

Dmitriy F. Kuznetsov (Peter the Great St.-Petersburg Polytechnic University, St.-Petersburg, Russia). **Recent results on a new approach to series expansion of iterated Stratonovich stochastic integrals with respect to components of a multidimensional Wiener process.**

Theorem 1 [1, Sect. 1.11, 2.1.4, 2.10-2.18]. *Assume that a CONS $\{\phi_j(x)\}_{j=0}^\infty$ in $L_2[t, T]$ ($\phi_0(x) = 1/\sqrt{T-t}$) and $\psi_1(\tau), \dots, \psi_k(\tau) \in C[t, T]$ are such that condition (2.689) [1, Sect. 2.10] is fulfilled. Then*

$$\begin{aligned} & \int_t^T \psi_k(t_k) \dots \int_t^{t_2} \psi_1(t_1) \circ d\mathbf{W}_{t_1}^{(i_1)} \dots \circ d\mathbf{W}_{t_k}^{(i_k)} = \\ & = \text{l.i.m.}_{p_1, \dots, p_k \rightarrow \infty} \sum_{j_1=0}^{p_1} \dots \sum_{j_k=0}^{p_k} C_{j_k \dots j_1} \zeta_{j_1}^{(i_1)} \dots \zeta_{j_k}^{(i_k)}, \end{aligned} \quad (1)$$

where $\zeta_j^{(i)} = \int_t^T \phi_j(\tau) d\mathbf{W}_\tau^{(i)}$ are i.i.d. $N(0, 1)$ -r.v.'s ($i \neq 0$), $k \in \mathbf{N}$, $d\mathbf{W}_\tau^{(i)}$ and $\circ d\mathbf{W}_\tau^{(i)}$ are the Itô and Stratonovich differentials, respectively, $C_{j_k \dots j_1}$ is the Fourier coefficient corresponding to the kernels $K(t_1, \dots, t_k) = \psi_1(t_1) \dots \psi_k(t_k) \mathbf{1}_{\{t_1 < \dots < t_k\}}$ ($k \geq 2$) and $K(t_1) = \psi_1(t_1)$, $t_1, \dots, t_k \in [t, T]$, $i_1, \dots, i_k = 0, 1, \dots, m$, $\mathbf{W}_\tau^{(i)}$ ($i = 1, \dots, m$) are independent standard Wiener processes, $\mathbf{W}_\tau^{(0)} = \tau$. Moreover, when $\{\phi_j(x)\}_{j=0}^\infty$ is a CONS of Legendre polynomials or trigonometric functions in $L_2[t, T]$ and $\psi_1(\tau), \dots, \psi_k(\tau) \in C^1[t, T]$, the expansion (1) is valid for $k = 1, \dots, 6$ ($p_1 = \dots = p_k = p$ for $k = 4, 5, 6$) and without condition (2.689) [1, Sect. 2.10].

Theorem 1 can be useful for constructing high-order strong numerical methods for systems of Ito stochastic differential equations with noncommutative noise.

References

- [1] D.F. Kuznetsov, Strong Approximation of Iterated Itô and Stratonovich Stochastic Integrals Based on Generalized Multiple Fourier Series. Application to Numerical Solution of Itô SDEs and Semilinear SPDEs. arXiv:2003.14184v39 [math.PR], 2023, 951 pp. DOI: <https://doi.org/10.48550/arXiv.2003.14184>