshold values were obtained in the class of threshold strategies.

Following [2] and [3], we examine the optimality in the class of all coordinated (not predictable!) controls and first consider discounted costs, and then (passing to the limit), consider the limit costs per time unit. We prove the optimality of threshold strategies and put forward an algorithm of successive construction of optimal thresholds. This algorithm is based on the fact that instead of evaluating the functionals corresponding to algorithm threshold strategies, we rather study their derivatives, and instead of smooth joints, we invoke convexity arguments, which are shown to be equivalent to twice smooth joints.

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E. A. Pchelintsev (Tomsk, Russia). **Adaptive estimation for a nonparametric regression with conditionally Gaussian Lévy noises.** [20]

We consider a problem of adaptive robust estimation for an unknown regression function $S(\cdot)$ based on the observations of the process, which is described by the stochastic differential equation

$$dy_t = S(t)\,dt + d\xi_t, \qquad 0 \leqslant t \leqslant n,$$

where $(\xi_t)_{0\leqslant t\leqslant n}$ is an unobserved noise modeled by a conditionally Gaussian Lévy process. For estimating the function S, a model selection procedure was proposed in [1] on the basis of weighted least squares estimates, which provided an adaptive solution to the nonparametric estimation problem by means of a nonasymptotic sharp oracle inequality for the quadratic risk. Since a nonparametric estimation usually has low quality, the problem of its improvement is relevant. Improved estimation

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for regression models in continuous time with pulse noise was made possible by the results of [2], [3]. In the present study, we construct an adaptive model selection procedure for estimating the function S on the basis of weighted improved least squares estimates with special weight coefficients that ensure asymptotic efficiency of the proposed estimate. Using the proposed procedure allows us to improve the quality of nonasymptotic estimation in nonparametric regression models.

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V. V. Rodochenko, O. E. Kudryavtsev (Rostov-on-Don, Russia)**. Pricing barrier options in stochastic volatility models admitting jumps by using a fast Wiener–Hopf factorization method.** [21]

We develop a new method allowing fast and accurate pricing barrier options for a wide class of stochastic volatility models admitting jumps. As an example we consider a down-and-out barrier put in the Bates model [1]. Choosing a suitable substitution for eliminating a correlation between the price and variance processes (analogously to the approach in [2]) and applying Carr's randomization procedure, it is possible to reduce computation of the arbitrage-free price of the option to solving recurrently a family of one-dimensional problems corresponding to vertices of a binomial tree.

Approximating the process of variance CIR by Markov chain, for each vertex we obtain a pair of problems with fixed variance, which can be solved by using the Wiener–Hopf factorization method. Since closed form expressions for the factors are not available, we apply approximate formulas obtained in the paper [3], admitting an efficient implementation by using the fast Fourier transform. The results of numerical experiments demonstrate fast convergence and accuracy of the method obtained.

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D. B. Rokhlin (Southern Federal University, Rostov-on-Don, Russia)**. Central limit theorem under uncertainty and the problem of prediction with expert strategies.** [22]

In the first part of this talk, we consider the description of the limits

$$\mathscr{L} := \lim_{n\to\infty} \sup_{A_0^{n-1}\in\mathfrak{A}_0^{n-1}} f\left(n^{-1/2}\sum_{j=0}^{n-1} A_j\xi_{j+1}\right). \tag{1}$$

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Typically, $(\xi_j)_{j=0}^{\infty}$ is a sequence of independent identically distributed d-dimensional random variables with zero mean and identity covariance matrix, or a sequence of independent one-dimensional zero mean random variables satisfying the Lindeberg condition. In the first case, $\mathfrak{A}_0^{n-1}$ is the class of $\sigma(\xi_0,\dots,\xi_{n-1})$-adapted sequences of $d\times d$ matrices, belonging to a compact set Λ. In the second case, $\mathfrak{A}_0^{n-1}$ is the class of adapted sequences with values in intervals $[\underline{a}_j,\overline{a}_j]$ such that their bounds satisfy some stabilization conditions as $j\to\infty$. The function f is assumed to be continuous and bounded. The sequences A_j describe the uncertainty of the model. They also can be regarded as opponent strategies under the game interpretation of the problem.

