On minimal total weight of k disjoint spanning trees

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We consider stochastic problem of extremal type on discrete structures. Let G_n denote a complete graph on n vertices with weighted edges: we assign an independent random weight with uniform distribution on $[0,1]$ to every edge. The problem is to find the distribution of minimal weight of a spanning tree in G_n. In 1985 A. Frieze proved that this random variable converges in probability to $\zeta(3) = \sum_{n=1}^{\infty} n^{-3}$. We consider the generalization of the problem concerning the minimal total weight of k disjoint spanning trees in G_n. The main result is formulated in the following theorem.

Theorem. *Let $k > 1$ be a fixed integer. Let $F(x)$ denote a distribution function of a positive random variable with $F(x) \sim a \cdot x$ as $x \to 0+$. Suppose that independent random weights with distribution $F(x)$ are assigned to the edges of G_n. Then the minimal total weight of k disjoint spanning trees in G_n converges in probability to the value*

$$\frac{1}{a} \int_{0}^{\infty} x \left(1 - \beta_k^2(x)\right) dx,$$

where $\beta_k(x) = 0$ for $x < 2c_k$, and otherwise is a solution of the equation $\beta_k(x) = 1 - \mathsf{P}(\xi \geq k)$, $\xi \sim \mathrm{Pois}(\beta_k(x)x)$. The value c_k is also a solution of the equation $c_k \frac{\mathsf{P}(\eta \geq k-1)}{\mathsf{P}(\eta \geq k)} = k$, $\eta \sim \mathrm{Pois}(2c_k)$.

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