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ABSTRACTS OF TALKS GIVEN AT THE 3RD INTERNATIONAL CONFERENCE ON STOCHASTIC METHODS*

(*Translated by A. R. Alimov*)

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The Third International Conference on Stochastic Methods (ICSM-3) was held June 3–9, 2018 in the village of Divnomorskoe (near the town of Gelendzhik) at the Raduga sports and fitness center of the Don State Technical University. Like ICSM-1 and -2, ICSM-3 was organized by the Steklov Mathematical Institute of RAS (Department of Theory of Probability and Mathematical Statistics), Moscow State University (Department of Probability Theory), and the Don State Technical University (Department of Higher Mathematics), the main university of Rostov-on-Don. The conference chairman was A. N. Shiryaev, a member of the Russian Academy of Sciences, who also headed the Organizing Committee and the Program Committee.

Members of the conference committees were as follows. **Organizing Committee:** I. V. Pavlov (Deputy Chairman), A. V. Bulinski, M. V. Zhiltukhin, F. S. Nasyrov, T. B. Tolozova, V. V. Shamraeva, and E. Eberlein; **Program Committee:** A. A. Gushchin (Deputy Chairman), Yu. E. Gliklikh, Yu. M. Kabanov, D. B. Rokhlin, and V. V. Ulyanov. Organizational issues were solved at the conference by the **Local Organizing Committee:** I. V. Pavlov (Chairman), A. G. Danekyants, N. P. Krasiy, S. I. Uglich, and I. V. Tsvetkova.

In addition to scientists from Russia, scientists from the USA, France, Germany, the Netherlands, Portugal, Saudi Arabia, Romania, Bulgaria, and Uzbekistan took part in the conference. Eighteen lectures and 46 talks were given. The themes of the lectures were as follows:

– *E. Eberlein*, Multiple curve interest rate modeling allowing for negative rates;

– *Yu. Kabanov*, On a multi-asset version on the Kusuoka limit theorem of option superreplication under transaction costs;

– *M. L. Esquível*, From an ordinary differential equation model to an open population Markov chain model, via stochastic differential equations; models for HIV infection in individuals and populations;

– *E. Lepinette, J. Baptiste, L. Carassus*, Pricing without martingale measure;

– *Yu. Gliklikh*, Investigation of completeness of stochastic flows generated by equations with current velocities;

– *Ya. Belopolskaya*, Systems of forward and backward nonlinear Kolmogorov equations;

– *N. Smorodina*, Approximation of an evolution operator by mathematical expectations of functionals of Poisson random fields;

– *F. Nasyrov*, A deterministic approach to stochastic maximum principle;

– *B. Dupire*, Functional Itô calculus and characterization of attainable claims;

– *M. Grigorova*, Doubly reflected BSDEs and nonlinear Dynkin games: Beyond right-continuity;

– *S. Borovkova*, Stochastic time change for asset price modeling;

– *V. Ulyanov*, Nonasymptotic estimates for the closeness of Gaussian measures on the balls;

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– *A. Bulinski*, Asymptotic behavior of entropy estimates;
– *A. Gushchin*, Joint distributions of increasing processes and their compensators, single jump martingales, and the Skorokhod embedding;
– *V. Afanasyev*, Boundary problems for a random walk in a random environment;
– *M. Platonova and K. Ryadovkin*, A branching random walk on graphene lattice;
– *A. Tikhomirov*, Local limit theorems for random matrices;
– *D. Rokhlin*, Q-learning in a stochastic Stackelberg game.

The Organizing Committee arranged photo sessions of the participants and an excursion to the village of Abrau-Durso and Pshad waterfalls.

Financial support from the Don State Technical University (B. Ch. Meskhi, rector), the Russian Foundation for Basic Research (grant 18-01-20032-g), and the Federal Agency for Scientific Organizations contributed immeasurably to the success of the conference.

A. N. Shiryaev, I. V. Pavlov

Following are the abstracts of the talks and lectures given at the conference.

V. I. Afanasyev (Moscow, Russia). **Boundary problems for a random walk in a random environment.** [1]

Let $\{X_k,\, k \geqslant 0\}$ be a random walk in a random environment. We assume that the random environment is a sequence of i.i.d. random vectors (p_i, q_i), $i \in \mathbf{Z}$, where $p_0 + q_0 = 1$, $p_0 > 0$, $q_0 > 0$. By definition, this means that for a fixed random environment, the sequence $\{X_k,\, k \geqslant 0\}$ is a discrete Markov chain starting from 0 with the set of states $\mathbf{Z}$ and transition probabilities p_{ij} such that $p_{i,i+1} = p_i$, $p_{i,i-1} = q_i$, $i \in \mathbf{Z}$.

Assume that

$$\mathbf{E} \ln \frac{q_0}{p_0} = 0, \quad \mathbf{E} \ln^2 \frac{q_0}{p_0} =: \sigma^2, \qquad \sigma^2 \in (0, +\infty). \tag{1}$$

We set $T_n = \min\{k \in \mathbf{N}\colon X_k = n\}$, where $n \in \mathbf{Z}$. The random variable $\ln T_n$, as $n \to \infty$, is well studied. In particular, limit theorems are established for this variable in both cases when condition (1) is satisfied and when $\mathbf{E} \ln^2(q_0/p_0)$ is infinite (see, for example, [1]).

Consider a two-boundary problem concerning the first exit of the sequence $\{X_k,\, k \geqslant 0\}$ from the interval $(-\lfloor an \rfloor, \lfloor bn \rfloor)$, where $a, b > 0$. The following results hold (see [2]).

THEOREM 1. *If condition* (1) *holds and if* $a, b > 0$, *then*

$$\lim_{n\to\infty} \mathbf{P}(T_{\lfloor bn \rfloor} < T_{-\lfloor an \rfloor}) = \frac{2}{\pi} \arctan \sqrt{\frac{a}{b}}.$$

Consider for $x, y > 0$ and $k, l \in \mathbf{Z}$ the triangle

$$S_{k,l}(x,y) = \left\{(u,v) \in \mathbf{R}^2\colon \frac{u}{x} + \frac{v}{y} \leqslant k + l + 1,\ u \geqslant kx,\ v \geqslant ly\right\}.$$

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We set $G(x,y)=\bigcup_{k,l\in\mathbf{Z}} S_{4k,2l+1}(x,y)$, $D(x,y)=\bigcup_{k,l\in\mathbf{Z}} S_{4k+2,2l+1}(x,y)$. Let ξ_1, ξ_2 be independent random variables with a standard normal distribution. We set

$$F(x,y)=\mathbf{P}\big((\xi_1,\xi_2)\in G(x,y)\big)-\mathbf{P}\big((\xi_1,\xi_2)\in D(x,y)\big).$$

THEOREM 2. *If condition* (1) *holds and if* $a,b>0$, *then, for all* $x>0$,

$$\lim_{n\to\infty}\mathbf{P}\biggl(\frac{\ln T_{\lfloor bn\rfloor}}{\sigma\sqrt{n}}<x\,\biggm|\,T_{\lfloor bn\rfloor}<T_{-\lfloor an\rfloor}\biggr)=2\pi\frac{F(x/\sqrt{b},x/\sqrt{a})}{\arctan(\sqrt{a/b})}.$$

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S. Albosaily (Rouen, France), **S. Pergamenshchikov** (Rouen, France; Tomsk, Russia). **Optimal investment and consumption for Ornstein–Uhlenbeck spread financial markets with power utility.**

We study an investment/consumption optimization problem for financial markets of Brownian motion type generated by the differences in risky financial assets for investor who can trade in one risk-free bond and multiple stocks. The goal of the investor is to allocate money in such a way that expected utility from terminal wealth is maximized. The model of the financial market "spread" we use is driven by Ornstein–Uhlenbeck processes

$$\mathrm{d}S_t=-\kappa S_t\,\mathrm{d}t+\sigma\,\mathrm{d}W_t, \tag{1}$$

where W_t is the standard Wiener process and σ is the $d\times d$ volatility matrix. The wealth process in this case is given by

$$\mathrm{d}X^\upsilon_t=(rX^\upsilon_t-\kappa_1\alpha_tS_t-c_t)\,\mathrm{d}t+\alpha_t\sigma\,\mathrm{d}W_t, \tag{2}$$

where r is the interest rate, $\kappa_1=\kappa+r$, $\upsilon=(\alpha_t,c_t)_{0\leqslant t\leqslant T}$ is the financial strategy, α_t is the investment in the risky asset (1), and c_t is the consumption. This model was proposed in [3] for a pure optimal investment of a one-dimensional problem. Several studies used the notion of spread to examine the behavior of financial markets. For example, for precious metals, the spread between gold and silver, and the spread between the gold futures market and the U.S. Treasury bill futures market have been examined [4]. Our goal is to maximize the following objective function:

$$\sup_{\upsilon\in\mathscr{V}}\mathbf{E}_{x,s}\biggl(\int_0^T c_u^\gamma\,\mathrm{d}u+\varpi(X^\upsilon_T)^\gamma\biggr), \tag{3}$$

where $\mathscr{V}$ is the set of all admissible financial strategies introduced in [1], $\varpi>0$ is some fixed constant, and $\mathbf{E}_{x,s}$ is the conditional expectation with respect to $X^\upsilon_0=x$ and $S_0=s$. For this problem, similarly to [2], we use the stochastic dynamic programming method and the Feynman–Kac representation. The main result is the following theorem.

