Pavlov I. V. (DSTU, Rostov-on-Don, Russia). **Martingales with mixed norm: results related to orthogonal series and the geometry of Banach spaces**

The report will present the results of the work [1] and some further progress. Let $f \in L_1 = L_1(\Omega, \mathcal{A}, P)$ and $f_n := E^P[f|\mathcal{F}_n]$, where $\mathcal{F}_n \uparrow \mathcal{A}$. Let $\mathbf{p} = (p_1, p_2, \ldots, p_n, \ldots)$ be an infinite dimensional vector $(1 \le p_k \le \infty, k = 1, 2, \ldots)$, $\|f\|_{\mathbf{p}} := \sup_n \left\| \ldots \left\| \|f_n\|_{p_n, \mathcal{F}_{n-1}} \right\|_{p_{n-1}, \mathcal{F}_{n-2}} \ldots \right\|_{p_1, \mathcal{F}_0}$, where $g_{p,\mathcal{F}} = \left(E[|g|^p \middle| \mathcal{F}] \right)^{1/p}$ if $p < \infty$ and $g_{\infty,\mathcal{F}} = \lim_{r \uparrow \infty} \uparrow \|g\|_{r,\mathcal{F}}$. If $\|f\|_{\mathbf{p}} < \infty$, we say $f \in L_{\mathbf{p}}$.

Theorem. Let $\mathbf{p} = (p_1, p_2, \ldots, p_n, \ldots)$ satisfy the condition $\sup_{n \ge 1} p_n < \infty$. Assume that there exists a subsequence $(n_k)_{k=0}^{\infty}$ $(n_0 = 0)$ of natural numbers and a sequence of r.v. $(\varphi_{n_k})_{k=0}^{\infty}$ $(\varphi_0 = 1)$, where φ_{n_k} is measurable with respect to $\mathcal{F}_{n_k}$ $(k \ge 0)$ such that for all $k \ge 1$:

1) φ_{n_k} does not depend on $\mathcal{F}_{n_{k-1}}$; 2) $E(\varphi_{n_k}) = 0$; 3) $\|\varphi_{n_k}\|_1 \ge \delta > 0$ (δ does not depend on k); 4) $\|\varphi_{n_k}\|_2 = 1$; 5) $\sup_{k \ge 1} \|\varphi_{n_k}\|_\infty < \infty$; 6) $\prod_{k=1}^{\infty} \|1 + \varphi_{n_k}\|_{p_{n_k}} = C < \infty$.

Then the Banach space $L_{\mathbf{p}}$ cannot be embedded in any Banach space with an unconditional basis.

In the report, in particular, examples will be given that implement the conditions of this theorem.

REFERENCES

[1] Pavlov, I.V. On properties of spaces with martingale mixed norm with summability exponents close to one. J Math Sci (2023). https://doi.org/10.1007/s10958-023-06327-y