

G. I. Beliavsky, N. V. Danilova (Southern Federal University, Rostov-on-Don, Russia). About one piecewise constant semimartingale.

In the presented work, a piecewise constant semimartingale W^h is studied. Process is constructed using a two-dimensional random sequence (τ_i, δ_i) , the first component $\tau_i = \tau_{i-1} + \nu_i$, $\tau_0 = 0$, ν_i - a sequence of independent and identically distributed random variables - passages of the Wiener process module, the second - a sequence of independent Rademacher random variables. New random process $W_{\tau_i} = W_{\tau_{i-1}} + h\delta_i$, $W_t^h = W_{\tau_{i-1}}$, $\tau_{i-1} \leq t < \tau_i$ replaces the Wiener process in the Ito process and in the stochastic differential equation. The use of a new random process instead of the Wiener process in the classical model of Black, Scholes and Samuelson made it possible to obtain a new formula for calculating the fair price of a payment.

The main result concerns the asymptotics of the Ito formula for a given semimartingale. Let

$$f(X_t) = f(X_0) + \int_0^t \left(f'(X_s)\mu_s + \frac{1}{2}f''(X_s)\sigma_s^2 \right) ds + \int_0^t f'(X_s)\sigma_s dW_s, \text{ let}$$

$$f(X_t^h) = f(X_0) + \int_0^t f'(X_{s-}^h)\mu_s ds + \int_0^t f'(X_{s-}^h)\sigma_s dW_s^h + \sum_{i=1}^{N_t^h} (f(X_{\tau_i-}^h + h\sigma_{\tau_i}\delta_i) - f(X_{\tau_i-}^h) - f'(X_{\tau_i-}^h)h\sigma_{\tau_i}\delta_i).$$

Theorem. If $|f^3(x)| \leq M$, if $|\sigma(x) - \sigma(y)| \leq L|x - y|$, then in $L_2 \lim_{h \rightarrow 0} f(X_t^h) = f(X_t)$.