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THEOREM 1. *Assume that* $\varpi \geqslant (16T/\pi)^{1-\gamma}$ *and* $0 < \gamma < 1/4$; *then the HJB equation*

$$\begin{cases} z_t(\varsigma,t) + H\big(\varsigma,t,\partial z(\varsigma,t),\partial^2 z(\varsigma,t)\big) = 0, & t \in [0,T], \\ z(\varsigma,T) = \mathbf{h}(x), \end{cases} \qquad \varsigma = (x,s),$$

has the solution defined by

$$z(\varsigma,t) = \varpi x^\gamma U(s,t) \quad \textit{and} \quad U(s,t) = \exp\left\{\frac{1}{2}s^2 g(t) + Y(s,t)\right\},$$

where Y *is a unique solution in* $\mathscr{X}$ *of*

$$\Psi_Y(s,t) = \frac{1}{2}\gamma Y_s^2(s,t) + \frac{1}{2}\sigma^2 g(t) + r\gamma + (1-\gamma)\varpi^{1/(\gamma-1)} \exp\left\{-\frac{1}{1-\gamma}\left(\frac{1}{2}s^2 g(t) + Y(s,t)\right)\right\} = 0$$

and is the fixed point for the Feynman–Kac mapping, i.e., $Y = \mathscr{L}_Y$.

By making use of this theorem the optimal investment and consumption strategies are found.

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U. A. Alekseeva (Ekaterinburg, Russia)**. On a relation between Brownian sheet, *Q*-Wiener, and cylindrical Wiener processes.** [2]

We are concerned with the problem of small transverse oscillations of a string under the influence of external random impulses. It is shown that the process describing external influences is a Brownian sheet $\{W(t,x),\, t \geqslant 0,\, x \in [0,l]\}$, i.e., a Gaussian random two-parameter function with zero mean and with $\mathrm{Cov}(W(t_1,x_1),W(t_2,x_2)) = \min\{t_1,t_2\}\min\{x_1,x_2\}$ (see [1]). It is proved that the Brownian sheet is a Q-Wiener process in $H = L_2[0,l]$ with $(Qh)(x) = \int_0^l K(x,y)h(y)\,dy$, where $K(x,y) = \min\{x,y\}$, and that its derivative $\frac{\partial W(t,x)}{\partial x}$ is a cylindrical Wiener process in H (see [2], [3]). The problem of oscillations leads to the stochastic equation

$$u_t(t,x) - g(x) = a\int_0^t \frac{\partial^2 u(\tau,x)}{\partial x^2}\,d\tau + b\,\frac{\partial W(t,x)}{\partial x}.$$

We also discuss the problem of weak and strong solvability of this equation in the space H.

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A. S. Asylgareev (Ufa, Russia)**. On comparison of solutions of stochastic differential equations driven by a multidimensional Wiener process.**

Consider two stochastic differential equations (hereinafter SDEs) with Stratonovich integrals driven by a multidimensional Wiener process $\overline{W}_t^{(n)} = (W_t^{(1)}, \dots, W_t^{(n)})$, which is defined on a filtered probability space $(\Omega, \mathscr{F}, (\mathscr{F}_t)_{t\geqslant 0}, \mathbf{P})$,

$$(1) \qquad d\xi_k^{(n)}(t) = \sum_{j=1}^{n} \sigma_{kj}^{(n)}\bigl(t, \xi_k^{(n)}(t)\bigr) * dW_t^{(j)} + b_k^{(n)}\bigl(t, \xi_k^{(n)}(t)\bigr)\,dt, \qquad k = 1, 2.$$

The purpose of the present study, which continues the paper [1], is a proof of comparison theorems for equations (1) in the case when the diffusion coefficients of the compared equations can be different. The classical comparison theorem for SDEs was proved by Skorokhod [2] for one-dimensional equations with Itô integral. The result of [2] was extended by Geiss and Manthey in [3] for the Itô's equations driven by multidimensional Wiener process, but in [3] the authors required that the diffusion coefficients of the equations under the corresponding components of the multidimensional Wiener process should coincide.

The approach used here is based on the fact that solutions of (1) can be represented in the form $\xi_k^{(n)}(t) = \widehat{D}_k^{(n)}\bigl(t, W_t^{(n)} + D_k^{(n-1)}(t, \overline{W}_t^{(n-1)})\bigr)$, where $\widehat{D}_k^{(n)}(t,u)$ are deterministic functions, and $\xi_k^{(n-1)}(t) = D_k^{(n-1)}(t, \overline{W}_t^{(n-1)})$ are solutions of the SDE driven by an $(n-1)$-dimensional Wiener process. The main result is the following.

Theorem 1. *Assume that the following conditions are satisfied for all* $t \geqslant 0$, $j = 1, \dots, n$:

(1) $\sigma_{2j}^{(j)}(t,v) > 0$ *for all* $v \in \mathbf{R}$;

(2) $\widehat{D}_2^{(j)}(t,u) \geqslant \widehat{D}_1^{(j)}(t,u)$ *for all* $u \in \mathbf{R}$;

(3) $D_2^{(0)}(t) \geqslant D_1^{(0)}(t)$ *with probability* 1.

Then $\xi_2^{(n)} \geqslant \xi_1^{(n)}$ *for all* $t \geqslant 0$ *a.s.*

Theorem 1 can be reformulated for the equations with the Itô integral because there is a transition formula between the Itô and Stratonovich integrals, which is valid under sufficient smoothness of the coefficients b and σ.

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V. S. Barbu, S. Beltaief, S. Pergamenshchikov (LMRS, University of Rouen Normandy, France). **Robust adaptive efficient estimation for a semi-Markov continuous time regression from discrete data [1], [2].**[3]

In this work we consider the nonparametric robust estimation problem for regression models in continuous time with semi-Markov noises, and we are interested in estimating an unknown function S on the basis of observations that can be in continuous or discrete time. This problem of nonparametric estimation in regression models is an important chapter of theoretical and applied statistics that has been considered in many frameworks. Our main goal is to develop nonparametric adaptive robust estimation, with the noise process with large dependence; to this end, we use a particular case of semi-Markov processes to model the dependent noises. The semi-Markov regression model in continuous time introduced in [1] is given by

$$\mathrm{d}y_t = S(t)\,\mathrm{d}t + \mathrm{d}\xi_t, \qquad 0 \leqslant t \leqslant n, \tag{1}$$

where $S(\,\cdot\,)$ is an unknown 1-periodic function defined on $\mathbf{R}$ with values on $\mathbf{R}$, $(\xi_t)_{t\geqslant 0}$ is the unobserved noise process $\xi_t = \varrho_1 L_t + \varrho_2 z_t$, where ϱ_1 and ϱ_2 are unknown coefficients, $(L_t)_{t\geqslant 0}$ is a Lévy process, while $(z_t)_{t\geqslant 0}$ is a particular case of a semi-Markov process (see, e.g., [3]) defined as $z_t = \sum_{i=1}^{N_t} Y_i$, where $(Y_i)_{i\geqslant 1}$ is an i.i.d. sequence of random variables with $\mathbf{E}Y_i = 0$, $\mathbf{E}Y_i^2 = 1$, and $\mathbf{E}Y_i^4 < \infty$. Here N_t is a general counting process defined as $N_t = \sum_{k=1}^{\infty} 1_{\{T_k \leqslant t\}}$ with $T_k = \sum_{l=1}^{k} \tau_l$, where $(\tau_l)_{l\geqslant 1}$ is an i.i.d. sequence of positive integrated random variables with mean $\mathbf{E}\tau_1 > 0$. We assume that the processes $(N_t)_{t\geqslant 0}$ and $(Y_i)_{i\geqslant 1}$ are independent of each another and are also independent of $(L_t)_{t\geqslant 0}$. Our problem is to estimate the unknown function S in the model (1) on the basis of observations $(y_{t_j})_{0\leqslant j\leqslant np}$, $t_j = j\Delta$, $\Delta = 1/p$, where the integer $p \geqslant 1$ is the observation frequency. We construct a series of estimators by projection and thus approximate the unknown function by a finite Fourier series. As we consider the estimation problem in an adaptive setting, i.e., in the situation when the regularity of the function is unknown, we develop a new adaptive method based on the model selection procedure proposed by Konev and Pergamenshchikov (2012). First, this procedure gives us a family of weighted least squares estimators $\widehat{S}_\lambda$, where the weight vector $\lambda = (\lambda(1), \ldots, \lambda(n))$ belongs to some finite set Λ from $[0,1]^n$. Second, we choose the best possible one by minimizing a cost function, $\widehat{\lambda} = \operatorname{argmin}_{\lambda\in\Lambda} J_n(\lambda)$, where J_n is a cost function that we consider. Using this weight coefficient $\widehat{\lambda}$ in $\widehat{S}_\lambda$ leads to the model selection procedure $\widehat{S}_* = \widehat{S}_{\widehat{\lambda}}$. Under general moment conditions on the noise distribution, we prove that there exists some constant $l^* > 0$ such that for any noise distribution Q and weight vector set Λ, for any periodic function S and for any $n \geqslant 1$, $p \geqslant 3$, and $0 < \delta \leqslant 1/6$, we have the sharp nonasymptotic oracle inequality

$$\mathscr{R}_Q(\widehat{S}_*, S) \leqslant \frac{1+3\delta}{1-3\delta} \min_{\lambda\in\Lambda} \mathscr{R}_Q(\widehat{S}_\lambda, S) + l^* \frac{\sigma_Q \nu}{\delta n},$$

where $\mathscr{R}_Q(\widetilde{S}_n, S) = \mathbf{E}_{Q,S}\|\widetilde{S}_n - S\|^2$ is the quadratic risk. We also prove the same type of oracle inequality for the robust risk $\mathscr{R}_n^*(\widetilde{S}_n, S) = \sup_{Q\in\mathscr{Q}_n} \mathscr{R}_Q(\widetilde{S}_n, S)$.

