

A simple model for targeting industrial investments with subsidies and taxes

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Consider an investor whose capital is divided into an industrial investment x_t and cash y_t . We use the following model [1] for the dynamics of these components: $x_0 = \bar{x}_0$, $y_0 = \bar{y}_0$,

$$\begin{aligned}x_{t+1} &= AL^\mu((1 - \beta)x_t + (1 + \delta)\alpha y_t)^\nu, \\y_{t+1} &= (1 - \alpha - c)y_t + (1 - \sigma)\beta x_t.\end{aligned}$$

Here $(\alpha, \beta, c) \in [0, 1]^3$, $\alpha + c \leq 1$, $L \geq 0$, are the parameters selected by the investor: α is the fraction of cash, intended for industrial investments, β is the withdrawn fraction industrial investments, c is the fraction of consumed capital, and L is the production factor (“labor”). The parameters $\delta \geq 0$, $\sigma \in [0, 1]$ are selected by the government: δ is fraction of industrial investments paid to the investor as a subsidy, σ is the fraction of withdrawn industrial capital paid by investor to the government due to the taxation. Finally, the positive constants A , μ , ν , where $\mu + \nu < 1$, are the parameters of the Cobb-Douglas production function.

We study a Stackelberg game, corresponding to the asymptotically stable equilibrium (x^*, y^*) of the mentioned dynamical system. For this equilibrium the investor (the follower) computes the revenue $cy^* - pL$, which is the difference between the follower consumption and the total cost of labor, and the government (the leader) computes the cost $\delta\alpha y^* - \sigma\beta x^*$, which is the difference between the amounts of subsidies and taxes. We present an explicit analytical solution of the specified Stackelberg game. Based on this solution, in particular, we introduce the notion of the fair industrial investment level x° , which is costless for the government.

Теорема 1 *The fair industrial investment level equals to*

$$x^\circ = (A\nu^\nu)^{1/(1-\mu-\nu)} \left(\frac{\mu}{p}\right)^{\mu/(1-\mu-\nu)} \left(\frac{\mu}{1-\nu}\right)^{\mu/(1-\mu-\nu)} \left(\frac{1}{\mu+\nu}\right)^{(\mu+\nu)/(1-\mu-\nu)}.$$

The tax and subsidy fractions, inducing x° , depend only on μ and ν .

We show that x° can produce realistic results by the case study of water production in Lahore. Parameter ν is explicitly related to follower’s reaction and can be regarded as known. Assuming that A , p are i.i.d. random variables, it is possible to estimate μ by using the maximum likelihood method.

ЛИТЕРАТУРА

1. Rokhlin, D.B., Ougolnitsky, G.A. A simple model for targeting industrial investments with subsidies and taxes // Mathematics 12, no. 6: 822, 2024.