In [1], [2] it was proved that the limit (1) can be expressed in terms of the viscosity solution of the G-heat equation: $\mathscr{L}=v(0,0)$, where

$$-v_t(t,x)-\frac{1}{2}\sup_{A\in\Lambda}\operatorname{Tr}\bigl(AA^T v_{xx}(t,x)\bigr)=0,\quad (t,x)\in[0,1)\times\mathbf{R}^d;$$
$$v(1,x)=f(x),\quad x\in\mathbf{R}^d.$$

A similar result was obtained by S. Peng in 2007 in the framework of his sublinear expectations theory. In this context, by the definition we have $u(0,0)=\widehat{\mathsf{E}}f(Y)$, where Y is a G-normally distributed random variable and $\widehat{\mathsf{E}}$ is the sublinear expectation functional.

In the second part of this talk we consider the problem of online prediction of individual sequences. The aim of the prediction is to make the cumulative error L_n after n rounds as close as possible to the cumulative error $\min_{1\leqslant i\leqslant N} L_n^i$ of the best expert from the given finite class. From the theory of online learning it is well known that typically the cumulative regret $R_n=L_n-\min_{1\leqslant i\leqslant N}L_n^i$ satisfies the inequality $R_n\leqslant C\sqrt{n}$. A quite precise upper bound of R_n in the case of randomized predictions was given in 2010 by the sequential Rademacher complexity of Rakhlin, Sridharan, and Tewari. In [3] it is mentioned that the asymptotic behavior of this quantity as $n\to\infty$ is determined by the limit of the form (1). In the language of sublinear expectations theory this limit coincides with the expected value of the largest order statistics of a multidimensional G-normal random variable.

Similar structures appear after taking the limits in the recurrence relations for the value function of a sequential prediction game. Passing to the limit as $n\to\infty$ one gets a nonlinear parabolic partial differential equation of Isaacs–Bellman type. Its smooth supersolutions of a special form correspond to the potential functions (known in online learning theory) and induce algorithms of weighting of expert opinions.

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A. I. Rytova (Moscow, Russia). **Asymptotics of numbers of particle in a branching random walk with heavy tails.** [23]

We consider a continuous time branching random walk on an integer lattice $\mathbf{Z}^d$, $d\geqslant 1$. Suppose that initially there is a single particle, which performs a random walk

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on sites of $\mathbf{Z}^d$ until reaching the special site $x_0 \in \mathbf{Z}^d$, where it can either die or give a random number of offspring, and then each one evolves independently according to the same rules. The underlying random walk is assumed to be symmetric, spatially homogeneous, and irreducible. Such branching random walks were studied by many authors (see, e.g., [1] and references therein). The main objects of interest are the local (i.e., determined at each site of $\mathbf{Z}^d$) and the total particles numbers. As a rule, such models are considered under the assumption that the variance of random walk jumps is finite; therefore, the new effects of infinite variance condition are expected. In [2] the transition intensities were subject to the relation

$$\lim_{\|z\|\to\infty} a(0,z)\|z\|^{d+\alpha} = H\left(\frac{z}{\|z\|}\right),$$

where $\alpha \in (0,2)$ and $H\colon \mathbf{S}^{d-1} \to \mathbf{R}$, $H(x) = H(-x)$, $x \in \mathbf{S}^{d-1}$, is a continuous positive function. As a result, all the properties of a random walk are saved, except the finiteness of the variance of jumps. Such a random walk may be transient even on low-dimensional lattices $d = 1, 2$ (see [2]). According to the scheme proposed in [1], the asymptotics of transition probabilities (see [3]), the equations for generating functions, and differential and integral equations for the moments of particle numbers are obtained for the case of infinite variance of jumps, and asymptotic behavior of their solutions is studied.

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S. M. Sitnik (Voronezh, Russia). **Turán-type inequalities and their applications in probability.** [24]

Turán-type inequalities, which secure the logarithmic convexity with respect to parameters or arguments of special functions, are known to be useful in various theoretical and applied problems. We mention the papers [1], [2], [3], [4], where such inequalities were obtained for various types of special functions: orthogonal polynomials, Bessel functions and their modifications, hypergeometric functions and their generalizations, Mittag-Leffler functions, etc.

This talk is concerned with applications of some results of [1], [2], [3], [4] to problems of stochastic mathematics, probability, mathematical statistics, and financial mathematics. These applications involve problems of estimation of parameters of joint Poisson distribution and maximum likelihood estimates for mixtures of Watson distributions, optimization of risk forecasts for one bank credit model, convergence of iteration algorithms in the blocking probabilities problem, and optimization of "tails" of Bernoulli distribution.

