A. A. Murzintseva, E. A. Pchelintsev (TSU, Tomsk, Russia). Robust portfolio optimization in financial market models with transaction $costs^1$.

We consider the problem of optimal control in the Black – Scholes financial market with transaction costs under logarithmic utilities. The risk-free asset evolves according to the equation $dB_t = r(t)B_tdt$, $B_0 = 1$, and risk asset – $dS_t = \mu(t)S_tdt + \sigma(t)S_tdW_t$, $S_0 > 0$. Here the interest rate r(t), the trend $\mu(t)$ and the volatility $\sigma(t)$ are continuous $[0,T] \to \mathbb{R}$ functions, $(W_t)_{t\geq 0}$ is a standard Wiener process. Let $\lambda = (r(t), \mu(t), \sigma(t))_{0\leq t\leq T}$ is a market parameter from some compact $\Lambda \subset \mathbf{C}^1([0,T], \mathbb{R}^3)$. The problem is solved by the method of stochastic dynamic programming. As in [1], a verification theorem is proved, which guarantees that the solution of the Hamilton-Jacobi-Bellman equation gives an optimal strategy $v^* = (\theta^*(t), c^*(t))_{0\leq t\leq T}$, for which explicit formulas for the share of investment in the risky asset and the level of consumption are obtained. Then, using the Leland – Lépinette approach [2], the resulting strategy is discretized to $\nu^{(n)}$, i.e. it is considered at the moments $t_k = kT/n$ for small transaction costs ($\kappa_n = o(n^{-1/2})$ as $n \to \infty$) and for large transaction costs ($\sqrt{n}\kappa_n$ does not tend to 0) – at the moments $t_k = (\frac{k}{n})^q T$, $k = \overline{1, n}$, where $q = q_n \geq 1$ such that $\lim_{n\to\infty} (\frac{q_n}{n} + \kappa_n) \ln q_n = 0$. The theorem has been proven.

Theorem. Strategy $\nu^{(n)}$ is asymptotically robust and optimal, i.e.

$$\lim_{n \to \infty} \sup_{\lambda \in \Lambda} \left| J_{\lambda}(x, \nu^{(n)}) - J_{\lambda}^{*}(x) \right| = 0,$$

where $J_{\lambda}^{*}(x)$ is optimal value of the objective function $J(x, v) = \mathbf{E}_{x,\lambda} \left[\int_{0}^{T} \ln(c_u X_u) du + \ln X_T \right], x > 0$ is an initial endowment and X_t is a portfolio wealth at the moment t.

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