Critical branching process evolving in non-favorable random environment *

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Let $\{Z_n, n = 0, 1, 2, ...\}$ be a critical branching process generated by a sequence $\{F_n(s),\ n = 1, 2, ...\}$ of independent i.i.d. random probability generating functions, $S_0 = 0, S_n = \log F_1'(1) + ... + \log F_n'(1), n \geq 1$ be the associated random walk, and $\varphi(n) \to \infty$ as $n \to \infty$ is a function such that $\varphi(n) = o(\sqrt{n})$.

Theorem 1 *If* $\mathbf{E} \log F_1'(1) = 0, \quad \sigma^2 = \mathbf{E}(\log F_1'(1))^2 \in (0, \infty),$ *then*

1) for any $y \in (0, 1]$

$$\lim_{n\to\infty} \mathbf{P}\left(\frac{1}{\varphi(n)} \log Z_n \leq y | S_n \leq \varphi(n), Z_n > 0\right) = y^2;$$

2) for any $t \in (0, 1)$ *and* $x \in [0, \infty)$

$$\lim_{n\to\infty} \mathbf{P}\left(\frac{1}{\sigma\sqrt{n}} \log Z_{nt} \leq x | S_n \leq \varphi(n), Z_n > 0\right) = \mathbf{P}\left(B_0^+(t) \leq x\right),$$

where $\left\{B_0^+(t), 0 \leq t \leq 1\right\}$ *is a Brownian excursion of unit length.*

*This work was supported by the Russian Science Foundation under grant no 19-11-00111, https://rscf.ru/project/19-11-00111/ .

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