Nasyrov F. S. (Ufa, Russia) Time Optimal Control for Stochastic Differential Equations.

Let $(\Omega, \mathcal{F}, (\mathcal{H}_t)_{t\geq 0}, P)$ be a filtered probability space completed with measure P. Suppose that (H_t) is generated by a random process $V(t)$, $t \geq 0$, with continuous realizations. The time optimal control is to minimize $J(x(,), u(.)) = T$, where $x(t)$ is the solution of the controlled SDE $x(t){=}x(0) + \int_0^t b(u(s), x(s))ds + \int_0^t \sigma(x(s)) * dV(s)$, $t \in [0, T]$, with fixed ends $x(0) = x_0$, $x(T) = x_T$, here the second integral on the right side is a symmetric integral ([1]) over the process $V(t)$, and $u(t) \in U$ are piecewise continuous H_t-adapted controls.

Theorem 1. *Let the functions $b(x, u)$ and $\sigma(x)$ be twice continuously differentiable for all its variables. Suppose that $\sigma(x) \neq 0$ for all x.*

Then the boundary value problem of the maximum principle for this time optimal control is solved as follows:

- *control $u(s) = u(x(s), \psi(s))$ is found from the condition* $(\psi(s), u(x(s), \psi(s))) = \max_{u\in U} b(x(s), \psi(s), u)$;
- *the adjoint variable $\psi(s) = \psi(x(s))$ is determined from the relation* $\psi(x)b(x, u(x, \psi(x))) = \frac{1}{2}$;
- *trajectory $x(s)$ is sought as function $x(s) = \varphi(C(s)+V(s))$, where the function $\varphi(.)$ is found from the relation $\int \frac{d\varphi}{\sigma(\varphi)} = C(s) + V(s)$, and $C(s)$ is the solution to the Cauchy problem for ODE* $\sigma(\varphi(V(s) + C(s)))C'(s) = b(\varphi(C(s) + V(s)), u(\varphi(C(s) + V(s)), \psi(\varphi(C(s) + V(s)))))$, $x_0 = \varphi(C(0) + V(0))$;
- *after the variables $x(.)$, $u(.)$ have been found, the time T that provides the minimum of the functional $J(x(,), u(.))$, is defined as the moment when the trajectory $x(s)$ first reaches the level x_T.*

LITERATURE

1. *Nasyrov F. S.* Local times, symmetric integrals and stochastic analysis. Moscow, Fizmatlit, 2011.