S.A. Gondin, E.A. Pchelintsev (National Research Tomsk State University, Tomsk, Russia). Quantile Hedging of Asian Options on (B,S)-Markets¹.

We develop a method for quantile pricing of Asian call options using the approach proposed in [1]. A continuous model of a financial (B, S)-market on the time interval [0, 1] with transaction costs is considered. It is assumed that the price of the risk-free asset is constant: $B_t = 1$ for all $t \in [0, 1]$, and the dynamics of the risky asset $(S_t)_{0 \le t \le 1}$ is described by the SDE: $dS_t =$ $S_t(mdt + \sigma dW_t)$. The problem of reducing the cost of the Asian-type option with payoff function $H = \left(\int_0^1 S_t dt - K\right)_+$ is studied. In the absence of transaction costs, the following result is obtained.

Theorem 1. Let $0 < \varepsilon < 1$. Then the quantile price of the Asian option is

$$C_{\varepsilon} = \mathbb{E}\left[H\mathbf{1}_{A}\right].$$

Here the expectation is taken with respect to a special measure from [2]. If $m \leq \sigma^2$, then $A = \{W_1 < b\}$, and if $m > \sigma^2$, then $A = \{W_1 < b_1\} \cup \{W_1 > b_2\}$. The values of b, b_1 and b_2 are determined from the equation $\mathbb{P}(A) = 1 - \varepsilon$.

Next, the case with transaction costs is investigated. Using Leland's approach [3] and replacing the volatility parameter with $\hat{\sigma}^2 = \sigma^2 + \sigma \sqrt{n}\kappa_n \sqrt{\frac{8}{\pi}}$, where κ_n is the proportional transaction coefficient and n is the number of portfolio revisions, the following result is obtained.

Theorem 2. If $\lim_{n \to \infty} \sqrt{n}\kappa_n = \infty$, then $\lim_{n \to \infty} \widehat{C}_{\varepsilon} = S_0$. If $\lim_{n \to \infty} \sqrt{n}\kappa_n = 0$, then $\lim_{n \to \infty} \widehat{C}_{\varepsilon} = C_{\varepsilon}$.

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