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V. V. Smorodina (St. Petersburg, Russia)**. Representation of solutions to initial-boundary value problems by mean values of functionals of processes reflecting from the boundary.** [25]

We consider the initial-boundary value problem for the equation $\partial u/\partial t = (\sigma^2/2)\Delta u$ in a bounded domain $D \subset \mathbf{R}^2$ with smooth boundary ∂D, the initial condition $u(0, x) = f(x)$, and the Neumann boundary condition $(\partial u/\partial n)|_{\partial D} = (\partial f/\partial n)|_{\partial D}$, where n denotes the outward unit normal vector to the boundary curve ∂D. For the solution to the initial-boundary value problem in the case when $(\partial f/\partial n)|_{\partial D} = 0$, we have the representation $u(t, x) = \mathbf{E}f(X_x(t))$, where $X_x(t)$ is a Wiener process $\sigma w(t)$ emitted from a point $x \in D$ and reflected from the boundary when it is reached. It should be noted that, in contrast to the process $X_x(t)$ which is stopped upon hitting the boundary, the construction of the process $\widetilde{X}_x(t)$ reflected from the boundary involves certain technical difficulties (because the paths of the Wiener process are nowhere differentiable) and is related to the so-called Skorokhod problem (see [1]). Solving the Skorokhod problem for a domain D means the construction, for each (nonrandom) continuous path emitted from an arbitrary point $x \in D$, of a version thereof which "reflects from the boundary." The Skorokhod problem is known to be solvable on a broad class of domains; for a detailed account of the results, see [2]. Two points are worth mentioning here. First, for a smooth curve, its "reflected" version (in the sense of Skorokhod) does not agree with its classical reflection (such that its normal component of the tangent vector of the curve changes its sign upon hitting the boundary of the domain). Second, the difficulties associated with solving the Skorokhod problem are exacerbated by the fact that it is by no means always convenient to deal with processes involving only continuous paths; processes with piecewise-constant paths are frequently more handy. This, in turn, calls for the need to solve again the Skorokhod problem.

We propose a new method of construction of a probabilistic representation of solutions to initial-boundary value problems based on the construction of a special extension on the initial function from the domain to the entire plane. We employ the Wiener process $\sigma w(t)$, but we make it "feel" the boundary of the domain by extending the initial function from the domain D onto the entire plane in a special (different) way.

For each fixed $x \in D$ we construct the representation F^x of the initial function f as a series of entire functions on $\mathbf{C}^2$ converging to the function f in any disk D_x centered at the point x and located completely in $\overline{D}$.

The probabilistic representation of the solution $u(t, x)$ to the initial-boundary value problem reads as $u(t, x) = \lim_{M\to\infty} \mathbf{E}F^x_M(x + \sigma w(t))$, where F^x_M denotes the Mth partial sum of the series defining the function F. For each fixed M the functions F_M in the last formula are entire analytic functions on $\mathbf{C}^2$; this enabled us to obtain a probabilistic representation of the solution for complex σ satisfying the condition $\operatorname{Re}\sigma^2 \geqslant 0$ (in particular, for the Schrödinger equation) also. It is also

[25] Supported by the Russian Science Foundation (grant 15-01-01453).

shown that in this probabilistic representation a Wiener process can be replaced by an appropriate random walk.

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A. I. Sukhinov (Rostov-on-Don, Russia)**, A. V. Nikitina** (Rostov-on-Don, Russia)**, V. V. Sidoryakina** (Taganrog, Russia)**, A. A. Semenyakina** (Taganrog, Russia)**. Justification and modeling of the turbulent exchange coefficients of reservoirs on the basis of stochastic method.** [26]

Stochastic methods are frequently useful for describing turbulent flows in reservoirs. Under this approach, various fluctuating variables are considered as random functions. At dissipative scales the turbulence is known to have an involved statistical structure due to strong intermittence. Field research of shallow reservoirs (the Sea of Azov and the Étang de Berre lagoon) produced data on velocity fluctuations of water flow at some points of reservoirs using WHS600 Sentinel Acoustic Doppler Current Profiler. Among various approximations of the vertical turbulent exchange coefficient, the best results were obtained for algebraic subgrid models where turbulence flows were defined as space- or (correlation) time-averaged products of fluctuations of the flow velocity components and the transported physical quantity.

