**Dmitriy F. Kuznetsov** (Peter the Great St.-Petersburg Polytechnic University, St.-Petersburg, Russia). **Latest results on a new approach to series expansion of iterated Stratonovich stochastic integrals. Multiplicities 1 to 8 and beyond.**

**Theorem 1** [1, Sect. 2.22, 2.27, 2.29, 2.31]. *Let* $\psi_1(\tau),\ldots,\psi_k(\tau)\in C[t,T]$ *and* $\{\phi_j(x)\}_{j=0}^{\infty}$ *is an arbitrary CONS in* $L_2[t,T]$. *Moreover, let one of the conditions* (2.1294), (2.1310), (2.1341) [1] *is fulfilled. Then*

$$\int_t^T \psi_k(t_k)\ldots\int_t^{t_2}\psi_1(t_1)\circ d\mathbf{W}_{t_1}^{(i_1)}\ldots\circ d\mathbf{W}_{t_k}^{(i_k)}=\underset{p\to\infty}{\text{l.i.m.}}\sum_{j_1,\ldots,j_k=0}^{p}C_{j_k\ldots j_1}\zeta_{j_1}^{(i_1)}\ldots\zeta_{j_k}^{(i_k)}, \tag{1}$$

*where* $k\in\mathbf{N}$, $\zeta_j^{(i)}=\int\limits_t^T\phi_j(\tau)d\mathbf{W}_\tau^{(i)}$ *are i.i.d.* $N(0,1)$*-r.v.'s* $(i\neq 0)$, $d\mathbf{W}_\tau^{(i)}$ *and* $\circ\, d\mathbf{W}_\tau^{(i)}$ *are the Itô and Stratonovich differentials, respectively,* $C_{j_k\ldots j_1}$ *is the Fourier coefficient corresponding to* $K(t_1,\ldots,t_k)=\psi_1(t_1)\ldots\psi_k(t_k)\mathbf{1}_{\{t_1<\ldots<t_k\}}$ $(k\geq 2)$ *and* $K(t_1)=\psi_1(t_1)$ $(k=1)$, $t_1,\ldots,t_k\in[t,T]$, $i_1,\ldots,i_k=0,1,\ldots,m$, $\mathbf{W}_\tau^{(i)}$ $(i=1,\ldots,m)$ *are independent standard Wiener processes,* $\mathbf{W}_\tau^{(0)}=\tau$.

**Theorem 2** [1, Sect. 2.1.4, 2.24, 2.32–2.34]. *Let* $\{\phi_j(x)\}_{j=0}^{\infty}$ *as in Theorem* 1. *Then the expansion* (1) *is valid without* (2.1294), (2.1310), (2.1341) [1] *in the following three cases.*

1. $\psi_1(\tau),\psi_2(\tau)\in C[t,T]$ $(k=1,2)$.
2. $\psi_1(\tau)=(\tau-t)^{p_1},\ldots,\psi_4(\tau)=(\tau-t)^{p_4}$, $p_1,\ldots,p_4=0,1,2,\ldots$ $(k=3,4)$.
3. $\psi_1(\tau),\ldots,\psi_6(\tau)\equiv 1$ $(k=5,6)$.

**Theorem 3** [1, Sect. 2.36, 2.37]. *Let* $\{\phi_j(x)\}_{j=0}^{\infty}$ *is a CONS of Legendre polynomials or trigonometric functions* (*Fourier basis*) *in* $L_2[t,T]$. *Then* (1) *is valid without* (2.1294), (2.1310), (2.1341) [1] *for the case* $\psi_1(\tau),\ldots,\psi_8(\tau)\equiv 1$ $(k=7,8)$.

Theorems 1–3 are useful for constructing high-order strong numerical methods for systems of Itô SDEs with non-commutative noise.

References

[1] D.F. Kuznetsov, Strong Approximation of Iterated Itô and Stratonovich Stochastic Integrals Based on Generalized Multiple Fourier Series. Application to Numerical Solution of Itô SDEs and Semilinear SPDEs. arXiv:2003.14184v63 [math.PR], 2025, 1208 pp. DOI: https://doi.org/10.48550/arXiv.2003.14184