Chistyakov A. E. (Don State Technical University, Rostov-on-Don), **Kuznetsova I. Y.** (Southern Federal University, Rostov-on-Don) **Stability estimation of the equation for calculating the pressure taking into account the collision time of the medium molecules**

A crucial task is to improve the accuracy of mathematical modeling of natural hazards, including storm surges and the transport of pollutants in a reservoir. In this paper, we propose to consider an approach to constructing a mathematical model of hydrodynamics based on the well-known connection between the kinetic and hydrodynamic descriptions of a continuous medium [1-2]. According to [1], in the case of a spatially one-dimensional layer, the scheme will be constructed under the following assumptions: 1) at the n-th time step in each spatial cell $x \in [x_i,\, x_{i+1}]$, there is a locally Maxwellian distribution f_{0i} that is constant for this cell

$$f_{0i} = \frac{\rho_i}{(2RP_i)^{3/2}} \exp\left(-\frac{\zeta - U_i}{2RP_i}\right), \tag{1}$$

where ρ is the density of the substance, R is the gas constant, P is the pressure, ζ is the velocity of the molecule, U_i is the macroscopic velocity; 2) during the time $t \in \left[t^n,\, t^{n+1}\right]$ the particles of the medium perform collisionless expansion; 3) at the *n+1* time step the distribution function f^{n+1}, obtained as a result of the expansion and is not Maxwellian is instantly Maxwellized, and the whole procedure is repeated again.

It is shown in [2] that when solving problems of hydrodynamics, the equation of continuity in the case of taking into account the time of collision of medium molecules takes the form

$$\rho'_t = \tau^* \rho''_{tt} + \nabla(\rho \mathbf{V}), \tag{2}$$

where $\tau^*\ h/c$ is the regularization parameter or the characteristic time between collisions of molecules, h is the computational grid step, c is the speed of sound, $\mathbf{V}$ is the velocity vector of the medium. In an adiabatic process, the speed of sound is determined from the expression: $c = \sqrt{(\partial P/\partial \rho)_S}$, where S is an index showing that the derivative is taken at constant entropy. In equation (2), the term $\tau^* \rho''_{tt}$ arises when the momentum transfer delay is taking into account in the case of representation of the collision time of molecules by a discrete function.

According to [3], the calculation of the pressure function will take the form:

$$\frac{1}{c^2} P''_{tt} - \Delta P = -\frac{\rho'_t + \nabla\left(\rho \tilde{\mathbf{V}}\right)}{\tau}, \tag{3}$$

with initial conditions $P\big|_{t=0} = P_0$, $P'_t\big|_{t=0} = P_1$. Here $\tau = t^{n+1} - t^n$ is the time step, $\tilde{\mathbf{V}}$ is the intermediate velocity field calculated without regard to pressure [4].

Using the Rote method for equation (3), an analytical solution was obtained [5]:

$$P''_{tt} = -\Lambda P, \quad P = \sum_i \alpha_i X_i, \quad \alpha_i(t) = \alpha_{i,0} \cos\left(\sqrt{\lambda_i} t\right) + \frac{\alpha_{i,1}}{\sqrt{\lambda_i}} \sin\left(\sqrt{\lambda_i} t\right), \tag{4}$$

where Λ is a self-adjoint, positive definite operator, X_i are the eigenvectors of the operator Λ forming an orthonormal basis, λ_i are the eigenvalues of the operator Λ, $\Lambda X_i = \lambda_i X_i$.

In general, the system of eigenvectors of the operator is unknown. In practice, equation (3) is solved by numerical methods. Solution (4) is necessary to estimate the approximation error of the numerical solution.

Theorem. The implicit difference scheme approximating the homogeneous equation (3) is absolutely stable, has the first order of accuracy, and the estimation of the calculation error has the form:

*The study was financially supported by the Grants Council of the President of the Russian Federation within the framework of the scientific project No. MD-3624.2021.1.1

$$\Psi_i^{n+1} = C^n \Psi_i^1 + \sum_{r=0}^{n-1} C^r \beta_i^{n-r}, \quad \cos\varphi = \frac{1}{\sqrt{1+k}}, \quad k = \frac{\lambda_i \tau^2}{\tau^*},$$

$$C^n = \left(\frac{1}{1+k}\cos(\varphi n) + \frac{2\sqrt{1+k}\exp\left(-1/\sqrt{1+k}\right) - 1}{k(1+k)}\sin(\varphi n) \right) \exp\left(\frac{n}{\sqrt{1+k}} \right),$$

$$\beta_i^n = \frac{2\alpha_{i,0}\cos\left(\sqrt{\lambda_i}t^n\right)\left(1-\cos\left(\sqrt{\lambda_i}\tau\right)\right) - \alpha_{i,0}k\cos\left(\sqrt{\lambda_i}t^n + \sqrt{\lambda_i}\tau\right)}{1+k} +$$
$$+ \frac{2\alpha_{i,1}\sin\left(\sqrt{\lambda_i}t^n\right)\left(1-\cos\left(\sqrt{\lambda_i}\tau\right)\right) - \alpha_{i,1}k\sin\left(\sqrt{\lambda_i}t^n - \sqrt{\lambda_i}\tau\right)}{(1+k)\sqrt{\lambda_i}}.$$

REFERENCES

1. *Chetverushkin, B. N.* Kinetic Shemes and Quasi-Gas Dynamic system of Equations. – CIMNE. – Barcelona, 2008. – 298 p.

2. *Chetverushkin, B. N.* Resolution limits of continuous media models and their mathematical formulations // Math. Models Comput. Simul. – 2013. – Vol. 5, Is. 3. – p. 266–279. DOI: 10.1134/S2070048213030034

3. *Sukhinov, A. I., Chistyakov, A. E., Kuznetsova I. Yu., Atayan, A. M., Nikitina, A. V.* Regularized difference scheme for solving hydrodynamic problems // Math. Models Comput. Simul. – 2022. – Vol. 34, Is. 2. – p. 85–100. DOI: 10.20948/mm-2022-02-07 (*in Russian*)

4. *Belotserkovskii, O. M., Gushchin, V. A., Konshin, V. N.* The splitting method for investigating flows of a stratified liquid with a free surface // Comput. Math. Math. Phys. – 1987. – Vol. 27, Is. 2. – p. 181–191.

5. *Sukhinov, A. I., Chistyakov, A. E.* Error solving the wave equation based on the scheme with weights // Math. Models Comput. Simul. – 2017. – Vol. 9, Is. 6. – p. 649–656. DOI: 10.1134/S2070048217060126