[3]Partially supported by RSF grant 14-49-00079 (National Research University "MPEI," Moscow, Russia), RFBR grant 16-01-0012, and the project Complex Systems, Territorial Intelligence and Mobility–Federation within the Large Scale Research Network *Logistics–Mobility–Digital Information* from the Region of Normandy, France.

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Ya. I. Belopolskaya (St. Petersburg Department of Steklov Mathematical Institute of Russian Academy of Sciences, St. Petersburg, Russia)**. Systems of forward and backward nonlinear Kolmogorov equations.** [4]

Systems of forward and backward nonlinear parabolic equations arise as mathematical models of various phenomena in physics, chemistry, biology, and many other fields. From the PDE point of view, the main difference between forward and backward systems is the fact that, commonly, forward systems have to be treated as systems with respect to measures (or their densities), while the backward systems should be treated as systems of equations with respect to functions. From the probabilistic point of view, this means that systems of the first type correspond to systems of forward Kolmogorov equations [1], [2], while systems of the second type correspond to systems of backward Kolmogorov equations [3], [4]. In this talk, we discuss the probabilistic interpretation of several systems of forward Kolmogorov equations — namely the MHD–Burgers system and the Brusselator system as particular cases of the form

$$\frac{\partial u_k}{\partial t} + \operatorname{div} f(u) = \frac{\sigma_k^2}{2}\Delta u, \quad u_k(0,x) = u_{0k}(x), \tag{1}$$

as well as nonlinear parabolic systems with cross-diffusion. For such systems, we construct a generalized solutions to the Cauchy problem in terms of the corresponding random processes and their multiplicative functionals [1], [2]. Moreover, we show that the Cauchy problem for such systems can be reduced to a closed stochastic system, which can be applied for construction of a numerical solution to the original problem. In particular, for the MHD–Burgers system, we have $f(u) = (u_1u_2, (u_1^2+u_2^2)/2)^*$, and the corresponding stochastic system has the form

$$d\widehat{\xi}_k(\theta) = -\sigma_k\, dw(\theta), \quad \hat{\xi}_k(0) = x, \tag{2}$$

$$d\widetilde{\eta}^k(\theta) = C_u^k(\widehat{\xi}(\theta))\widetilde{\eta}(\theta)\, dw(\theta), \quad \widetilde{\eta}^k(0) = 1, \tag{3}$$

$$u^k(t,x) = \mathbf{E}\widetilde{\eta}^k(t)u_{0k}(\widehat{\xi}); \qquad k = 1,2. \tag{4}$$

Theorem 1. (1) *Assume that there exists a regular generalized solution to the Cauchy problem for the the MHD–Burgers system. Then this solution admits a stochastic representation of the form* (4).

(2) *Let $u_{0k} > 0$ and $\nabla u_{0k} \in \mathbf{L}^2$. Then there exists an interval $[0,T]$ such that, for all $t \in [0,T]$, there exists a unique solution of system* (2)–(4)*. Moreover, the function $u^k(t,x)$ of form* (4) *satisfies the Cauchy problem for the MHD–Burgers system in the sense of Schwartz distribution theory.*

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We also discuss the probabilistic interpretation of the Cauchy problem solutions for parabolic systems with cross-diffusion [3], [4].

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V. A. Bovkun (Ekaterinburg, Russia). **Connection between infinite-dimensional stochastic problems and deterministic problems for probabilistic characteristics.**

We consider the Cauchy problem for the infinite-dimensional stochastic equation

$$X(t) = \xi + \int_0^t \mathscr{A}(s, X(s))\, ds + \int_0^t B(s, X(s))\, dW(s), \qquad t \in [0, T], \tag{1}$$

with an operator $\mathscr{A} = \mathscr{A}(t,x) = Ax + F(t,x)$, $t \in [0,T]$, $x \in H$, where A is the generator of a C_0-semigroup in a Hilbert space H, $F\colon [0,T] \times H \to H$ and $B\colon [0,T] \times H \to H$ are (in general) nonlinear mappings, $\{W(t),\, t \geqslant 0\}$ is an H-valued Q-Wiener process with respect to the filtration $\{\mathscr{F}_t,\, t \geqslant 0\}$ on a probability space $(\Omega, \mathscr{F}, \mathbf{P})$, and ξ is an $\mathscr{F}_0$-measurable H-valued random variable. It is known that under additional requirements on F and B, there is a unique Markov process $\{X(t),\, t \geqslant 0\}$ which is a mild solution of the problem (see, for example, [1]).

In [2], first, a connection between the global moments of the first and second orders of the mild solution and the coefficients of the stochastic equation (1) was established. Second, under the assumption of continuity of the trajectories of the process $\{X(t), t \geqslant 0\}$ and the existences of its local moments, infinite-dimensional analogues of the direct and inverse Kolmogorov equations for the probabilistic characteristics of the process were obtained. In this talk, we prove that the process $\{X(t), t \geqslant 0\}$ has continuous trajectories and finite local moments of the first and second orders which coincide with the corresponding global moments. The following result holds.

Theorem. *Let A be a generator of a C_0-semigroup of operators, and let mappings F and B be continuous in t on $[0,T]$ and satisfy the Lipschitz condition and the sublinear growth condition. Also let $\delta > 0$, and let f be a bounded twice Fréchet differentiable functional on H. Then, for any $x \in D(A)$,*

$$\mathbf{P}(t + \Delta t, y \mid t, x) = o(\Delta t), \quad \|y - x\|_H > \delta,$$

$$\int_{\|y-x\|_H \leqslant \delta} f'(x)(y-x)\mathbf{P}(t+\Delta t, dy \mid t, x) = f'(x)\mathscr{A}(t,x)\Delta t + o(\Delta t),$$

$$\int_{\|y-x\|_H \leqslant \delta} f''(x)[y-x]^2\mathbf{P}(t+\Delta t, dy \mid t, x) = \operatorname{Tr}[f''(x)B(t,x)QB^*(t,x)]\Delta t + o(\Delta t),$$

where $\mathbf{P}(t', \mathscr{B} \mid t, x)$ is the transition probability of the mild solution $\{X(t), t \geqslant 0\}$.

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A. V. Bulinski (Moscow, Russia). **Asymptotic behavior of entropy estimates.** [5]

The concept of entropy is fundamental in physics and mathematics. Significant contributions to the development of this notion were made by L. Boltzmann, J. Gibbs, M. Plank, C. Shannon, A. N. Kolmogorov, Ya. G. Sinai, A. Rényi, C. Tsallis, and A. S. Holevo. We are mainly concerned with statistical estimates of the Shannon differential entropy. For applications, see, e.g., [1], [2]. There are mutually complementary approaches to entropy estimation. We refer, e.g., to the works by L. F. Kozachenko and N. N. Leonenko (1987), P. Hall and S. C. Morton (1993), A. B. Tsybakov and E. C. Van der Meulen (1996), E. G. Miller (2003), L. Paninski (2003), D. Stowell and M. D. Plumbley (2009), K. Sricharan et al. (2013), E. Archer et al. (2016), A. Charzyńska and A. Gambin (2016), and S. Delattre and N. Fournier (2017).

In parallel with out survey in this field, we consider recent studies [3], [4]. Let $X, X_1, X_2, \dots$ be a sequence of i.i.d. random vectors with values in the space $\mathbb{R}^d$ and having density f w.r.t. the Lebesgue measure μ. Recall that the well-known Kozachenko–Leonenko estimates of the Shannon differential entropy $H(X) := -\int_{\mathbf{R}^d} f(x)\log f(x)\,\mu(dx)$ have, for $N \in \mathbb{N}$ $(N>1)$, the form

$$H_N := d\log\overline{\rho}_N + \log V_d + \gamma + \log(N-1),$$

where $\overline{\rho}_N := (\rho_{1,N}\cdots\rho_{N,N})^{1/N}$, $\rho_{i,N}$ is the Euclidean distance from X_i to its nearest neighbor in the sample $\{X_1,\dots,X_N\}\setminus\{X_i\}$, and $\gamma := -\int_{(0,\infty)} e^{-t}\log t\,dt \approx 0.5772$ and $V_d := \pi^{d/2}/\Gamma(d/2+1)$ are the Euler constant and the volume (i.e., the Lebesgue measure) of the unit ball in $\mathbb{R}^d$, respectively. Under wide conditions (involving an analogue of the Hardy–Littlewood maximal function), it was proved in [3] that $\lim_{N\to\infty}\mathbf{E}H_N = H(X)$. It was also established in [3] that, under wide conditions, $\lim_{N\to\infty}\mathbf{E}(H_N - H(X))^2 = 0$. In particular, these assertions also hold for any nondegenerate Gaussian vector X. The above results are noteworthy, since, e.g., in [6] it was indicated that the available proofs of the asymptotical unbiasedness and L^2-consistency of H_N, as proposed in various papers of other authors, should be corrected.

