Application of semi-Markov processes with a common phase space of states and hidden Markov models based on them to the modeling of systems

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To model systems for various purposes [1,2] use semi-Markov processes (SMP) with a common phase space of states. When constructing a semi-Markov model, it is necessary to introduce a phase space of states of the system, which can be finite, discrete-continuous. In the system operation for which a semi-Markov model is constructed, it is not always possible, when changing its states, to fully obtain the information contained in the encoding of states, but it is possible to obtain some signal (information) associated with the SMP states and the embedded Markov chain (EMC). In this case, the SMP and EMC states can be considered hidden (unobservable). There are problems of evaluating the characteristics and predicting the states of the semi-Markov model and its correction based on the received vector of signals. The solution of these problems is possible based on the use of the apparatus of the theory of hidden Markov models (HMM) [3,4].

The paper presents semi-Markov models with a discrete-continuous phase space of states of a queuing system (QS) $GI/\overline{G}/N/0$ and a system with a component-wise instantly replenished time reserve. Based on these semi-Markov models and using their EMCs, we construct HMMs and use them to solve the above problems.

We consider the QS $GI/\overline{G}/N/0$. Let the received requirement service time by the k-th device be a random variable (RV) α_k with the probability distribution function (PDF) $F_k(x)$, $k=\overline{1,N}$; the time between the receipt of the requirements is RV β with the PDF $G(x)$.

To construct a SMP $\xi(t)$ describing the functioning of the QS $GI/\overline{G}/N/0$, we use the phase space of states by the following form:

$$E=\left\{i\bar{d}\bar{x}, i\bar{d}\bar{x}z, i\bar{x} : i=\overline{1,N}, \bar{d}=(d_1,...,d_k,...,d_N), \bar{x}=(x_1,...,x_k,...,x_N)\right\}, \quad (1)$$

where i is the number of the device in which the state change occurred, $x\geq 0$ is the time elapsed since the beginning of the requirement service by the k-th device; $z>0$ is the time elapsed since the last request was received;

$$d_k=\begin{cases}0, & \text{if k}-th \text{ device is free,}\\ 1, & \text{if k}-th \text{ device is busy by servicing.}\end{cases}$$

We use the semi-Markov model to find the QS $M/\overline{G}/N/0$ stationary characteristics, in this case $G(x)=1-e^{-bx}, x\geq 0$. Let $|\bar{d}|$ - the number of the vector $\bar{d}$ components equal to zero (the number of free devices).

Theorem. The stationary distribution of the EMC $\{\xi_n; n\geq 0\}$ has the form:

$$\rho(i\bar{x})=\rho_0 b\prod_{k=1}^{N}\overline{F}_k(x); \rho\left(i\bar{d}\bar{x}\right)=\rho_0\frac{|\bar{d}|!}{b^{|\bar{d}|}}\prod_{k:d_k=1}\overline{F}_k(x_k), d_i=1; \quad (2)$$

$$\rho\left(i\bar{d}\bar{x}\right)=\rho_0\frac{\left(|\bar{d}|-1\right)!}{b^{|\bar{d}|-1}}\prod_{k:d_k=1}\overline{F}_k(x_k), d_i=0, \quad (3)$$

where the constant ρ_0 is from the normalization condition.

Denote by D the set of all binary vectors $\bar{d}$, describing the states of QS $M/\overline{G}/N/0$ devices, and let D_+ be a subset of D, $D_-=D\backslash D_+$. Then the average stationary residence time (T_{D_+}) of QS $M/\overline{G}/N/0$ in the states corresponding to D_+ is determined by the formula:

$$T_{D_+}=\frac{\sum\limits_{\bar{d}\in D_+}|\bar{d}|!E^{|\bar{d}|}\beta\prod\limits_{k:d_k=1}E\alpha_k}{\sum\limits_{\bar{d}\in D_+}\left[\sum\limits_{\substack{j:d_j=1,\\ \bar{d}^{(j)}\in D_-}}|\bar{d}|!E^{|\bar{d}|}\beta\prod\limits_{\substack{k:d_k=1,\\ k\neq j}}E\alpha_k+\sum\limits_{\substack{j:d_j=0,\\ \bar{d}^{(j)}\in D_-}}\left(|\bar{d}|-1\right)!E^{|\bar{d}|-1}\beta\prod\limits_{k:d_k=1}E\alpha_k\right]}, \quad (4)$$

where $E\alpha_k$ – mathematical expectation of RV α_k, $E\beta = \frac{1}{b}$.

The considered semi-Markov model uses to find the stationary characteristics of the QSs $GI/G/1/0$ and $GI/G/2/0$.

The paper also provides a semi-Markov model of a system with a component-wise instantly replenished time reserve and its stationary characteristics [5].

During constructing the HMM for the transition to a discrete set of states, we use the stationary phase enlargement algorithm proposed by V.S. Korolyuk and A.F. Turbin [1].

HMMs are used to analyze the above systems functioning and predict their states based on the received vector of signal.

References

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