A REMARK ABOUT ITO'S FORMULA FOR WIENER PROCESS.

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We extend the classical Ito's formula for a smooth function V of the Wiener process to the case when $v = V' \in L_{2,loc}(R)$. We show that in this general case the Ito's correction term has a classical form if $v' = V''$ is treated in the sense of generalized function theory (in a distribution sense). Earlier, in [1], under the same conditions, another form of the correction term was obtained.

Theorem 1. *Let $v \in L_{2,loc}(\mathbb{R})$, the function V is an antiderivative of v ($V' = v$), distribution v' is a derivative of v in a distribution sense. Let $\{v_\varepsilon\}$ be a family of absolute continuous functions such that for every $N > 0$ the condition $\lim_{\varepsilon\to 0} \|v_\varepsilon - v\|_{L_2[-N,N]} = 0$ holds. Under these conditions we have: 1. There exists $\lim_{\varepsilon\to 0+} \int_0^t v'_\varepsilon(w(\tau))\, d\tau$ (in probability) and this limit does not depends on the family v_ε. For this limit we use the notation $\int_0^t v'(w(\tau))\, d\tau$. 2. The Ito's formula is valid $V(w(t)) = V(w(0)) + \int_0^t v(w(\tau))\, dw(\tau) + \frac{1}{2}\int_0^t v'(w(\tau))\, d\tau$.*

References

[1] H. Föllmer , Ph. Protter , A. N. Shiryayev, Quadratic Covariation and an Extension of Ito's Formula , *Bernoulli,* 1/2(1995), 149–169.

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