We next turn to the new statistical estimates of the conditional Shannon entropy, which were introduced in [4]. The mixed-pair model (X, Y) is studied, where X and Y take values in $\mathbb{R}^d$ and an arbitrary finite set M, respectively. Such models cover, e.g., the logistic regression. In contrast to the Kozachenko–Leonenko estimates of the unconditional entropy, the estimates proposed here are constructed by means of certain k_N-nearest neighbor statistics (N is the sample cardinality) and a random number of observations contained in certain balls with random centers and random radii. The asymptotic unbiasedness and L^2-consistency of the new estimates are also

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established under simple conditions. Unlike [5], our construction does not call for the existence of a topological structure in a set M.

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A. E. Chistyakov (Rostov-on-Don, Russia)**. Stochastic modeling of turbulent flows in coastal systems on a supercomputer.**[6]

Stochastic methods are often used for describing turbulent flows in waters, various fluctuating values being considered as random functions. The turbulence on dissipative scales has involved statistical structure due to strong intermittency. Field studies of coastal systems were performed on an example of the Azov Sea. As a result, data on water velocity pulsations in certain water body points were obtained using the WHS600 Sentinel ADCP (Acoustic Doppler Current Profiler) [1].

The correlations of the products of deviations of the flow velocity components are as follows:

$$K_{ZZx}(z) = -\overline{u'w'} \Big/ \frac{\partial \overline{u}(z)}{\partial z}, \quad K_{ZZy}(z) = -\overline{v'w'} \Big/ \frac{\partial \overline{v}(z)}{\partial z},$$
$$\nu(z) \equiv K_{ZZ}(z) = \sqrt{K_{ZZx}^2 + K_{ZZy}^2},$$

where $\nu(z)$ is the coefficient of turbulent exchange, and u', v', w' are the pulsations of velocity vector components.

A stochastic model was developed and numerically implemented on a multiprocessor computer system for calculation of the vertical turbulent exchange coefficient in a coastal system on an example of the Azov Sea. The model is based on the definition of turbulent flows as space-averaged (correlation) multiplication of deviations of flow velocities components and a transported physical quantity. A numerical experiment showed that the mechanisms of vertical turbulent exchange are suppressed on large scales of vertical grids in numerical simulation of hydrodynamic processes of the coastal system.

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E. G. Chub (Rostov-on-Don, Russia). **Synthesis of a stochastic controllable information-measuring complex.**

Consider the stochastic model of the gyrostabilizer of the information-measuring complex on a perturbed base in the observer–object form [1], [2]

$$\dot{Y} = F_1(Y,t) + F_2(Y,t)\zeta + F_3 M(Z_a), \qquad Z = H(Y,t) + W_a,$$

where $Y = (\alpha, \beta, \gamma, w^{\mathrm{T}})^{\mathrm{T}}$ is the state vector; α, β, γ are turning angles; w is the vector of random overloads perturbing the base of the gyrostabilizer along the corresponding axes, as described in the general case by a system of stochastic nonlinear differential equations in the form of Langevin $\dot{w} = F_w(w,t) + \xi$; ξ is white Gaussian noise (WGN) with zero expectation and a known intensity matrix; F_1, F_2, F_3, H are known functions; $\zeta = (W,\xi)^{\mathrm{T}}$; W are perturbations acting on the gyrostabilizer, which are approximated in the general case by a WGN with zero expectation and a known intensity matrix; $M(Z_a)$ is the vector of control moments, as formed from the readings of accelerometers; and W_a is the accelerometer interference vector, as approximated in the general case by a WGN with zero expectation and a known intensity matrix. To determine the current vector function of control moments that ensures the minimum deviation of the trihedron of the gyroscopic coordinate system with respect to the astronomical coordinate system with minimal costs for the formation of the control vector, we introduce the criterion function $\Theta = \widehat{\alpha}^2 + \widehat{\beta}^2 + \widehat{\gamma}^2 + \int_0^t M^T Q M\, dx$, where $\widehat{\alpha}$, $\widehat{\beta}$, $\widehat{\gamma}$ are estimates of the turning angles, and Q is a weight matrix. Then the suboptimal vector of control moments can be found as follows:

$$M = -Q^{-1} N \Phi^{\mathrm{T}}(\widehat{\beta}, \widehat{\gamma})(\widehat{\alpha}, \widehat{\beta}, \widehat{\gamma})^{\mathrm{T}};$$

here, N, Φ are known matrices. In this case, the computational costs of synthesizing the control vector are determined mainly by the costs of integrating the system of equations, which can easily be realized by calculators of the information-measuring complex [2].

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A. G. Danekyants, N. V. Neumerzhitskaia (Rostov-on-Don, Russia). **Generalization of a result on the existence of weakly interpolating martingale measures.** [7]

Let $\Omega = \{\omega_1, \omega_2, \dots\}$ be a countable set, $\mathscr{F}_0 = \{\Omega, \varnothing\}$, and $\mathscr{F}_1$ be the set of all subsets of Ω. Consider a process $Z = (Z_k, \mathscr{F}_k)_{k=0}^1$ and denote $a := Z_0$, $b_i := Z_1(\omega_i)$, $i = 1, 2, \dots$. Let $\mathbf{F} = (\mathscr{F}_k)_{k=0}^1$, and let $\mathscr{P}(Z, \mathbf{F})$ be the set of probability measures P on $(\Omega, \mathscr{F})$ such that $P(\omega_i) > 0$, $i = 1, 2, \dots$, and the process $Z = (Z_k, \mathscr{F}_k, P)_{k=0}^1$ is a martingale. We suppose that $\mathscr{P}(Z, \mathbf{F}) \neq \varnothing$. We are interested in the question of the existence in the set $\mathscr{P}(Z, \mathbf{F})$ of the so-called weakly interpolating martingale

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measures, which are defined as follows. Let $\{n_1, n_2, \dots\}$ be a transposition of the sequence $1, 2, \dots,$ and let $\mathscr{G}_0 = \{\Omega, \varnothing\}$, $\mathscr{G}_i = \sigma\{\omega_{n_1}, \omega_{n_2}, \dots, \omega_{n_i}\}$. It is clear that $\mathscr{G}_\infty = \mathscr{F}_1$. A measure $P \in \mathscr{P}(Z, \mathbf{F})$ is called a weakly interpolating martingale measure (see [1], [2]) if, for any such permutation, the process $Y_i = E^P[Z_1 \mid \mathscr{G}_i]$, $i = 1, 2, \dots,$ admits a unique martingale measure in $\mathscr{P}(Z, \mathbf{F})$ (which is automatically equal to P). Such measures P make it possible to interpolate incomplete financial markets into complete ones and, therefore, to construct perfect hedges. In proving the existence of such measures, there are two fundamentally different cases to consider: (1) the sequence $\{b_i\}_{i=1}^\infty$ *contains a finite number* of different values [1] (a new result in this direction is presented in the abstracts of I. V. Pavlov and I. V. Tsvetkova at ICSM-3); (2) this sequence *contains a countable number* of different values [2]. Under condition (2) we proved the following.

THEOREM. *Let a be an irrational number, and a sequence $\{b_i\}_{i=1}^\infty$ contain only a finite number of irrational numbers, while the other terms are rational. If $\{b_i\}_{i=1}^\infty$ does not contain a finite collection $\{b_{i_j}\}_{j=1}^k$ such that $a = d_0 + d_1 b_{i_1} + \dots + d_k b_{i_k}$ for some rational numbers $d_0, d_1, \dots, d_k$, then the set $\mathscr{P}(Z, \mathbf{F})$ contains weakly interpolating martingale measures.*

Note that in the case when all $\{b_i\}_{i=1}^\infty$ are rational, we obtain the result of [2].

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Dupire B. (New York, USA)**. Functional Itô calculus and characterization of attainable claims [1].**

Path dependence is of paramount importance in finance as it can be present in the dynamics of the assets or in the definition of claim payoffs. We first review the functional Itô calculus which is the framework needed for path dependence and allows for the extension of many results to the non-Markov case. We show that it is possible to characterize which contingent claims can be replicated by either

(1) a dynamic trading of the underlying asset,
(2) a static position in European options, or
(3) a combination of the former two.

We show that the answer lies in the properties of the intrinsic value functional, which attributes to each asset price path up a current date before maturity the payoff obtained by freezing the asset price until maturity. More precisely, it depends on the behavior of the second functional space derivative of the intrinsic value.

We illustrate the power of the result by applying the associated algorithm to a variety of path dependent claims.

