Cramér type moderate deviations for random fields

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(joint work with Hailin Sang and Yimin Xiao)

Let $\{X_{nj}, n \in \mathbb{N}, j \in \mathbb{Z}^d\}$ be a random field with zero means defined on a probability space $(\Omega, \mathcal{F}, P)$. Suppose that for each n, there is a disc on the complex plane $\mathbb{C}$ centered at the origin $z = 0$ with a finite radius H_n within which the cumulant generating function $L_{nj}(z) = \log \mathbb{E}e^{zX_{nj}}$ of X_{nj} is analytic and

$$|L_{nj}(z)| \leq c_{nj} \quad \text{for all } z \in \mathbb{C} \text{ with } |z| < H_n. \tag{1}$$

Denote

$$S_n = \sum_{j \in \mathbb{Z}^d} X_{nj}, \quad B_n = \sum_{j \in \mathbb{Z}^d} var(X_{nj}), \quad C_n = \sum_{j \in \mathbb{Z}^d} c_{nj}$$

and let

$$F_n(x) = P(S_n < x\sqrt{B_n}).$$

Assume that S_n is well-defined, $B_n < \infty$ for each $n \in \mathbb{N}$, $B_n H_n^2 \to \infty$ as $n \to \infty$ and that $C_n = O(B_n H_n^2)$. Then, denoting by $\Phi(z)$ the cumulative distribution function of standard normal distribution, the following theorem establishes exact moderate deviation for the random filed X_{nj} under Cramér's condition (1):

Theorem If $x \geq 0$ and $x = o(H_n\sqrt{B_n})$, then

$$\frac{1 - F_n(x)}{1 - \Phi(x)} = \exp\left\{\frac{x^3}{H_n\sqrt{B_n}} \lambda_n\Big(\frac{x}{H_n\sqrt{B_n}}\Big)\right\}\left(1 + O\left(\frac{x+1}{H_n\sqrt{B_n}}\right)\right),$$

where

$$\lambda_n(t) = \sum_{k=0}^{\infty} \beta_{kn} t^k$$

is a power series that stays bounded uniformly in n for sufficiently small values of $|t|$ and the coefficients β_{kn} only depend on the cumulants of X_{nj} $(n \in \mathbb{Z}, j \in \mathbb{Z}^d)$.