Convergence and mixing rate bounds for Markov chains and switched diffusions

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Let X_n be a homogeneous Markov chain (MC) on a finite state space with a irreducible & aperiodic transition probability matrix Q. The following characteristics are compared: Markov - Dobrushin's ergodic coefficient $\kappa(Q)$, its m-steps analogue $\kappa(Q^m)$, the spectral bound $|\lambda_2(Q^m)|$ due to [1, Ch. XIII, (96)] based on the second eigenvalue of Q, and the spectral radii $r(V)$ & $r(V_m)$ of the sub-stochastic matrices V & resp. V_m constructed via a markovian coupling [2, Sec. 2].

Theorem 1. *For Q specified earlier, for the invariant measure μ and $\mu_n^x := \mathcal{L}(X_n)$*

$$\lim_{m\to\infty}(\kappa(Q^m))^{1/m} = \lim_{m\to\infty}(r(V_m))^{1/m} = |\lambda_2(Q)| \ \&\ \limsup_{n\to\infty}\frac{1}{n}\ln\|\mu_n^x-\mu\|_{TV} \le \ln|\lambda_2(Q)|.$$

For infinite state spaces this result is still an open problem. This equivalence is important because $\kappa(Q^m) \ge r(V_m) \ge |\lambda_2(Q^m)|$, and in examples both inequalities here are usually strict (cf. [2, Section 4]). The analogues of the first two characteristics κ and r also work in the non-homogeneous cases, unlike the one based on the value $|\lambda_2(Q)|$. Similar non-uniform bounds of convergence towards the invariant regime may be established for various non-compact state spaces. The natural extension of these bounds is Kolmogorov's mixing coefficient, which will be estimated in the talk in examples such as Wright – Fisher's diffusions and diffusions with switching.

References

[1] F.R. Gantmacher, The Theory of Matrices. vol. 1, 2, AMS Chelsea Publ., Providence, Rhode Island, USA, 2000. doi: 10.1126/science.131.3408.1216-a

[2] A.Yu. Veretennikov, M.A. Veretennikova, On improved bounds and conditions for the convergence of Markov chains, Izv. Mathem., 2022, 86(1), 92-125.

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