A simple model for targeting industrial investments with subsidies and taxes

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Consider an investor whose capital is divided into an industrial investment x_t and cash y_t. We use the following model [1] for the dynamics of these components: $x_0 = \overline{x}_0$, $y_0 = \overline{y}_0$,

$$x_{t+1} = AL^{\mu}((1-\beta)x_t + (1+\delta)\alpha y_t)^{\nu},$$
$$y_{t+1} = (1-\alpha-c)y_t + (1-\sigma)\beta x_t.$$

Here $(\alpha, \beta, c) \in [0,1]^3, \alpha + c \leq 1, L \geq 0$, are the parameters selected by the investor: α is the fraction of cash, intended for industrial investments, β is the withdrawn fraction industrial investments, c is the fraction of consumed capital, and L is the production factor ("labor"). The parameters $\delta \geq 0$, $\sigma \in [0,1]$ are selected by the government: δ is fraction of industrial investments paid to the investor as a subsidy, σ is the fraction of withdrawn industrial capital paid by investor to the government due to the taxation. Finally, the positive constants A, μ, ν, where $\mu + \nu < 1$, are the parameters of the Cobb-Douglas production function.

We study a Stackelberg game, corresponding to the asymptotically stable equilibrium (x^*, y^*) of the mentioned dynamical system. For this equilibrium the investor (the follower) maximizes the revenue $cy^* - pL$, which is the difference between the follower consumption and the total cost of labor, and the government (the leader) minimizes the cost $\delta\alpha y^* - \sigma\beta x^*$, which is the difference between the amounts of subsidies and taxes, under an additional constraint $x^* = \underline{x}$. We present an explicit analytical solution of the specified Stackelberg game. Based on this solution, in particular, we introduce the notion of the fair industrial investment level x°, which is costless for the government.

Теорема 1 *The fair industrial investment level equals to*

$$x^\circ = (A\nu^{\nu})^{1/(1-\mu-\nu)} \left(\frac{\mu}{p}\right)^{\mu/(1-\mu-\nu)} \left(\frac{\mu}{1-\nu}\right)^{\mu/(1-\mu-\nu)} \left(\frac{1}{\mu+\nu}\right)^{(\mu+\nu)/(1-\mu-\nu)}.$$

The tax and subsidy fractions, inducing x°, depend only on μ and ν.

We show that x° can produce realistic results by the case study of water production in Lahore. Parameter ν is explicitly related to follower's reaction and can be regarded as known, and μ can be estimated by the maximum likelihood method.

ЛИТЕРАТУРА

1. Rokhlin, D.B., Ougolnitsky, G.A. A simple model for targeting industrial investments with subsidies and taxes // Mathematics 12, no. 6: 822, 2024.