N. Ratanov (Chelyabinsk State University). **Fractional telegraph equation and fractionally integrated telegraph processes** [1]**.**

The fractionally integrated telegraph process $\mathcal{F}^\alpha(t)$ is defined by

$$\mathcal{F}^\alpha(t) = I^\alpha[c_{\varepsilon(\cdot)}](t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1} c_{\varepsilon(s)}\mathrm{d}s, \qquad 0 < t < \infty.$$

Here $\alpha \in (0, 1]$ is the order of integration.

Such processes are widely used in various fields, including very specific areas such as models of the physio-chemical mechanism triggering muscle contraction, see, for example, [1].

The mean $\mathfrak{M}^{(1)}(t)$ and the second moment $\mathfrak{M}^{(2)}(t)$ of $\mathcal{F}^\alpha(t)$ can be expressed explicitly.

Futhermore, in the symmetric case $c_0 = -c_1 = c > 0$ $\lambda_0 = \lambda_1 = \lambda > 0$, we obtain the following asymptotics.

Theorem 1. *For $\lambda t \to +\infty$, we have*

$$\mathfrak{M}^{(1)}(t) \sim \frac{t^{\alpha-1}}{2\Gamma(1+\alpha)\Gamma(\alpha)} \cdot \frac{c}{\lambda}$$

and

$$\mathfrak{M}^{(2)}(t) \sim \begin{cases} \frac{t^{2\alpha-1}}{(2\alpha-1)\Gamma(1+\alpha)\Gamma(\alpha)^2} \cdot \frac{c^2}{\lambda}, & \text{if } \alpha > 1/2, \\ \frac{2\ln(2\lambda t)}{\pi^{3/2}} \cdot \frac{c^2}{\lambda}, & \text{if } \alpha = 1/2, \\ \frac{2K_\alpha}{\Gamma(1+\alpha)\Gamma(\alpha)} \cdot \frac{c^2}{\lambda^{2\alpha}}, & \text{if } 0 < \alpha < 1/2, \end{cases}$$

where

$$K_\alpha = \int_0^\infty u^{2\alpha-1}\mathrm{e}^{-2u}\Phi(\alpha, 1+\alpha; 2u)\mathrm{d}u.$$

The process $\mathcal{F}^\alpha$ defined above is not a Markov process. However, it can be expressed using the Kac-Ornstein-Uhlenbeck process, which is Markov, see [2].

Theorem 2. *The fractionally integrated telegraph process $\mathcal{F}^\alpha = \mathcal{F}^\alpha(t)$ is represented through the infinite-dimensional Kac-Ornstein-Uhlenbeck process $Z_\beta(t)$, $\beta \in (0, \infty)$,*

$$\mathcal{F}^\alpha(t) = \frac{1}{\Gamma(1-\alpha)\Gamma(\alpha)}\int_0^\infty \beta^{-\alpha} Z_\beta(t)\mathrm{d}\beta.$$

Here $Z_\beta(t)$ is defined by the integral equation,

$$Z_\beta(t) = -\beta\int_0^t Z_\beta(s)\mathrm{d}s + \mathcal{T}(t), \qquad t \geq 0,$$

where $\mathcal{T}(t) = \int_0^t c_{\varepsilon(s)}\mathrm{d}s$ is the integrated telegraph process.

REFERENCES

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[2] Ratanov N. Ornstein-Uhlenbeck processes of bounded variation. *Methodology and Computing in Applied Probability*, 2021; 23, 925–946.

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