At large scales of vertical grids, mechanisms of vertical turbulent exchange are suppressed during numerical modeling, which calls for sufficiently small scales of vertical resolution. The vertical turbulent exchange coefficient for the Sea of Azov and the Étang de Berre lagoon is evaluated on the basis of statistical data for the velocity field of the water flow and by using Monin and Smagorinskii's subgrid models [1].

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T. S. Turova (Lund, Sweden)**. Coulomb systems in physics and biology.**

We consider the system of particles on a finite interval with pairwise nearest neighbors Coulomb interaction and external force. This model was introduced by Malyshev [1] to study the flow of charged particles in a network-like media on a rigorous mathematical level. In the future, we hope to use this model to study (electric) neuronal activity. In [1] it was proved that at the zero temperature case (ground states) there are phase transitions in the structure of the configurations of charges under different strengths of external force.

The local structure of Gibbs configurations at positive temperature but without an external force was analyzed in [2], where interactions of more general form were also considered. It was proved that, for any positive temperature, the configurations remain strongly localized at the minima of the energy.

Continuing the studies of [2], we consider Gibbs configurations in the presence of an external electric field. In [3], we derive the asymptotics for the mean and the variances of the distances between the neighboring charges. Due to its simplicity, this

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model features a "precise resolution": we are able to distinguish five different phases. This, combined with the results of [1], suggests continuity of the distribution at zero temperature. We prove that for the weak force the charges remain densely and almost equally spaced over the entire interval (as observed for the zero force case in [2]), at the critical value of external force they occupy only a finite part of the interval, and when the force is above the critical value all the charges collapse at one end of the interval. Note that here the phase transitions (along the strength of the external force) are observed at any positive temperature.

The methods we use in [3] develop the probabilistic approach of [2]; however, it is applied now under the inhomogeneous setting.

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S. I. Uglich (Rostov-on-Don, Russia). **On optimization of quasilinear systems with several random priorities.** [27]

We are concerned with optimization of quasilinear models describing the interaction (in a unique system) of various competitive structures with due consideration of random prioritization of outsiders by a referee who makes decisions based on expert recommendations.

In the present talk, we consider a model assuming that a system involves three structures and that priorities α_i $(i = 1, 2, 3)$ are arbitrarily uniformly distributed random variables with ranges in the interval $[0, 1]$. Let $F = \mathbf{E}(F_1^{\alpha_1} F_2^{\alpha_2} F_3^{\alpha_3})$ be the objective function of the referee, where F_i are linear functions of n variables considered on the domain where they are positive. Two cases are considered: when random variables are independent and when $\alpha_1 + \alpha_2 + \alpha_3 = 1$ (in the case of priorities; see [1] and [2]). Using special computer procedures we are able to show that in each of these cases the function F assumes its maximum $F_{\max}$ at the intersection of hyperplanes $\sum_{i=1}^n a_{ij} x_i = t_j^\star$, $j = 1, 2$, where $t_1^\star$, $t_2^\star$ are roots of a system of two transcendental equations depending on the negative parameters c_1 and c_2 (obviously, the system is case-specific). The parameters c_j are coefficients of the representation of the function $\sum_{i=1}^n a_{i3} x_i$ involved in F_3 in terms of the functions $\sum_{i=1}^n a_{ij} x_i$, which are components of F_i, $i = 1, 2$. The function $F_{\max}(c_1, c_2)$ is minimized numerically. It is shown that this function assumes its minimum value. This value can be interpreted as an optimal sum which can be allocated by a referee for normal operation of the system.

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I. V. Tsvetkova (Rostov-on-Don, Russia)**. Numerical evaluation of the canonical hedge for incomplete markets with countable state set.** [28]

