Nikita Ratanov (Chelyabinsk State University, Russia). **Stochastically resetted Kac-Ornstein-Uhlenbeck processes.** [1] Consider the Kac-Ornstein-Uhlenbeck process $\mathbb{X}$, [1-2], which obey the stochastic "Langevin" equation $\mathbb{X}(t) = \mathbb{A}(t) - \int_0^t \mathbb{X}(s)\mathrm{d}\mathbb{G}(s)$, $t \geq 0$, where $\mathbb{A}(t) = \int_0^t a_{\varepsilon(s)}\mathrm{d}s$ and $\mathbb{G}(t) = \int_0^t \gamma_{\varepsilon(s)}\mathrm{d}s$ are two telegraph processes based on a common homogeneous Markov process $\varepsilon(t) \in \{0,1\}$, $t \geq 0$, with two states, switching times $\{\tau_n\}_{n\geq 1}$ and a switching parameter λ.

We assume a complete reset of $\mathbb{X}$ after each state switch: $\mathbb{X}(\tau_n) = 0$.

Here we prove that process $\mathbb{X}$ weakly converges to r.v. $\mathbb{X}^*$ as $t \to \infty$. The distribution of limit $\mathbb{X}^*$ is determined by the signs of parameters. For example, in completely attracting case, $\gamma_0, \gamma_1 > 0$, the following result holds.

Theorem. *Let $\gamma_0, \gamma_1 > 0$ and $\rho_0 < \mathbb{X}(0) = 0 < \rho_1$.*

The process $\mathbb{X}(t)$ converges in distribution in the Skorohod space $D[\rho_0,\ \rho_1]$:

$$\mathbb{X}(t) \xrightarrow{D} \mathbb{X}^*, \qquad t \to \infty,$$

where the distribution of $\mathbb{X}^*$ is determined by the probability density function

$$\mathscr{P}^*(x) = \frac{\lambda}{2a_1}(1 - x/\rho_1)^{-1+\lambda/\gamma_1}\mathbb{1}_{\{0<x<\phi_1(t)\}} - \frac{\lambda}{2a_0}(1 - x/\rho_0)^{-1+\lambda/\gamma_0}\mathbb{1}_{\{\phi_0(t)<x<0\}}.$$

Here $\rho_0 = a_0/\gamma_0$, $\rho_1 = a_1/\gamma_1$ and $\phi_0(t) = \rho_0(1 - \mathrm{e}^{-\gamma_0 t})$, $\phi_1(t) = \rho_1(1 - \mathrm{e}^{-\gamma_1 t})$.

The proof is based on explicit formulae for the distribution of $\mathbb{X}(t)$ and Prohorov's theorem, see e.g. [3].

References:

1. N.Ratanov. Kac-Ornstein-Uhlenbeck processes: stationary distributions and exponential functionals. Methodology and Computing in Applied Probability, 1-19, 2022.

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3. P.Billingsley. Convergence of Probability Measures. 2d Ed, Wiley, 1999.

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