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Asymptotic properties of some random variables defined by stopping times of Brownian motion.

In [1] and [2] expressions were found for the Laplace transforms of certain stopping times of Brownian motion with drift (related to the drawdown and drawup of the process). However, they are rather cumbersome and complex for further analysis. Nevertheless, as recently proved by the first author, their study can be reduced to the study of three families of random variables whose Laplace transforms have a much simpler form:

$$\mathbf{E}e^{-\lambda X^a} = \frac{a\sqrt{2\lambda}}{\operatorname{sh}(a\sqrt{2\lambda})}, \ \mathbf{E}e^{-\lambda Y^{a,y}} = \frac{a\operatorname{sh}(y\sqrt{2\lambda})}{y\operatorname{sh}(a\sqrt{2\lambda})}, \ \mathbf{E}e^{-\lambda Z^{a,z}} = e^{z/a}e^{-z\sqrt{2\lambda}\operatorname{cth}(a\sqrt{2\lambda})},$$

where $a > y > 0$, $z > 0$. The following statement holds.

Theorem.

$$\lim_{t\to+\infty}\frac{\log \mathrm{P}\{X^a > t\}}{t} = \lim_{t\to+\infty}\frac{\log \mathrm{P}\{Y^{a,y} > t\}}{t} = \lim_{t\to+\infty}\frac{\log \mathrm{P}\{Z^{a,z} > t\}}{t} = -\frac{\pi^2}{2a^2}.$$

The proof of the theorem makes essential use of Tauberian theorem 2 from [3] and criterion 2 from [4].

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