We consider a statistical $(1, Z)$-market on $(\Omega, \mathbf{F})$, where $\mathbf{F} = (\mathscr{F}_k)_{k=0}^1$, $\mathscr{F}_0 = \{\Omega, \varnothing\}$, $\mathscr{F}_1 = \sigma(B_1, B_2, \dots)$. Let $Z = (Z_k, \mathscr{F}_k)_{k=0}^1$ be an $\mathbf{F}$-adapted random process (discounted stock value). If the market under consideration is incomplete, then a transition to a complete one is affected by construction of an interpolation market. To this end, we consider a special Haar interpolating filtration $\mathbf{H} = (\mathscr{H}_n)_{n=0}^\infty$, where $\mathscr{H}_0 = \mathscr{F}_0$, $\mathscr{H}_1 = \sigma\{B_{n_1}\}$, $\mathscr{H}_2 = \sigma\{B_{n_1}, B_{n_2}\}$, $\dots$, $\mathscr{H}_\infty = \sigma\{B_{n_1}, B_{n_2}, \dots\} = \mathscr{F}_1$, and $\{n_i\}_{i=1}^\infty$ is an arbitrary fixed permutation on $\mathbb{N}$. Let a martingale measure P have the weak property of universal Haar uniqueness [1]. Consider a martingale Haar interpolation $Y = (Y_n, \mathscr{H}_n, P)_{n=0}^\infty$ of the random process Z: $Y_n = \mathbf{E}^P[Z_1 \,|\, \mathscr{H}_n]$. The market interpolating the original one is complete, and hence for any financial commitment there is a replicating self-financing brief bag $\pi = (\beta_n, \gamma_n)_{n=0}^\infty$. Quantile hedging is used for practical calculations of the components of π. To this end, from any arbitrarily small ε (computational accuracy) one evaluates the computational horizon N: $\sum_{i=1}^N \mathbf{P}(B_{n_i}) > 1 - \varepsilon$.

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A. E. Chistyakov (Rostov-on-Don, Russia)**, E. F. Timofeeva** (Stavropol, Russia)**. Processing of full-scale experiments for evaluating parameters of sea waves based on stochastic approaches.** [29]

The problem of theoretical investigation of wave processes in stochastic inhomogeneous media calls for the study of various phenomena accompanying propagation of waves and in finding stochastic characteristics of wave fields. In the study of wave hydrodynamic processes occurring in shallow reservoirs, we perform the following full-scale experiment: a measurement mechanism is submerged at various depths and wave oscillations are performed within a minute using a video camera. The data obtained in the experiment require processing. Primary processing of video materials with the aim of finding the (time-dependent) water surface elevation function is performed using the pattern recognition machinery. The algorithm developed and its numerical implementation allow one to sufficiently and accurately evaluate the level elevation function. The following parameters of wave processes are obtained using statistical and spectral methods: spectrum, medium frequency. We also verify the conjectures that the spectrum of the level elevation function has normal and log-normal distributions. It is shown that wave processes can be described by three quantities: the expectation (the wave period), the variance, and the maximal spectrum value. These quantities are evaluated during processing of data obtained from the full-scale experiment. The values obtained are used as initial data for the previously developed mathematical models of wave hydrodynamic processes [1].

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[29]Supported by the Russian Science Foundation (grant 17-11-01286).

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V. V. Shamraeva (Rostov-on-Don, Russia). **Existence of special interpolating martingale measures admitting transformation of incomplete financial markets into complete ones.** [30]

Let $(\Omega, \mathbf{F})$ be a filtered space $\mathbf{F} = (\mathscr{F}_0, \mathscr{F}_1)$, where $\mathscr{F}_0 = \{\Omega, \varnothing\}$ and $\mathscr{F}_1 = \sigma\{B_i,\, i \in \mathbf{N} = \{1, 2, \dots\}\colon \bigcup_{i=1}^{\infty} B_i = \Omega, B_i \cap B_j = \varnothing\ (i \neq j)\}$. Consider the random process $Z = (Z_n, \mathscr{F}_n)_{n=0}^{1}$, where $Z_0 := a$, $Z_1|_{B_i} := b_i$. By $\mathscr{P}(Z, \mathbf{F})$ we denote the set of nondegenerate martingale measures of the process Z.

Definition. *We say that* $P \in \mathrm{NBC}$ *(*P *satisfies the* barycenter noncoincidence condition*) if the series* $\sum_{i=1}^{\infty} b_i p_i$ *is absolutely convergent and* $\sum_I b_i p_i / \sum_I p_i \neq \sum_J b_j p_j / \sum_J p_j$ *for any* $I, J \subset \mathbf{N}$ *such that* $I \cap J = \varnothing$, $|I| \leqslant |J|$.

In [1], [2] it was proved that in the case of a finite σ-algebra $\mathscr{F}_1$ the set NBC is nonempty if $\mathscr{P}(Z, \mathbf{F}) \neq \varnothing$ and $a \neq b_i$ for any i. So far, the question about nonemptiness of NBC in the case of countably generated $\mathscr{F}_1$ remained open and no example of a martingale measure satisfying NBC was available. The following theorem gives a partial answer to this problem.