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E. Eberlein (Freiburg, Germany). **Multiple curve interest rate modeling allowing for negative rates [1], [2].**

The global financial crisis which began in August of 2007 had a lasting effect on financial markets. In particular, the fixed income markets changed in a fundamental way. As a consequence of a new perception of risk, a number of interest rates, which until then had been roughly equivalent, drifted apart. The basic rates, which are relevant for the interbank market, became tenor-dependent after market participants became aware of credit, liquidity, and funding risks in this market segment. These risks had been assumed to be negligible. In the new reality, classical modeling approaches, which are based on arbitrage considerations assuming tenor independence, can no longer reflect the market behavior. More sophisticated approaches, so-called multiple curve models, are needed to take the increased diversity of risks into account.

We develop a multiple curve forward process as well as a multiple curve forward rate (HJM-type) model. In both approaches, time-inhomogeneous Lévy processes are used as drivers. Negative interest rates are taken into account in a natural way. We derive valuation formulas for standard interest rate financial products such as caps, floors, swaptions, and digital interest rate options. A number of calibration results is presented where we also consider data in the setting of a two price economy, thus exploiting explicitly bid and ask quotes.

This project is a joint work with Christoph Gerhart (Freiburg) and Zorana Grbac (Paris).

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M. L. Esquível (New University of Lisbon, Portugal). **From an ordinary differential equation model to an open population Markov chain model, via stochastic differential equations; models for HIV infection in individuals and populations [1], [2], [3].** [8]

We present an initial exploration of a method for the association of an open population Markov chain model—with a finite number of states—to some phenomena that may be, by force of its intrinsic characteristics, best modeled by an ODE, at least in some average sense. The ODE model presented here is formulated as a dynamic change between two regimes; one regime is of mean reverting type and the other is of inverse logistic type. For the general purpose of defining an open Markov chain model for a human population, we associate an Itô process to the ODEs by means of the addition of Gaussian noise terms which may be thought to model nonessential characteristics of the phenomena with small and undifferentiated influences. The next step consists of discretizing the Itô processes and using the sequence of values obtained to define, by simulation, trajectories that, in turn, may define transitions of a finite valued Markov chain if the state space of the Itô process is partitioned according to some rule. We detail the application of these ideas to the study of the evolution of

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a Portuguese population newly diagnosed with HIV. For that purpose the state space of the Itô process is partitioned in six infection classes. We detail the evolution of the population in these classes under two different projections for the evolution of the newly diagnosed cases. The method presented here connects the model for the evolution of the HIV viral load and the CD4 leucocytes count to a Markov chain open model for the Portuguese HIV-diagnosed population.

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Yu. E. Gliklikh (Voronezh, Russia)**. Investigation of completeness of stochastic flows generated by equations with current velocities.**[9]

This is a joint talk with T. A. Shchichko. The main purpose of the talk is to find conditions (sufficient, and necessary and sufficient) for completeness of the stochastic flow generated by an equation given in terms of the so-called current velocities (the Nelson symmetric mean derivatives).

The preliminaries can be found in [1], [2], [3], [4].

Let a Borel vector field $v(t,x)$ and a field of symmetric nonnegative semidefinite matrices $\alpha(t,x)$ be given on $\mathbb{R}^n$. The equation with current velocities (the Nelson symmetric mean derivatives) is a system in $\mathbb{R}^n$ of the form

$$\begin{cases} D_{\mathrm{S}}\xi(t) = a(t,\xi(t)), \\ D_2\xi(t) = \alpha(t,\xi(t)), \end{cases} \tag{1}$$

where D_{S} is the symmetric mean derivative (current velocity) and D_2 is the quadratic mean derivative. According to [5], if a and α are smooth and satisfy (together with the first derivatives of the field α) Itô-type estimates, α is positive definite, and the initial value is a random variable with density that is smooth and nowhere vanishes, then (1) has a well-posed solution for $t \in [0,\infty)$. Our purpose is to find conditions for the existence of a solution for $t \in [0,\infty)$ (completeness of the flow) without the requirement that the Itô-type estimates be satisfied.

Recall that a function $\varphi : \mathbb{R}^n \to \mathbb{R}$ is called proper if the preimage $\varphi^{-1}(K)$ of any relatively compact set in $\mathbb{R}$ is relatively compact in $\mathbb{R}^n$.

Theorem 1. *Let a smooth positive proper function φ exist on $\mathbb{R}^n$ such that $\mathscr{L}(t,x)\varphi < C$ for a certain $C > 0$ for all $t \in \mathbf{R}$, $x \in \mathbf{R}^n$, where $\mathscr{L}$ is the generator of flow $\xi(s)$. Then the flow $\xi(s)$ is complete.*

Theorem 2. *Let a smooth positive proper function u exist on $\mathbb{R}^n$ such that $\widetilde{\mathscr{L}}u < C$ for a certain constant $C > 0$, where $\widetilde{\mathscr{L}}$ is the generator of inverse flow $\widetilde{\eta}(t)$. Then the forward flow $\eta(t)$ is continuous at infinity on $[0,T]$.*

Consider the direct product $\mathbf{R}^n_+ = [0,T] \times \mathbf{R}^n$.

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THEOREM 3. *A necessary and sufficient condition needed for both the forward flow* $\xi(s)$ *and the inverse flow* $\widetilde{\xi}(s)$, *as generated by equation* (1), *to be both continuous at infinity and complete on* $[0,T]$ *is that there exist smooth positive proper functions* $u(t,x)$ *and* $\widetilde{u}(t,x)$ *on* $\mathbb{R}^n_+$ *such that the inequalities* $(\partial/\partial t+\mathscr{A})u<C$ *and* $(-\partial/\partial t+\widetilde{\mathscr{A}})\widetilde{u}<\widetilde{C}$ *be satisfied for some positive constants* C *and* $\widetilde{C}$, *where* $\mathscr{A}$ *and* $\widetilde{\mathscr{A}}$ *are generators of the forward and inverse flows, respectively.*

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M. Grigorova (Bielefeld University, Germany)**. Doubly reflected BSDEs and nonlinear Dynkin games: Beyond right-continuity [1], [2], [3].**

We formulate a notion of a doubly reflected backward stochastic differential equation (DRBSDE) with standard Lipschitz driver g in the case where the lower barrier ξ and the upper barrier ζ are completely irregular. To simplify the presentation, we place ourselves in the case of a Brownian filtration. The given processes ξ and ζ are optional (not necessarily right-continuous) with $\xi\leqslant\zeta$ and $\xi_T=\zeta_T$ and such that $\mathbf{E}(\operatorname{ess\,sup}_{\tau\in\mathscr{T}_0}\xi_\tau^2)<\infty$ and $\mathbf{E}(\operatorname{ess\,sup}_{\tau\in\mathscr{T}_0}\zeta_\tau^2)<\infty$. Here $\mathscr{T}_0$ denotes the set of stopping times such that $0\leqslant\tau\leqslant T$ a.s., where $T>0$ is a fixed terminal horizon. The solution of the DRBSDE with parameters (g,ξ,ζ) is a process with six components (Y,Z,A,C,A',C'). In comparison with the case of right-continuous barriers ξ and ζ, in our general case there is an additional "push-up" process C and an additional "push-down" process C' (which satisfy suitably defined minimality conditions or Skorokhod-type conditions). After defining the DRBSDE, we prove the following two theorems.

THEOREM 1. (i) *There exists a solution to the DRBSDE with parameters* (g,ξ,ζ) *if and only if the so-called Mokobodzki condition is satisfied*; *that is*, *there exist two nonnegative strong supermartingales* H *and* H' *such that* $\xi\leqslant H-H'\leqslant\zeta$.

(ii) *If a solution to the DRBSDE exists*, *it is unique.*

We denote by $\mathscr{E}^g$ the nonlinear g-expectation induced by the (nonreflected) BSDE with driver g. We consider the nonlinear Dynkin game with g-expectation and payoff processes ξ and ζ. The upper value $\overline{V}$ and lower value $\underline{V}$ of the game are defined by

$$\overline{V}(0):=\inf_{\sigma\in\mathscr{T}_0}\sup_{\tau\in\mathscr{T}_0}\mathscr{E}^g_{0,\tau\wedge\sigma}[\xi_\tau\mathbf{1}_{\tau\leqslant\sigma}+\zeta_\sigma\mathbf{1}_{\sigma<\tau}],$$

$$\underline{V}(0):=\sup_{\tau\in\mathscr{T}_0}\inf_{\sigma\in\mathscr{T}_0}\mathscr{E}^g_{0,\tau\wedge\sigma}[\xi_\tau\mathbf{1}_{\tau\leqslant\sigma}+\zeta_\sigma\mathbf{1}_{\sigma<\tau}].$$

THEOREM 2. *We assume that* ξ *and* $-\zeta$ *are right-uppersemicontinuous* (*but not necessarily right-continuous*) *and satisfy Mokobodzki's condition. Then*, *the above*

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nonlinear Dynkin game has a value, that is, $\overline{V}(0) = \underline{V}(0)$. *Moreover, this common value coincides with* Y_0, *where* Y *is the first component of the solution of the DRBSDE with parameters* (g, ξ, ζ).

If ξ and ζ are not right-uppersemicontinuous, the above nonlinear Dynkin game problem might not have a value. In this case, we formulate an "extension" of the above game over a set of "stopping strategies" larger than the set of stopping times and show that the solution Y of the DRBSDE is equal to the value of the "extended" game. This characterization then proves useful in establishing a comparison result and a priori estimates with universal constants for the DRBSDE.

The talk is based on a joint work with P. Imkeller, Y. Ouknine, and M.-C. Quenez.

