Probability characteristics of the networks of the bistable Hodgkin-Huxley-type of models with noise

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Hodgkin-Huxley-type of models are used for mathematical modeling of the dynamics of a single neuron [1]. These models demonstrate various types of oscillatory behavior, including stable equilibrium (resting state), spike and bursting oscillations (oscillatory activity). These models are described by ordinary nonlinear differential equations and are characterized by various phenomena, including multistability, when several dynamic modes coexist in the system [2]. The influence of noise on multistable systems is one of the classical problems considered in the framework of the theory of nonlinear dynamical systems [3].

In the frame of this work, we will consider the features of a network of Hodgkin-Huxley-type of models with bistability in the presence of noise. As a base model, we consider the modified Sherman model [4]:

$$\begin{aligned}
&\tau\dot{V} = -g_{Ca}m_\infty(V)(V - V_{Ca}) - g_K n(V - V_K) - g_{K2}p_\infty(V - V_K) - g_S S(V - V_K),\\
&\tau\dot{n} = \sigma(n_\infty(V) - n),\\
&\tau_S\dot{S} = S_\infty(V) - S.
\end{aligned} \tag{1}$$

Here, the dynamic variable V is the membrane potential, n is interpreted as the probability of opening potassium channels, and S is a slow variable in the system that can describe the concentration of calcium ions in the cell. In general form we will use dynamical variable $x = (V, n, S)$.

The sigmoidal functions m_∞, n_∞, and S_∞ describe the opening probabilities of fast and slow potassium channels:

$$\omega_\infty(V) = [1 + exp\frac{V_\omega - V}{\theta_\omega}]^{-1}, \omega = m, n, S. \tag{2}$$

The function p_∞ describes opening probability of pathological potassium channel, which provide bistability in the model:

$$p_\infty(V) = [exp\frac{V - V_\omega}{\theta_\omega} + exp\frac{V_\omega - V}{\theta_\omega}]^{-1}. \tag{3}$$

The conductivities of calcium and potassium ion channels correspond to the following values: $g_{Ca} = 3.6$, $g_K = 10.0$ and $g_{K2} = 0.2$. The Nernst potentials (threshold potentials for ion channel activation) are fixed as follows: $V_{Ca} = 25$ mV and $V_K = 75$ mV. Another parameters fixed as: $\tau = 0.02$, $\tau_S = 35$, $\sigma = 0.93$, $V_m = -20.0$, $\theta_m = 12.0$, $V_n = -16.0$, $\theta_n = 5.6$, $V_S = -35.0$, $\theta_S = 10.0$, $V_p = -47.0$, $\theta_p = 1.0$. Model (1) demonstrates the bistability between the equilibrium state and the bursting attractor. For initial condition $x_1 = (-49.084, 0.027105, 0.19648)$ stable equilibrium exists, for initial conditions $x_2 = (-49, 0.02, 0.17)$ bursting attractor can be obtained.

Theorem. For any $x \in R^3$ exists and unique solution of system (1) with initial condition x_1 or x_2. Stable equilibrium point x_1 is stable focus.

In [5] it is shown that when noise is added to the system, classical switching between attractors is not observed; when a certain noise level is reached, the system from the equilibrium state passes to the bursting attractor and then remains on it. This feature is due to the fact that the attractor is distant from the equilibrium state in the direction of the dynamic variable S.

Now we consider a network of similar oscillators whose dynamics was studied in [6] with the addition of white noise. In this case it possible to find certain interval of coupling strength when stable equilibrium point will dominate.

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