Theorem. *Let* $b_1 < a < b_2 < b_3 < b_4 < \cdots$, $b_i - b_{i-1} \geqslant b_{i-1}$ *for any* $i \geqslant 2$. *Then* $\mathrm{NBC} \neq \varnothing$.

If the inequality in the definition of NBC is satisfied only for I and J such that $|I| = 1$ and if $\mathbf{N} \setminus J$ is finite, then we arrive at the definition of the *weakened barycenter noncoincidence condition* and of the corresponding set WNBC. The papers [2], [3] give sufficient conditions for WNBC to be nonempty.

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S. Ya. Shatskikh, L. E. Melkumova (Samara, Russia). **Maximum-likelihood method in de Finetti's theorem.** [31]

On the space $\{\mathbf{R}^\infty, \mathscr{B}(\mathbf{R}^\infty)\}$, we consider "coordinate" random variables $e_k(\mathbf{z}) = z_k$, $\mathbf{z} = (z_1, \dots, z_n, \dots) \in \mathbf{R}^\infty$, $k = 1, \dots, \infty$. For a conditional distribution function $F(x|y)$ satisfying the "classical regularity conditions" [1], we introduce a family of probability measures $\mathscr{P} = \{\mu_g\}$ with symmetric finite-dimensional distributions

$$(*)\qquad \mu_g\{\mathbf{z}\colon e_k(\mathbf{z}) \leqslant x_k,\, k = \overline{1, n}\} = \int_0^\infty \prod_{k=1}^{n} F(x_k|y) g(y)\, dy, \qquad n = 1, \dots, \infty,$$

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where $g(y)$ are probability densities that are positive on $[0,+\infty)$. According to de Finetti's theorem, $(*)$ follows from the conditional independence and identical distribution of the infinite sequence of exchangeable random variables with respect to some random variable $s^*(\mathbf{z})$ (or the σ-algebra generated by it). There are various approaches towards construction of the random variable $s^*(\mathbf{z})$ (see [2], [3], [4]).

On the basis of representation $(*)$, we consider the construction of $s^*(\mathbf{z})$ using the maximum-likelihood method.

Using the likelihood function

$$L(y; e_1(\mathbf{z}), \dots, e_n(\mathbf{z})) = \prod_{k=1}^{n} F(e_k(\mathbf{z})|y), \qquad y \in [0,\infty),$$

we introduce the sequence of maximum likelihood estimators (MLE)

$$s_n(\mathbf{z}) \in \operatorname*{argmin}_{y\in[0,\infty)} L\bigl(y; e_1(\mathbf{z}), \dots, e_n(\mathbf{z})\bigr).$$

THEOREM. *Under the "classical regularity conditions" there exist an MLE sequence* $\{s_n(\mathbf{z})\}$ *and a statistics* $s^*(\mathbf{z})$ *with the following properties*:

(1) $\mu_g\{\mathbf{z}\colon s_n(\mathbf{z}) \to s^*(\mathbf{z})\} = 1$ *for any* $\mu_g \in \mathscr{P}$;

(2) $\mu_g\{\mathbf{z}\colon s^*(\mathbf{z}) \leqslant x\} = \int_0^x g(y)\,dy$ *for any* $\mu_g \in \mathscr{P}$;

(3) *for complete families of densities* $\{g(y)\}$ (*positive*, *gamma-*), *the statistics* $s^*(\mathbf{z})$ *is complete for families of probabilities* μ_g;

(4) *the following relations hold*:

$$\mu_g\{\mathbf{z}\colon e_k(\mathbf{z}) \leqslant x_k,\ k{=}1,\dots,n \mid s^*(\mathbf{z}) = y\} = \prod_{k=1}^{n} \mu_g\{\mathbf{z}\colon e_k(\mathbf{z}) \leqslant x_k \mid s^*(\mathbf{z}) = y\}$$
$$= \prod_{k=1}^{n} F(x_k|y), \qquad y \in [0,\infty),$$

and there is no dependency on the density $g(y)$.

Examples of MLE are considered for densities $f(x|y)$ from one-parameter exponential families [5].

