Fixed-width conference interval estimation of functionals of an unknown distribution function.

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Consider a sequence of independent, identically distributed random variables $\xi_1, \xi_2, ..., \xi_n$ with a distribution function $F(x)$. In order to estimate functional $\theta(F)$ of this distribution function $F(x)$ we consider a statistical estimators $\theta_n(F) = \theta_n(\xi_1, \xi_2, ...\xi_n)$, which have finite expectations and, therefore, can be represented in the following form, $\theta_n(F) = \theta(F) + \sum_{k=1}^{n} Y_n(F, \xi_k) + Z_n$, where $Y_n(F, \xi_k), 1 \leq k \leq n$ and $Z_n = E(\theta_n(F)) - \theta(F)$ which satisfy the following condition: $n^{\alpha} \sum_{k=1}^{n} Y_n(F, \xi_k) \xrightarrow{D} N(0, \sigma^2(F))$ and $n^{\alpha} Z_n \xrightarrow{P} 0$ as $n \to \infty$, here $0 < \sigma^2(F) < \infty$, $\alpha > 0$.

Let be $0 < \gamma < 1$, $a = \Phi^{-1}((1 + \gamma)/2)$, $\Phi(y) = (2\pi)^{-1/2} \int_{-\infty}^{y} e^{-t^2/2} dt$ and $V_n^2 = V_n^2(\xi_1, \xi_2, ..., \xi_n)$ consistent estimators of $\sigma^2(F)$. Introduce the stopping time

$$N_{\varepsilon} = inf(n \geq 1 : n \geq (a^2 V_n^2/\varepsilon^2)^{2\alpha^{-1}}), \varepsilon > 0.$$

Theorem 1. 1) If $V_n^2 \xrightarrow{P} \sigma^2(F)$ as $n \to \infty$, then $N_{\varepsilon}/n_{\varepsilon} \xrightarrow{P} 1$ as $\varepsilon \to 0$. 2) If $n^{\alpha}(E(\theta_n(F)) - \theta(F)) \to 0$ as $n \to \infty$ and $\xi_{\varepsilon}(t)$, $t \in [0, 1] \xrightarrow{J} W(t)$, $t \in [0, 1]$ as $n \to \infty$, here $\xi_{\varepsilon}(t) = \sigma^{-1}(F) n_{\varepsilon}^{\alpha} \sum_{k=1}^{[n_{\varepsilon} t]} Y_{n_{\varepsilon}}(F, \xi_k)$, $t \in [0, 1]$ is the Wiener process, symbol $\xrightarrow{J}$ denotes convergence in Skorokhod J-topology, then $\sigma^{-1}(F) N_{\varepsilon}^{\alpha}(\theta_{N_{\varepsilon}}(F) - \theta(F)) \xrightarrow{D} N(0, 1)$ as $\varepsilon \to 0$ and $\lim_{\varepsilon \to 0} P(\theta(F) \in I(N_{\varepsilon})) \geq \gamma$.

REFERENCES

[1] Rakhimova G. G.,"Application of limit theorems for superposition of random functions to sequential estimation", In the book: Silvestrov S., Rancic M., Malyarenko A. (eds.) Stochastic Processes and Applications. Chapter 7, p.148-154. Springer, Proceedings in Mathematics & Statistics, 271, Springer, Cham, 2018.