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A. A. Gushchin (Steklov Mathematical Institute, Moscow, Russia). **Joint distributions of increasing processes and their compensators, single jump martingales, and the Skorokhod embedding.** [10]

Let us denote by $\mathbb{W}^*$ the class of all Borel probability measures $\mu = \mu(dx, dy)$ on $\mathbb{R}^2_+$ satisfying

$$\int x\,\mu(dx,dy) = \int y\,\mu(dx,dy) \quad \text{and} \quad \int_{\{y\leqslant\lambda\}} x\,\mu(dx,dy) \leqslant \int (y\wedge\lambda)\,\mu(dx,dy) \quad \forall\,\lambda\geqslant 0.$$

If, additionally, we have equality in the inequality for each $\lambda \geqslant 0$, then the corresponding class is denoted by $\mathbb{W}^*_e$.

Let us recall that, for an increasing process $A = (A_t)_{t\geqslant 0}$, the family of random variables $C_t = \inf\{s \geqslant 0\colon A_s > t\}$ is the change of time generated by A. For a progressively measurable process X, the time-changed process $X \circ C = (X_{C_t})_{t\geqslant 0}$ is well defined if X_t converges a.s. to a finite limit X_∞ as $t \to \infty$ on the set $\{A_\infty < \infty\}$.

Here a single jump martingale is understood in the narrow sense; namely, $M = (M_t)_{t\geqslant 0}$ is said to be a single jump martingale if it has the form

$$M_t = W \wedge t - V\mathbf{1}_{\{t\geqslant W\}},$$

where a pair (V, W) of random variables has a joint distribution from $\mathbb{W}^*_e$. It is easy to show that M is indeed a martingale (e.g., with respect to the filtration that it generates). Let us note that the process $A_t := W \wedge t$ is continuous and hence predictable, and therefore it is the compensator of the increasing process $X_t := V\mathbf{1}_{\{t\geqslant W\}}$.

The following proposition is the initial point for further analysis.

Proposition 1 [1]. *Let* X *be a nonnegative local submartingale with the Doob–Meyer decomposition* $X = M + A$, $X_0 = M_0 = A_0 = 0$. *If* $\mathbf{P}(A_\infty < \infty) = 1$, *then* X_t

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converges a.s. to a finite limit X_∞ *as* $t \to \infty$ *and* $\mathrm{Law}(X_\infty, A_\infty) \in \mathbb{W}^*$. *Moreover,* $\mathrm{Law}(X_\infty, A_\infty) \in \mathbb{W}_e^*$ *if and only if* $(X - A) \circ C$ *is a single jump martingale, where* C *is the change of time generated by* A.

In particular, if N, $N_0 = 0$, is a local martingale such that its running maximum $\overline{N}_t := \sup_{s\leqslant t} N_s$ is continuous and $\mathbf{P}(\overline{N}_\infty < \infty) = 1$, then $\mathrm{Law}(\overline{N}_\infty - N_\infty, \overline{N}_\infty) \in \mathbb{W}^*$. Moreover, it is clear that for any $\mu \in \mathbb{W}_e^*$, there exists a single jump martingale M such that $\mathrm{Law}(\overline{M}_\infty - M_\infty, \overline{M}_\infty) = \mu$. Therefore, for any local martingale satisfying the above conditions and such that $\mathrm{Law}(\overline{N}_\infty - N_\infty, \overline{N}_\infty) \in \mathbb{W}_e^*$, there exists a single jump martingale M such that the joint distribution $\mathrm{Law}(M_\infty, \overline{M}_\infty)$ coincides with $\mathrm{Law}(N_\infty, \overline{N}_\infty)$. On the other hand, if we embed M into a Brownian motion according to the first Monroe's theorem, then one can construct a Skorokhod embedding τ, i.e., a Brownian motion B and a minimal stopping time τ such that the joint distribution $\mathrm{Law}(B_\tau, \overline{B}_\tau)$ is again the same as $\mathrm{Law}(N_\infty, \overline{N}_\infty)$.

This leads to the natural question of whether there are similar representations for distributions in $\mathbb{W}^*$. It turns out that, for any measure $\mu \in \mathbb{W}^*$, one can construct a locally integrable increasing process X with continuous compensator A and such that $\mathrm{Law}(X_\infty, A_\infty) = \mu$. However, X can be chosen as an increasing process with a single jump if and only if

$$\int (x-y)^+ \,\mu(dx,dy) \geqslant \int (y-x)^+ \,\mu(dx,dy).$$

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A. S. Holevo (Steklov Mathematical Institute, Moscow, Russia)**. Quantum dynamical semigroups: Nonstandard generators, stochastic representations.**[11]

Quantum dynamical semigroups are a noncommutative analogue of (sub-)Markov semigroups in classical probability: while the latter are semigroups of positive normalized maps in functional spaces, the former are semigroups of corresponding maps in operator algebras [1]. These semigroups satisfy the *Markovian master equations* (M.m.e.), which are a noncommutative generalization of the Kolmogorov–Chapman equations.

Let K, L_j be linear operators that are defined on a dense domain $\mathscr{D}$ of a Hilbert space $\mathscr{H}$ and satisfy the condition

$$\sum_j \|L_j\psi\|^2 \leqslant 2\,\mathrm{Re}\langle\psi|K|\psi\rangle, \qquad \psi \in \mathscr{D}; \tag{1}$$

in particular, K is accretive: $\mathrm{Re}\langle\psi|K|\psi\rangle \geqslant 0$, $\psi \in \mathscr{D}$. We assume that K is a maximal accretive operator. Then there exists a unique minimal solution T_t, $t \geqslant 0$, to the Cauchy problem of the *backward* quantum M.m.e.

$$\frac{d}{dt}\langle\varphi|T_t[X]|\psi\rangle = \sum_j \langle L_j\varphi|T_t[X]|L_j\psi\rangle - \langle K\varphi|T_t[X]|\psi\rangle - \langle\varphi|T_t[X]|K\psi\rangle, \tag{2}$$

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where $\varphi, \psi \in \mathscr{D}$, $X \in \mathfrak{L}(\mathscr{H})$, satisfying the condition $T_0[X] = X$, which is a dynamical semigroup on the algebra $\mathfrak{L}(\mathscr{H})$ of all bounded operators in $\mathscr{H}$ (see [2]).

If, in addition, L_j are closable and such that $\sum_j \|L_j^*\psi\|^2 < \infty$ for $\psi \in \mathscr{D}^*$, where $\mathscr{D}^*$ is an essential domain for K^*, then the predual semigroup $S_t^0 = (T_t)_*$ is a minimal solution of the *forward* M.m.e.

$$\frac{d}{dt}\langle\varphi|S_t^0[\omega]|\psi\rangle = \sum_j \langle L_j^*\varphi|S_t^0[\omega]|L_j^*\psi\rangle - \langle K^*\varphi|S_t^0[\omega]|\psi\rangle - \langle\varphi|S_t^0[\omega]|K\psi\rangle, \tag{3}$$

where $\varphi, \psi \in \mathscr{D}^*$, $\omega \in \mathfrak{T}(\mathscr{H})$, and $\mathfrak{T}(\mathscr{H}) = \mathfrak{L}(\mathscr{H})_*$ is the Banach space of trace-class operators ω in $\mathscr{H}$. There is a classical probabilistic representation

$$\langle\varphi|T_t[X]|\psi\rangle = \mathbf{E}\langle\varphi(t)|X|\psi(t)\rangle$$

via *weak-topology* solutions of the stochastic integral equation of the form

$$\psi(t) = \psi + \int_0^t \sum_j L_j\psi(s)\,dW_j(s) - \int_0^t K\psi(s)\,ds,$$

where $W_j(t)$, $j = 1, 2, \ldots$, are independent standard Wiener processes [2].

A semigroup is *standard* if it is a minimal solution of backward M.m.e. in the above sense. We consider two cases of dynamical semigroups obtained by singular perturbations of the generator of a standard semigroup [3]. We first describe a generalization of an example [4] of a standard dynamical semigroup which does not satisfy the forward M.m.e. Second, we consider an improved construction of a nonstandard dynamical semigroup.

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P. N. Ievlev (St. Petersburg Department of Steklov Mathematical Institute of Russian Academy of Sciences, St. Petersburg, Russia). **Probabilistic representation of the Cauchy problem solution for the multidimensional Schrödinger equation.** [12]

Consider the Cauchy problem for the Schrödinger equation

$$-i\frac{\partial u}{\partial t} = \frac{1}{2}\Delta u,$$

where Δ is the Laplacian in $\mathbf{R}^d$. In [1], a method of probabilistic representation of the solution to the Cauchy problem for the Schrödinger equation was derived based on

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a probabilistic representation of the solution to the Cauchy problem for the heat equation. Namely, it was proposed to consider the one-dimensional Schrödinger equation as the heat equation

$$\frac{\partial u}{\partial t} = \frac{\sigma^2}{2}\frac{\partial^2 u}{\partial x^2}$$

but with complex $\sigma = \exp(i\pi/4)$. An attempt at direct use of the well-known probabilistic representation involves several difficulties. In order to circumvent these issues, some "operations" were introduced [1] that led to the notion of "generalized random variables" (with special emphasis on the fact that this notion is not mathematically rigorous). It has turned out that a "generalized random variable" in the sense of [1] can be looked upon as a random functional. In the present study, we also use the notion of a random functional, but, as distinct from [2], we choose a different space of sampling functions and a different set of operations over functionals. Using the objects just defined, the principal result of [1] can be easily extended to the multi-dimensional setting. Namely, we construct a family of probabilistic semigroups $\{P^t_\varepsilon\}_{\varepsilon>0}$, which converges L_2-strongly to the semigroup $P^t = \exp(it\Delta/2)$ that corresponds to the solution of the Schrödinger equation.

