

Dmitriy F. Kuznetsov (Peter the Great St.-Petersburg Polytechnic University, St.-Petersburg). **Expansions of iterated Itô and Stratonovich stochastic integrals. The case of arbitrary CONS in $L_2[t, T]$.**

Theorem 1 [1, P.205]. *Let $\{\phi_j(x)\}_{j=0}^\infty$ be a CONS in $L_2[t, T]$, $\psi_1(\tau), \dots, \psi_k(\tau) \in L_2[t, T]$. Then for all $k \in \mathbf{N}$ and $i_1, \dots, i_k = 0, 1, \dots, m$*

$$\int_t^T \psi_k(t_k) \dots \int_t^{t_2} \psi_1(t_1) d\mathbf{W}_{t_1}^{(i_1)} \dots d\mathbf{W}_{t_k}^{(i_k)} = \text{l.i.m.}_{p_1, \dots, p_k \rightarrow \infty} \sum_{j_1=0}^{p_1} \dots \sum_{j_k=0}^{p_k} C_{j_k \dots j_1} \times \\ \times \left(\prod_{l=1}^k \zeta_{j_l}^{(i_l)} + \sum_{r=1}^{\lfloor k/2 \rfloor} (-1)^r \sum'_{s=1}^r \mathbf{1}_{\{i_{g_{2s-1}} = i_{g_{2s}} \neq 0\}} \mathbf{1}_{\{j_{g_{2s-1}} = j_{g_{2s}}\}} \prod_{l=1}^{k-2r} \zeta_{j_{q_l}}^{(i_{q_l})} \right),$$

where $\mathbf{W}_\tau^{(i)}$, $i = 1, \dots, m$ are ind. stand. Wiener proc., $\mathbf{W}_\tau^{(0)} = \tau$, $d\mathbf{W}_\tau^{(i)}$ is the Itô differential, $[x]$ is an integer part of x , \sum' is the sum with respect to all permutations of the set $(\{\{g_1, g_2\}, \dots, \{g_{2r-1}, g_{2r}\}\}, \{q_1, \dots, q_{k-2r}\})$, braces mean an unordered set, parentheses mean an ordered set, $\{g_1, g_2, \dots, g_{2r-1}, g_{2r}, q_1, \dots, q_{k-2r}\} = \{1, 2, \dots, k\}$, $\prod_\emptyset \stackrel{\text{def}}{=} 1$, $\sum_\emptyset \stackrel{\text{def}}{=} 0$, $\zeta_j^{(i)} = \int_t^T \phi_j(\tau) d\mathbf{W}_\tau^{(i)}$ are i.i.d. $N(0, 1)$ -r.v.'s ($i \neq 0$), $C_{j_k \dots j_1}$ is the Fourier coef. of $K(t_1, \dots, t_k) = \psi_1(t_1) \dots \psi_k(t_k) \times \mathbf{1}_{\{t_1 < \dots < t_k\}}$, $k \geq 2$ and $K(t_1) = \psi_1(t_1)$, $t_1, \dots, t_k \in [t, T]$, $\mathbf{1}_A$ is the indicator of A .

Theorem 2 [1, Sect.2.18–2.30]. *Let $\{\phi_j(x)\}_{j=0}^\infty$ be a CONS in $L_2[t, T]$. Then*

$$\int_t^T \psi_k(t_k) \dots \int_t^{t_2} \psi_1(t_1) \circ d\mathbf{W}_{t_1}^{(i_1)} \dots \circ d\mathbf{W}_{t_k}^{(i_k)} = \text{l.i.m.}_{p_1, \dots, p_k \rightarrow \infty} \sum_{j_1=0}^{p_1} \dots \sum_{j_k=0}^{p_k} C_{j_k \dots j_1} \prod_{l=1}^k \zeta_{j_l}^{(i_l)},$$

where $\psi_1(\tau), \psi_2(\tau) \in L_2[t, T]$ for $k = 1, 2$ and $\psi_1(\tau), \dots, \psi_k(\tau) \in C[t, T]$, $p_1 = \dots = p_k = p$ for $k \geq 3$; $\circ d\mathbf{W}_\tau^{(i)}$ is the Stratonovich differential; another notations as in Theorem 1.

References

- [1] D.F. Kuznetsov, <https://arxiv.org/abs/2003.14184> (v52 – 2024).