BRANCHING WIENER PROCESS WITH SINGULAR BRANCHING RATE.

SMORODINA N.V.

We construct branching one-dimensional Wiener process $X_x(t)$, starting from single particle at $x \in \mathbb{R}$, the branching intensity of which is a distribution $q_\alpha(x) = -|x|^{-1-\alpha}$, where $\alpha \in (0, 1/2)$. The distribution q_α acts on a test function f as $\int (f(0) - f(x))|x|^{-1-\alpha}\, dx$. For this process we consider operator semigroup P^t, acting on the function $\varphi \in L_2(\mathbb{R})$ as $[P^t\varphi](x) = \mathbb{E}\sum_{y \in X_x(t)} \varphi(y)$, where the summation is carried out over all particles of the branching process $X_x(t)$, existing in the system at time t. It is shown that the generator of the semigroup P^t is the operator $\mathcal{A} = \frac{1}{2}d^2/dx^2 + q_\alpha(x)$, understanding in the sense of the paper [1]. The spectrum $\sigma(\mathcal{A})$ of the operator $\mathcal{A}$ consists of a negative semi-axis $(-\infty, 0]$ and a unique positive eigenvalue λ_0, which corresponds to a positive eigenfunction $\varphi_0,\ \|\varphi_0\|_2 = 1$.

Theorem 1. *Suppose that $\varphi \in L_2(\mathbb{R})$, by φ_1 denote its projection onto the orthogonal complement to φ_0. Then for any $t > 0$ the inequality holds $\|P^t\varphi - e^{\lambda_0 t}(\varphi, \varphi_0)\varphi_0\|_2 \leqslant \|\varphi_1\|_2$.*

References

[1] A. M. Savchuk, A. A. Shkalikov, Sturm-liouville operators with singular potentials, *Mathematical Notes,* 1999, Volume 66, Issue 6, Pages 741–753

PDMI RAS, SPbSU St.-Petersburg, Russia.
E-mail address: smorodina@pdmi.ras.ru