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D. V. Ivanov (Moscow, Russia)**. Problems of reachability of conditional bounds of the expected maxima of independent random variables.**

We are concerned with expected maxima of an arbitrary number n of random i.i.d. variables $X_1,\dots,X_n$,

$$\mu_n = \mathbf{E}\max\{X_1,\dots,X_n\}.$$

Probability distributions with zero mean and variance of 1, and with given value of the expected maximum of m independent random variables of this distribution, are considered. Classical inequalities for the expected maxima can be found in [2]. We study the problem of reachability of the boundaries, as obtained in [2]. Namely, we determine the ranges of μ_m such that these bounds are reachable. The boundaries are clarified in the cases where this problem is still open. In addition, the boundaries reachability condition is studied, provided that the expected maxima of m and p random variables, respectively, are known. In particular, for $n=4$, $p=3$, $m=2$, we get the bound

$$\mu_4 \leqslant 2\mu_3 - \frac{6}{5}\mu_2 + \frac{1}{5\sqrt{7}}\sqrt{1-20\mu_3^2+60\mu_3\mu_2-48\mu_2^2},$$

which is reachable on the domain

$$\begin{aligned}&86025\mu_2^4 - 211200\mu_3\mu_2^3 + 200000\mu_3^2\mu_2^2\\&\quad - 4102\mu_2^2 - 86400\mu_3^3\mu_2 + 5040\mu_3\mu_2 + 14400\mu_3^4 - 1680\mu_3^2 + 49 \leqslant 0\end{aligned}$$

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for the distribution with a specific cubic generalized inverse distribution function $x(F)$. It is also demonstrated that in a number of cases the resulting bound is better than its analogue from [3]. This problem has applications in queuing theory, insurance, finance, and other fields.

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M. I. Kadomtsev, A. A. Lyapin (Rostov-on-Don, Russia)**. Stochastic methods of analysis of nonstationary signals in structure health monitoring.** [13]

The problem of analysis of dynamic response of structures to external stochastic loads in structure health monitoring is considered. The object of the study is the construction design. The aim of the study is to verify the presence or absence of structural damage. To achieve this goal, we use an approach involving statistical processing of data obtained using a network of vibration sensors, and recognition of statistical patterns using a supervised neural network. Application of Bayes's theorem yields $\mathbf{P}(C_i|\{x_i\}) = p(\{x_i\}|C_i)\mathbf{P}(C_i)/p(\{x_i\})$, where the vectors $\{x_k\}$, $k = 1, \ldots, N$, are known and acquired from sensors in parallel with oscillation recording for each class C_k; $p(\{x\})$ is the unconditional density function, which can be computed from the training set if necessary by concatenating all the feature vectors over all classes; and $P(C_i)$ is the prior probability of finding an example from class C_i without considering any measurement information. This allows us to construct the a priori conditional probability density functions $p(\{x\}|C_i)$, which specify the probability that a measurement vector $\{x\}$ can arise from the class C_i. As always, such a statement of the problem makes this approach suitable for dealing with supervised learning problems. In contrast to the methods discussed in [2], statistical parameters of amplitude and energy distribution in the frequency domain are taken as input data for the neural network. The data obtained from sensors developed in ASA DSTU are used for the analysis. In addition, use is made of the data obtained from modeling a structure [1] by the finite element and boundary element methods for different soil structures.

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A. V. Korolev (Moscow, Russia)**. On nonuniform averages of stochastic flows in the ergodic theorem.** [14]

Consider nonuniform averagings of the form $F_t(x) = \int_0^\infty f(T_{ts})\rho(s)\,ds$ for an ergodic dynamical system T_t, $t \geqslant 0$, on a probability space (X, μ), where ρ is a probability density on $[0, +\infty)$. Consider the stochastic differential equation $d\xi_t^x = A(\xi_t^x)\,dw_t + b(\xi_t^x)\,dt$, $\xi_0^x = x$. Suppose that μ is the corresponding T_t-invariant probability measure, $f \in L_p(\mu)$, $\nu = \rho(s)\,ds$, where $\rho \in L_q[0, +\infty)$.

We also assume that one of the following conditions is fulfilled: (1) the density ρ has bounded support on $[a, b]$; (2) $p > 1$, and there exists a nondecreasing function β on $[0, +\infty)$ such that $\beta \geqslant 0$, $\beta \in L_q[0, +\infty)$, and $\rho(t) \leqslant \beta(t)$ on $[t_0, +\infty)$ for some t_0. Then

$$\lim_{T\to\infty} \int_0^{+\infty} f(\xi_{ts}^x(w))\rho(s)\,ds = \int_X f\,d\mu$$

for every $x \in X$, for P-almost all $w \in W$ and certain assumptions on A and b.

For a detailed account, see [1], [2].

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Y. O. Koroleva, A. V. Korolev (Moscow, Russia)**. On a hydrodynamic problem in a domain with random roughness.** [15]

We study a three-dimensional incompressible flow of lubricant in a thin domain bounded by two moving surfaces $x_3 = \varepsilon h^{\pm}(x_1, x_2, t)$, $(x_1, x_2) \in \omega$, $t \in [0, T]$, with rough random structure. It is assumed that the parameter $\varepsilon > 0$ characterizes the thickness of the gap between the surfaces. The roughness of the surfaces is assumed to be an ergodic stochastic process, and hence the pressure is also a stochastic process, which in the limit as $\varepsilon \to 0$ satisfies the stochastic Reynolds equation

$$D_t h + \operatorname{div}\biggl(-\frac{h^3}{12\nu}\nabla p^* + \frac{h}{2}(v^+ + v^-)\biggr) = 0 \quad \text{for } \omega \times (0, T).$$

Here ν is the viscosity of the lubricant, $v^{\pm}$ are given velocities of the surfaces, $h \equiv h^+ - h^- = h_0 + h_s$, and h_0 is the film thickness corresponding to smooth surfaces, while the term h_s describes the roughness of the surfaces and is assumed to be a randomly varying field of mean 0.

The results obtained generalize those derived in the case of a similar lubrication problem in a domain with smooth deterministic boundaries (see [1], [2]).

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N. P. Krasiy (Don State Technical University, Rostov-on-Don, Russia)**. A refinement to the theorem on the existence and uniqueness of the maximum point in problems of optimization of quasilinear models with independent priorities.** [16]

In [1], conditions were found that guarantee the existence and uniqueness of the maximum point of the objective function for an optimizer which sets priorities (independent random variables $\alpha_j \in [0;1]$, $j = 1,\dots,k$) between k differently directed structures interacting in the same system. In this case, the objective function is of the form $F = \prod_{j=1}^{k} \mathbf{E} u_j^{\alpha_j}$, where $u_j = \sum_{i=1}^{n} a_{ij} x_i + b_j$, $a_{ij} \in \mathbf{R}$, $b_j \in \mathbf{R}$, $x = (x_1,\dots,x_n) \in \mathbf{R}^n$. We discuss the possibility of relaxing the conditions of Theorem 1 of [1]. As a result, the following refined formulation of the theorem on the existence and uniqueness of the maximum point of the function F is obtained.

Theorem. *Let the following conditions be satisfied*:

(1) $\mathbf{P}(0 < \alpha_j < 1) > 0$, $j = 1,\dots,k$;

(2) *the system of vectors* $\mathbf{a}_j = (a_{1j},\dots,a_{nj})$, $j = 1,\dots,k$, *is linearly dependent, and each vector is expressed in terms of the others with negative coefficients* $-c_j$, $c_j > 0$, $j = 1,\dots,k-1$;

(3) $c_k = \sum_{j=1}^{k-1} c_j b_j + b_k > 0$.

Then the function F *for* $u_k = -\sum_{j=1}^{k-1} c_j u_j + c_k$ *has in the positivity domain a unique stationary point, which is the point of the local* (*and also global*) *maximum.*

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O. E. Kudryavtsev (Rostov-on-Don, Russia)**. Monte Carlo method and Wiener–Hopf factorization in option pricing problems in Lévy models.** [17]

An application of the Monte Carlo method is very time consuming in the case of Lévy models for pricing options with payoff depending not only on the final price, but also on the price supremum (infimum). An approach combining the Monte Carlo method and the Wiener–Hopf factorization was proposed in [1]. This method is applicable for pricing exotic options in Lévy models admitting an explicit factorization. It should be noted that the time randomization used in [1], which in essence is equivalent to the partition of sample path on n parts, results in a slow convergence of the method (of order $O(n^{-1})$).

In the present research, the Monte Carlo method [1] is generalized to a wider class of Lévy processes. Construction of the characteristic functions of supremum (infimum) processes uses approximate formulas for Wiener–Hopf factors [2] with the inverse Laplace transform applied to them. The Laplace transform is inverted by using the Gaver–Stehfest algorithm. Thus, in the method considered, a simulation

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of the supremum (infimum) processes is implemented directly, which results in an essential gain in the computing speed.

