Dmitriy F. Kuznetsov (Peter the Great St.-Petersburg Polytechnic University, St.-Petersburg). **Expansions of iterated Itô and Stratonovich stochastic integrals. The case of arbitrary CONS in $L_2[t,T]$.**

Theorem 1 [1, P.205]. *Let $\{\phi_j(x)\}_{j=0}^{\infty}$ be a CONS in $L_2[t,T]$, $\psi_1(\tau),\ldots,\psi_k(\tau)\in L_2[t,T]$. Then for all $k\in\mathbf{N}$ and $i_1,\ldots,i_k=0,1,\ldots,m$*

$$\int_t^T\psi_k(t_k)\ldots\int_t^{t_2}\psi_1(t_1)d\mathbf{W}_{t_1}^{(i_1)}\ldots d\mathbf{W}_{t_k}^{(i_k)}=\underset{p_1,\ldots,p_k\to\infty}{\text{l.i.m.}}\sum_{j_1=0}^{p_1}\ldots\sum_{j_k=0}^{p_k}C_{j_k\ldots j_1}\times$$
$$\times\left(\prod_{l=1}^{k}\zeta_{j_l}^{(i_l)}+\sum_{r=1}^{[k/2]}(-1)^r\sum{}'\prod_{s=1}^{r}\mathbf{1}_{\{i_{g_{2s-1}}=\ i_{g_{2s}}\neq 0\}}\mathbf{1}_{\{j_{g_{2s-1}}=\ j_{g_{2s}}\}}\prod_{l=1}^{k-2r}\zeta_{j_{q_l}}^{(i_{q_l})}\right),$$

where $\mathbf{W}_\tau^{(i)}$, $i=1,\ldots,m$ are ind. stand. Wiener proc., $\mathbf{W}_\tau^{(0)}=\tau$, $d\mathbf{W}_\tau^{(i)}$ is the Itô differential, $[x]$ is an integer part of x, $\sum{}'$ is the sum with respect to all permutations of the set $(\{\{g_1,g_2\},\ldots,\{g_{2r-1},g_{2r}\}\},\{q_1,\ldots,q_{k-2r}\})$, braces mean an unordered set, parentheses mean an ordered set, $\{g_1,g_2,\ldots,g_{2r-1},g_{2r},q_1,\ldots,q_{k-2r}\}=\{1,2,\ldots,k\}$, $\prod_\emptyset\overset{\text{def}}{=}1$, $\sum_\emptyset\overset{\text{def}}{=}0$, $\zeta_j^{(i)}=\int_t^T\phi_j(\tau)d\mathbf{W}_\tau^{(i)}$ are i.i.d. $N(0,1)$-r.v.'s $(i\neq 0)$, $C_{j_k\ldots j_1}$ is the Fourier coef. of $K(t_1,\ldots,t_k)=\psi_1(t_1)\ldots\psi_k(t_k)\times\mathbf{1}_{\{t_1<\ldots<t_k\}}$, $k\ge 2$ and $K(t_1)=\psi_1(t_1)$, $t_1,\ldots,t_k\in[t,T]$, $\mathbf{1}_A$ is the indicator of A.

Theorem 2 [1, Sect.2.18–2.30]. *Let $\{\phi_j(x)\}_{j=0}^{\infty}$ be a CONS in $L_2[t,T]$. Then*

$$\int_t^T\psi_k(t_k)\ldots\int_t^{t_2}\psi_1(t_1)\circ d\mathbf{W}_{t_1}^{(i_1)}\ldots\circ d\mathbf{W}_{t_k}^{(i_k)}=\underset{p_1,\ldots,p_k\to\infty}{\text{l.i.m.}}\sum_{j_1=0}^{p_1}\ldots\sum_{j_k=0}^{p_k}C_{j_k\ldots j_1}\prod_{l=1}^{k}\zeta_{j_l}^{(i_l)},$$

where $\psi_1(\tau),\psi_2(\tau)\in L_2[t,T]$ for $k=1,2$ as well as $\psi_1(\tau),\ldots,\psi_k(\tau)\in C[t,T]$ and $p_1=\ldots=p_k=p$ for $k=3,4,5$ or for $k\ge 6$, but if condition (2.1302) [1, P.777] *is satisfied in the latter case, $\circ d\mathbf{W}_\tau^{(i)}$ is the Stratonovich differential, another notations as in Theorem* 1.

References

[1] D.F. Kuznetsov, https://arxiv.org/abs/2003.14184 (v53 — 2024).