Chubatov A. A. (Sirius University of Science and Technology, Sochi, Russia) **A numerical algorithm to solve fully nonlinear parabolic PDEs based on forward-backward SDEs and neural networks.**

We consider the Cauchy problem for a fully nonlinear PDE

$$\frac{\partial u}{\partial t} + f(x, u, \nabla u, \nabla^2 u) = 0, \quad u(T, x) = h(x), \quad (t, x) \in [0, T] \times \mathbb{R}^d, \qquad (1)$$

and include it into a system of quasilinear parabolic equations with a diagonal principal part

$$\frac{\partial v_k}{\partial t} + \frac{1}{2}\operatorname{Tr}[AA^+ \nabla^2 v_k] + \psi_k(x, u, v, \nabla v) = 0, \quad k = 0, \ldots, d,$$
$$v_0(T, x) = h(x), \quad v_k(T, x) = \nabla_{x_k} h(x), k = 1, \ldots, d \qquad (2)$$

where $AA^+ = 2\nabla_\gamma f(x, u, v, \gamma)$, $v_0 = u, v_1 = \nabla_{x_1} u, \ldots, v_d = \nabla_{x_d} u, \gamma_{kj} = \nabla_{x_j} v_k$.

`Condition C.` *We say that condition C holds iff*
1. $f(x, u, v, \gamma)$ and $h(x)$ are C^1-smooth and bounded functions;
2. $\nabla_\gamma f > 0$;
3. function $\psi(x, u, v, \gamma)$ satisfies the condition of the classical existence and uniqueness theorem of an FBSDE solution [1].

`Theorem 1.` *Assume that condition C holds. Then*
1. There exists an FBSDE associated with (2);
2. FBSDE solution generates a viscosity solution to (1);
3. There exists a stochastic optimal problem equivalent to this FBSDE.

An algorithm to obtain a numerical solution of the optimization problem based on neural network technique is developed.

References

[1] Ma J. and Yong J. Forward-Backward stochastic differential equations and their applications. Springer. 2007.