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A. N. Shiryaev (Steklov Mathematical Institute of Russian Academy of Sciences, Moscow, Russia)**, E. A. Feinberg** (Stony Brook University, USA)**. On forward and backward Kolmogorov equations of general jump Markov processes.** [32]

To define a general jump Markov process, we first introduce a Q-function $q(x,t,B)$ satisfying the following two conditions:

(a) for $x \in X$, $t \in [T_0, T_1)$ the function $q(x,t,\cdot\,)$ is a signed measure on the space of states $(X, \mathscr{B}(X))$ (a standard Borel space) and has the properties $q(x,t,X) \leqslant 0$ and $0 \leqslant q(x,t,B \setminus \{x\}) < \infty$ for each $B \in \mathscr{B}(X)$;

(b) for each $B \in \mathscr{B}(X)$ function $q(x,t,B)$ is measurable in (x,t).

We also assume that the function q is locally L_1-bounded; that is,

(c) $\int_{T_0}^{s} q(x,t)\,dt < \infty$ for each $s \in (T_0, T_1)$, where $q(x,t) = -q(x,t,\{x\})$.

Let Ω be the space of all sequences $\omega = (t_0, x_0, t_1, x_1, \dots)$, $t_0 = T_0$, and let $(t_n, x_n)_{n\geqslant 0}$ be a multivariate point process on $(\Omega, \mathscr{F})$. With each Q-function q satisfying conditions (c) one may associate the random (predictable) measure ν: for $t \in [T_0, T_1)$ and $B \in \mathscr{B}(X)$

$$\nu(\omega, [T_0, t), B) = \int_{T_0}^{t} \sum_{n\geqslant 0} I(t_n < s \leqslant t_{n+1}) q(x_n, s, B \setminus \{x_n\})\,ds.$$

The measure ν and the probability measure γ on X define (Jacod, 1975) the probability measure P on $(\Omega, \mathscr{F})$ such that $P(x_0 \in B) = \gamma(B)$, $B \in \mathscr{B}(X)$; additionally, ν is the compensator of the random measure μ of the multivariate point process $(t_n, x_n)_{n\geqslant 0}$.

The process

$$X_t(\omega) = \sum_{n\geqslant 0} I(t_n \leqslant t < t_{n+1}) x_n, \qquad t \in [T_0, T_1),$$

is called a general jump Markov process.

Consider the functions (Feller) $\overline{P}^{(0)}(u,x;t,B) = I(x \in B) e^{-\int_u^t q(x,s)\,ds}$,

$$\overline{P}^{(n)}(u,x;t,B) = \int_u^t \int_X e^{-\int_u^t q(x,\theta)\,d\theta} q(x,s,dy \setminus \{x\}) \overline{P}^{(n-1)}(s,y;t,B)\,ds, \quad n \geqslant 1.$$

We also set $\overline{P}(u,x;t,B) = \sum_{n=0}^{\infty} \overline{P}^{(n)}(u,x;t,B)$.

We show that $\overline{P}(u,x;t,B)$ is a transition function and satisfies (under condition (c)) the backward Kolmogorov equation

$$\frac{\partial}{\partial u} P(u,x;t,B) = q(x,u) P(u,x;t,B) - \int_X q(x,u,dy \setminus \{x\}) P(u,y;t,B).$$

If $\overline{P}(u,x;t,B) = 1$ for all u,x,t, then $\overline{P}$ is the unique solution of this equation.

It is also shown that, under the local boundedness q (that is, when $\sup_{t\in[T_0,s)} < \infty$, $s \in [T_0,T_1)$, $x \in X$), the function $\overline{P}$ is a *minimal* solution to the forward Kolmogorov equation

$$\frac{\partial}{\partial t} P(u,x;t,B) = -\int_B q(y,t)\,P(u,x;t,dy) + \int_X q(y,t,B \setminus \{y\})\,P(u,x;t,dy).$$

Backward and forward equations are studied in detail.

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L. L. Shiryaeva (Samara, Russia)**. On properties of the three-parameter Grubbs' copula-function.**

Consider the statistics

$$T_{n,(1)} = (\overline{X} - \min\{X_i\})/S \quad \text{and} \quad T_n^{(1)} = (\max\{X_i\} - \overline{X})/S,$$

