Branching process evolving in extremal random environment *

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Let $\{Z_0 = 1, Z_n, n \geq 1\}$ be a critical branching process generated by a sequence $\{F_n(s), n \geq 1\}$ of independent i.i.d. random probability generating functions, $\{S_0 = 0, S_n = \log F_1'(1) + ... + \log F_n'(1), n \geq 1\}$ be the associated random walk, and $\{B_s, 0 \leq s < \infty\}$ is a standard Brownian motion.

Theorem 1 *If* $\mathbf{E}\log F_1'(1) = 0, \quad \sigma^2 = \mathbf{E}(\log F_1'(1))^2 \in (0,\infty)$, *and* $\min(m,n) \to \infty$ *in such a way that* $m = o(n)$. *Then*

1) if $\varphi(n) = o(\sqrt{m})$, *then for any* $z \in [0,\infty)$

$$\lim_{n\to\infty} \mathbf{P}\left(\frac{1}{\sqrt{m}}\log Z_{n-m} \leq z | S_n \leq \varphi(n), Z_n > 0\right) = 1 - e^{-z^2/2};$$

2) if $\varphi(n) \sim T\sqrt{m}, T \in (0,\infty)$, *then for any* $z \in [0,\infty)$

$$\lim_{n\to\infty} \mathbf{P}\left(\frac{1}{\sqrt{m}}\log Z_{n-m} \leq z | S_n \leq \varphi(n), Z_n > 0\right) = \frac{1}{T^2}\sqrt{\frac{2}{\pi}}\int_0^z wdw \int_0^T e^{-(t-w)^2/2}\mathbf{P}\left(\inf_{0\leq s\leq 1} B_s \geq 0 | B_0 = w, B_1 = t\right) dt;$$

3) if $\sqrt{m} = o(\sqrt{\varphi(n)}), \varphi(n) = o(\sqrt{n})$, *then for any* $z \in (-\infty,\infty)$

$$\lim_{n\to\infty} \mathbf{P}\left(\frac{1}{\sqrt{m}}(\log Z_{n-m} - S_n) \leq z | S_n \leq \varphi(n), Z_n > 0\right) = \mathbf{P}(B_1 \leq z).$$

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