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On the stability in probability of solutions of certain stochastic differential equations of the second order

Over the last five decades the theory of differential equations (SDE) has significantly developed in the works of numerous authors. Many fundamental results concerning the stability property of SDE were obtained in pioneering works of Kushner [1] and [2] Khasminskii by Lyapunov-like functions method (see also surveys [3], [4]).

In the present talk we derive sufficient conditions of asymptotic stability in probability in the large of equilibrium position of the second-order SDEs which are typical in the theory of nonlinear dynamics.

Let us present one of the results.

Consider a stochastic generalized Rayleigh's equation written as a two-dimensional system of Ito's SDEs

$$dx(t) = y(t)dt,\ dy(t) = \left[-f(y) - g(x)\right]dt + \sigma(y)d\xi, \tag{1}$$

where the functions f, g and σ satisfy the Lipschitz condition, and $f(0) = g(0) = \sigma(0) = 0$.

Theorem. *Suppose that there exist numbers* $b > 0$ *and* σ_0 *such that*

1) $f(y)/y > b$ *for all* y,
2) $xg(x) > 0$ *for all* $x \neq 0$,
3) $\int_0^x g(s)ds \to +\infty$ *for* $|x| \to \infty$,
4) $0 < \sigma(y)/y < \sigma_0^2$, *and* $\sigma_0^2 < 2b$.

Then the trivial solution $(x(t) \equiv 0,\ y(t) \equiv 0)$ *of the system* (1) *is asymptotically stable in probability in the large.*

The proof of the above theorem is based on the use of Lyapunov-like method of auxiliary functions developed for SDEs [1, 2].

References

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