which are studentized deviations of measurements from the mean values calculated from n-sample (see [1]). It is assumed that in a normally distributed sample $\{X_i\}_{i=1}^n$ there is one abnormal observation X_{out} (with unknown number). So, the emission X_{out} differs from the remaining observations by the shift parameter α and the scale parameter $\nu > 0$. Let $G_{n,(1)}(x;\alpha,\nu) = \mathbf{P}(T_{n,(1)} < x)$, $G_n^{(1)}(x;\alpha,\nu) = \mathbf{P}(T_n^{(1)} < x)$, $\Upsilon_n(x,y;\alpha,\nu) = \mathbf{P}(\{T_{n,(1)} < x\} \cap \{T_n^{(1)} < y\})$. Recursive relations for describing Grubbs' statistic distribution functions $T_{n,(1)}$ and $T_n^{(1)}$ can be found in [2]. A three-parameter copula is extracted from the joint distribution $\Upsilon_n(\,\cdot\,)$ using inversion. Graphs of the modeled values from the copula are analyzed. It is shown that a copula is capable of describing negative dependencies between random variables. In the case $\alpha = 0$, the copula becomes symmetric. The effect of copula parameters n, α, and ν on the coefficients of its tail dependence is examined. It is shown that there exists a domain in which the copula agrees with the lower Fréchet–Hoeffding bound. The effect of copula parameters on the boundary of this domain is studied.

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E. L. Shishkina (Voronezh, Russia)**. Fractional Euler–Poisson–Darboux equation and random walks.** [33]

The "fictitious particle walk" Markov random process, such that the change in the motion direction is subject to an inhomogeneous Poisson process, leads to a general hyperbolic second-order equation with coefficients depending on time (see [1]). In the case of a slow Markov random process, we get the following problem for the fractional Euler–Poisson–Darboux equation:

$$\left(\frac{\partial^2}{\partial t^2} + \frac{2\alpha}{t}\frac{\partial}{\partial t}\right)^{\beta} p(x,t) = \frac{\partial^2 p(x,t)}{\partial x^2}, \qquad \beta \in \left(0, \frac{1}{2}\right), \tag{1}$$

$$p(0,t) = \delta_{2\alpha}(t), \qquad p_t(0,t) = 0, \tag{2}$$

where $t > 0$, $x \in \mathbf{R}$, $p = p(x,t)$ is the random walk law of particles in the space $\mathbf{R}^n$, α/t is the intensity of the inhomogeneous Poisson process, $\alpha > 0$, the fractional Bessel derivative $(\partial^2 u/\partial t^2 + (2\alpha/t)\partial u/\partial t)^\beta$ is defined in [2], and the weighted distribution $\delta_{2\alpha}$ is defined in [3]. The main result of this talk is as follows.

Theorem. *The solution to problem* (1), (2) *reads as*

$$p(x,t) = [(\mathbf{H}^{-1})_\xi \operatorname{ch}(x\xi^\alpha)](t),$$

where $(\mathbf{H}^{-1})_\xi$ *is the explicit inverse transform with the Fox function in the kernel.*

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A. L. Yakymiv (Moscow, Russia)**. On the order logarithm of a random *A*-permutation.**

Let S_n be the group of all permutations of $\{1, 2, \dots, n\}$. By $M(\sigma)$ we denote the order of a permutation σ from S_n. The following transparent example demonstrates one of the advantages of the probabilistic approach to combinatorial problems. In [1] it was shown that the random variable $\ln M(\sigma_n)$ is asymptotically normal with the mean $(\ln^2 n)/2$ and variance $(\ln^3 n)/3$, where σ_n is random permutation uniformly distributed on S_n. Indeed, according to [2], the order of permutations from S_n varies in the following broad range: from 1 (the order of the identical permutation) to $\exp\{(1+\delta(n))\sqrt{n \ln n}\}$ (the maximal order), where $\delta(n) \to 0$. From [1] it follows that there exists a sequence $\varepsilon(n) \downarrow 0$ such that a random permutation taken "at random" from S_n has an order from $\exp\{(1/2 - \varepsilon(n)) \ln^2 n\}$ to $\exp\{(1/2 + \varepsilon(n)) \ln^2 n\}$ with probability tending to 1. This remark is much more informative than the previous one. For a survey of further investigations in this area, see [3]. We fix a set $A \subseteq \mathbf{N}$. A permutation σ is called an A-permutation if the lengths of all cycles in σ lie in A. Let T_n be the class of all A-permutations from S_n and let a random permutation τ_n be uniformly distributed on T_n. The following assertion is valid. If the sequence $\{|T_n|/n!\}$ is RO-variable at infinity with lower exponent exceeding -1, then the random variable $\ln M(\tau_n)$ is asymptotically normal with the mean $\sum_{i=1}^{n} \chi\{i \in A\}(\ln i)/i$ and the variance $\sum_{i=1}^{n} \chi\{i \in A\}(\ln^2 i)/i$.

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