Litvinov V. N., **Gracheva N. N.**, **Rudenko N. B.** (Don State Technical University, Rostov-on-Don; Azov-Black Sea Engineering Academy, Zernograd, Russia). **Probabilistic estimates for solving grid equations on heterogeneous computing systems.**

Solving problems of mathematical physics using implicit schemes is reduced to solving systems of linear algebraic equations (SLAE) of high dimension (10^9 and more). Placing all the necessary data in RAM and efficient software implementation of the developed numerical methods are possible only with the use of modern high-performance computing systems built on the basis of a heterogeneous architecture. Estimation of the solution time for grid equations is of a probabilistic nature. This is due to the peculiarities of the functioning of complex hardware and software complex in multi-threaded mode and data caching algorithms.

The aim of the study is to determine the functional dependences of the calculation time of the SLAE by the modified alternating-triangular iterative method (MATM) on the dimension of fragments of a uniform three-dimensional computational grid. Research was carried out for the most time-consuming stages of solving grid equations by the MPTM method, including the solution of SLAEs with lower and upper triangular matrices [1].

The first experiment was carried out on the K-60 computing cluster of the Keldysh Institute of Applied Mathematics. As a result, the dependence of the data transfer time between the computing nodes of the cluster T_{nn} on the number of transmitted elements N_{el} is determined

$$T_{nn} = 13.57 + 6.03 \cdot N_{el} \cdot 10^{-3}. \quad (1)$$

The coefficient of determination of the regression equation was 0.8.

In the second experiment, the calculation time for one fragment of the computational grid was measured by the MATM method with a different number of parallel flows. The number of streams varied from 1 to 32. For 32 streams, the statistical characteristics of the experimental data were obtained: the minimum value is 4096 us, the maximum value is 4188 us, the average value is 4143.9 us, the 90th percentile is 4170 us, the variance is $696,29$ us^2, standard deviation is 26.39 us. According to experimental data, the least square deviation was obtained when calculating one fragment by 32 parallel streams, equal to 26.39 us.

As a result of the third experiment, a regression equation was obtained, which makes it possible to determine the calculation time by the MATM method on a graphics accelerator.

$$T_{GPU} = a - b \cdot Y - c \cdot ln(k) - d \cdot ln(Y), \quad (2)$$

where k is the ratio of the number of flows along the Ox axis to the number of flows along the Oz axis, Y - number of grid nodes along the Oy axis. The coefficient of determination was 0.86; $a = 26; b = 0.0002; c = 0.16; d = 0.77$.

Among the results of the study is the following theorem.

Theorem 1. *The calculation time for the step of solving a SLAE with a lower triangular matrix by the MATM is determined by the formula* $T_{matm} = \sum_{s=1}^{N_s} \max(\mathbf{T}_s)$, *where* s, N_s *are the step number and the number of steps of the parallel-pipeline computing process, respectively;* $\mathbf{T}_s$ *- a vector containing the values of the time spent on computing fragments of the computational grid by all calculators at step* s .

REFERENCES

1. *Sukhinov, A.* Computational Aspects of Solving Grid Equations in Heterogeneous Computing Systems / A. Sukhinov, V. Litvinov, A. Chistyakov [et al.] // Lecture Notes in Computer Science. – 2021. – Vol. 12942 LNCS. – P. 166-177. – DOI 10.1007/978-3-030-86359-3_13.

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