Martynov Gennady. Cramer-von-Mises test for a Mixture of distributions.(IITP RAS)

The report examines the problem of applying goodness-of-fit criteria such as QM and KS to a five-parameter family of mixtures of normal distribution

$$F(x;\theta) = \rho G(x;m_1,\sigma_1) + (1-\rho)G(x;m_2,\sigma_2),$$

where $\theta = (m_1, \sigma_1, m_2, \sigma_2, \rho)$, $-\infty < x < \infty$, $-\infty < m_1 < \infty$, $\sigma_1 > 0$, $-\infty < m_2 < \infty$, $\sigma_2 > 0$, $\rho > 0$ and G is the standard normal distribution function.

Theorem 1. *The covariance function of the corresponding parametric empirical process under some regularity conditions is*

$$C(t,\theta) = \min(t,\tau) - t\tau - q^\top(t;\theta)I^{-1}(\theta)q(\tau;\theta),$$

where $q(s;\theta) = ((d/d\theta)F(x;\theta))|_{x=F^{-1}(s;\theta)}$, *and* $I(\theta)$ *is the Fisher information marix.*

When calculating the statistics of the mentioned tests, the EML of the parameter vector θ is used. The limit distribution depends on five unknown parameters. Therefore, when determining critical levels, instead of the unknown value of the θ parameter, its estimate is used.