Accumulation process asymptotics for random works in domain with borders

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Consider a counting system of particles in the band $\Pi = Z \times \{0, 1, 2, ..., N-1\}$. At the initial moment of time exactly one particle is located at each point of the set $\Pi_+ = \mathbb{N} \times \{0, 1, 2, ..., N-1\}$. The dynamics of each particle is determined by a homogeneous and discrete in time, irreducible and non-periodic Markov chain $\xi_t = (\xi_t^1, \xi_t^2)$ with a state space Π and with transition probabilities

$p_{l_1 j_1}^{lj} = P(\xi_{t+1} = (l_1, j_1) | \xi_t = (l, j))$, $(l, j), (l_1, j_1) \in \Pi$, satisfying the conditions: $p_{l_1 j_1}^{lj} = 0$ for $|l - l_1| > 1$ and $p_{l_1 j_1}^{lj} = p_{l_1 - l, j_1}^{0j}$. It is assumed that random walks of different particles are independent and identically distributed. Denote by $n(t)$ the number of particles that have entered the set $\Pi_0 = \{0\} \times \{0, 1, 2, ..., N-1\} \subset \Pi$ at least once by time t. Define an ergodic Markov chain with the set states $\{0, 1, 2, ..., N-1\}$ and with transition probabilities $q_{j_1}^j = p_{l+1 j_1}^{lj} + p_{l j_1}^{lj} + p_{l-1 j_1}^{lj}$.We denote by π_j the stationary distribution of this chain, $v = \sum_{j=0}^{N-1} \pi_j m_j$, $m_j = E(\xi_{t+1}^1 - \xi_t^1 | \xi_t^1 = 0, \xi_t^2 = j), j \in \{0, 1, 2, ..., N-1\}$.

Theorem 1.

1) In case $v < 0$ exists $\frac{n(t)}{t} \xrightarrow[t\to\infty]{P} -Nv$, $t \to \infty$

2) In case $v = 0$ exists $\frac{En(t)}{\sqrt{t}} \xrightarrow[t\to\infty]{} c$, where constant $c > 0$ could be found explicitly.

Literature

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