

Grechko A. S. (InWise Systems, LLC, Rostov-on-Don, Russia), **Kudryavtsev O. E.** (InWise Systems, Southern Federal University, Rostov-on-Don, Russia). **Monte Carlo methods for pricing options in Lévy models**¹.

In this paper, we consider an approach that allows us to simulate the extremum of the Lévy process or the joint distribution of the final position of the Lévy process X_t and its supremum $\bar{X}_T (= \sup_{0 \leq s \leq T} X_s)$ (or infimum $\underline{X}_T (= \inf_{0 \leq s \leq T} X_s)$). The proposed approach allows us to construct in a universal form efficient Monte Carlo methods for pricing options in Lévy models into which machine learning algorithms can be incorporated as auxiliary blocks. In particular, the paper proves the following theorem, which can be used to calculate $F_{X|\bar{X}}(x, y, T) = \Pr(X_T < x | \bar{X}_T = y)$ – the conditional distribution function of the value X_T relative to the position of the supremum process \bar{X}_T .

Theorem 1. Let T_q be a random time distributed exponentially with an intensity parameter $q > 0$. For non-positive z we define

$$F^-(z, q) = q^{-1} \Pr(\underline{X}_{T_q} < z).$$

Let us fix an even natural number $N = 2n$ and determine the points q_k in accordance with the Gaver-Stehfest algorithm: $q_k = \frac{k \ln(2)}{T}$, $k = 1, \dots, N$.

Then $F_{X|\bar{X}}(x, y, T) \approx \frac{\sum_{k=1}^N \omega_k \cdot F^-(x-y, q_k) \hat{p}_{q_k}^+(y)}{p_T^+(y)}$, $y \geq 0, x \leq y$, where $p_t^+(x)$ и $\hat{p}_q^+(x)$ – the probability densities of \bar{X}_t and \bar{X}_{T_q} , respectively; the weight coefficients $\omega_k := \frac{(-1)^{n+k}}{k \cdot n!} \sum_{j=\lceil (k+1)/2 \rceil}^{\min\{k, n\}} j^{n+1} C_n^j C_{2j}^j C_j^{k-j}$, $[x]$ – integer part x and $C_L^K = \frac{L!}{(L-K)!K!}$ – the number of all K -combinations of a given set of L elements.

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