Suchkova D. A. (UUST, Ufa, Russia). On the Cauchy problem for generalized stochastic and deterministic Korteweg-de Vries equations.

Let $V(t), t \in [0, T]$, be a random process with continuous realizations with probability 1. For the generalized stochastic Korteweg-de Vries (GKdV) equation $d(u)_t + (f(u))_x * dV(t) + u_{xxx}dt = 0$ (1), with noise in the nonlinear term in the form of a symmetric integral [1], where $u(x, 0) = u_0$, $(x, t) \in R \times [0, T]$, the following result is obtained. Via $Ai(z) = \frac{1}{\pi} \int_0^{\infty} \cos\left(\frac{y^3}{3} + yz\right) dy$ will denote the Airy function of the first kind.

Theorem. Let the function
$$u(x, t, V(t)) = (3t)^{-\frac{1}{3}} \int_{-\infty}^{+\infty} Ai\left(\frac{x-y}{(3t)^{\frac{1}{3}}}\right) u(y, 0, V(t)) dy$$
 (2),

where u(y, 0, V(t)) is the solution to the Cauchy problem for the equation $\frac{\partial}{\partial v}u(x, t, v)|_{v=V(t)} + f'(u(x, t, V(t)))\frac{\partial}{\partial x}u(x, t, V(t)) = 0$ for t = 0, with the initial condition $u(x, 0, 0) = u_0(x)$, $f(u) \in C^2(R)$, $u_0(x) \in C^3(R)$, then the Cauchy problem for the original equation (1) has a solution u(x, t, V(t)) [2], which is represented as relation (2).

Note. The case when noise affects both the nonlinear and dispersion terms, cases with noise in the dispersion term or on the right-hand side of the equation are studied in [3], [4].

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