The Monte Carlo method suggested can be optimized by means of a parallel computing application based on nVidia CUDA API for simulating sample paths of a joint distribution of the asset price and the maximal (minimal) price.

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E. Lepinette, J. Baptiste, L. Carassus (Paris, France)**. Pricing without martingale measure.**

Our first motivation is to characterize the minimal super-hedging prices of a European claim under a very weak no-arbitrage condition. It appears that it is possible to solve the problem first in a two-step model without supposing any no-arbitrage condition. The key tool is a new theorem stating that a conditional essential supremum is actually a pointwise supremum, i.e., just a deterministic supremum once $\omega \in \Omega$ is fixed. For this two-step model, the minimal price of any nonnegative payoff is either nonnegative or $-\infty$. In the second case, that corresponds to the existence of negative prices for the zero claim and, more generally, for any arbitrarily fixed Call option. Therefore, the no-arbitrage condition we consider is the nonnegativity of the price of any fixed Call option. Clearly, this condition is observed in practice. Moreover, it is weaker than the classical no-arbitrage condition equivalent to the existence of a risk-neutral probability measure. Finally, we propose a rather general model allowing one to reiterate the arguments of the two-step model and deduce the minimal super-hedging prices. This contribution is innovative in the sense that we may consider models which do not admit any martingale measure under a very natural condition observed in real markets.

D. I. Lisovskii (Moscow, Russia)**. Sequential hypothesis testing problem for stationary Gauss–Markov processes.**[18]

We consider continuous in probability stationary Gauss–Markov processes. According to the classical Doob's result [1], such a class coincides with the class of stationary Ornstein–Uhlenbeck processes. It is assumed that there are two hypotheses to distinguish,

$$H_0\colon\quad dX_t = \theta(\mu - X_t)\,dt + \sigma\,dB_t, \qquad X_0 \sim \mathscr{N}\left(\mu, \frac{\sigma^2}{2\theta}\right),$$

$$H_1\colon\quad dX_t = \gamma(\mu - X_t)\,dt + \sigma\,dB_t, \qquad X_0 \sim \mathscr{N}\left(\mu, \frac{\sigma^2}{2\gamma}\right),$$

where $\sigma > 0$, $\mu \in \mathbf{R}$ is a mean function of X, the parameters $\theta, \gamma > 0$ stand for the speed of mean-reversion of the observable process, and $B = (B_t)_{t\geqslant 0}$ is a standard Brownian motion independent of the initial value X_0 (all the processes and random variables are assumed to be given on some probability space $(\Omega, \mathscr{F}, \mathbf{P})$).

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Following A. Wald, we assume that any decision rule $\Delta = \Delta(\tau, d)$ is given by the following pair: a stopping moment τ, which is a Markov time adopted to the natural filtration $\mathbb{F}^X$ generated by the observable process X, and a function of the final decision d, which is an $\mathscr{F}_\tau^X$-measurable random variable that can take only two different values such that each can be identified with a decision in favor of the hypothesis H_0 or H_1. According to Liptser and Shiryaev [2], a decision rule $\Delta^* = \Delta(\tau^*, d^*)$ is said to be optimal in the class of all sequential schemes provided the probabilities of a wrong terminal decision are given and to be fixed if it minimizes the Kullback–Leibler divergence in this class.

We are concerned with the SPRT (sequential probability ratio test), which is known [3], [4] to be optimal for a number of models. We show [5] that for the above problem the SPRT ceases to be optimal but is still asymptotically optimal only in the most interesting cases: when the error probabilities of the first and second kinds tend to zero and when the tested parameters go off to infinity but the distance between them is fixed.

This talk is based on joint work with A. N. Shiryaev.

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A. V. Makarova, V. A. Gorlov (Voronezh, Russia)**. Stochastic inclusions with current velocities having decomposable right-hand sides.**

A natural analogue of standard physical velocity of a deterministic curve is the current velocity (the symmetric mean derivative of a random process, which was introduced by Edward Nelson). If the current velocity and the quadratic derivative in the mean are given, then under certain conditions it is possible to construct a process with a prescribed current velocity and quadratic derivative. Yu. E. Gliklikh and S. V. Azarina showed that the solution exists under very stringent conditions when the multivalued current velocity and a single-valued quadratic derivative are given (see [1]). Hence, of special importance is the further investigation of inclusions of this kind for more general settings involving the current velocity and the quadratic derivative. An existence theorem for stochastic differential inclusions defined in terms of the so-called current velocities is obtained. The right-hand sides involving the current velocity and the quadratic derivative are multivalued, lower semicontinuous, and decomposable.

Theorem 1. *Let multivalued fields* $\mathbf{v}$ *and* $\boldsymbol{\alpha}$ *on* $\mathscr{T}^n$ *be lower semicontinuous and uniformly bounded and have closed decomposable images of points. Consider a random element* ξ_0 *with values in* $\mathcal{T}^n$ *such that the density* ρ_0 *with respect to the Euclidean volume form* Λ_E *is smooth and vanishes nowhere. Then, for the initial condition* $\xi(0) = \xi_0$*, the inclusion has a well-posed solution on the entire interval* $t \in [0, T]$.

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G. V. Martynov (IITP RAS, Moscow, Russia)**. Notes on the Cramér–von Mises test with estimated parameters.** [19]

We consider various ways of applying the Cramér–von Mises test for testing the hypothesis that the distribution of the observed random variable belongs to a parametric family. The universal martingale method of Khmaladze is well known for the transformation of empirical processes (see [1]). This method leads to statistics with a limiting distribution that does not depend on the parametric family and also on the unknown value of the parameter. The previously known methods, however, can be easily applied to the most well known families of distributions of the form $\{G((x-m)/\sigma),\ -\infty < x, m < \infty,\ \sigma > 0\}$, $\{R((x/\beta)^\alpha),\ \alpha, x, \beta > 0\}$, or $\{R((x/\beta)^\alpha), \alpha, x, \beta > 0\}$ (see [3]). These families include, for example, normal, lognormal, Weibull, Pareto, exponential, and double exponential distributions, as well as other distributions. The aim of the present paper is to describe the method of using the Cramér–von Mises test in the case when the limiting distribution of statistics depends on an unknown parameter. In this case, specially calculated tables are used during testing of the hypothesis with the approximation of an unknown parameter by its estimate. The results of paper [2] can be used here. The corresponding error in the significance level is also analyzed.

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E. Yu. Mashkov (Kursk, Russia)**. On solvability of singular stochastic Leontief-type equation with impulse action II.** [20]

By a stochastic Leontief-type equation we mean a special class of stochastic differential equations in the Itô form such that both their left- and right-hand sides contain rectangular real matrices that form a singular pencil (see [1]). In addition, the right-hand side contains, first, a deterministic summand, which depends only on time, and second, an impulse action (see [2]). It is assumed that the diffusion coefficient of the system is given by a matrix depending only on time. For investigation of the equation it is required to consider derivatives of the free terms (including the Wiener process) of sufficiently high orders. In this connection, to differentiate the Wiener process, we apply the machinery of Nelson mean derivatives (see [3]) of random processes, which makes it possible to avoid using the theory of distributions in the study of the equation.

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THEOREM. *Under the above hypotheses, analytic formulas hold for the solutions of the equation in terms of symmetric derivatives in the mean of random processes.*

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V. V. Misyura (Rostov-on-Don, Russia)**. Application of online teaching methods for predicting the flow of events in stochastic models with uncertain parameters.** [21]

The talk, which is based on the paper [1], is concerned with online learning technology for forecasting the flow of random events in stochastic models with uncertain drift and volatility in discrete time. We follow the approach where, instead of a single model, one considers a set of possible models as experts or decision rules from pattern recognition. Experts make predictions of the future outcome. We define the forecast as a real number from the interval $[0,1]$. We assume that the loss function $l(y,z)$ is defined, where $y \in \{0,1\}$, $z \in [0,1]$. The predictive algorithm observes the estimates from experts and assesses their effectiveness. The predictive algorithm is based on problems of linear and nonlinear programming. The goal is to minimize the difference between the loss of the algorithm and the loss of the best mixed model that has the minimum amount of loss. The following main result is proved. If $l(1,x)$ and $l(0,x)$ are convex functions, then the minimax local forecast z_{t+1} is a solution of the following equation: $l(1,z) - l(0,z) = \sum_{i=1}^{t} l\big(y_i,(x_{u_*^1})_i\big) + l\big(1,(x_{u_*^1})_{t+1}\big) - \big[\sum_{i=1}^{t} l\big(y_i,(x_{u_*^0})_i\big) + l\big(0,(x_{u_*^0})_{t+1}\big)\big]$. The computational experiment is carried out on the basis of models with indeterminate drifts and volatility, which is used as an alternative to the Black–Scholes model [2]. The forecast results show the expedience and effectiveness of the method proposed.

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F. S. Nasyrov (Ufa, Russia)**. A deterministic approach to stochastic maximum principle.**

The solution of the minimization problem of the objective functional

$$J_E(u(\cdot)) = \mathbf{E}\{J(u(\cdot))\}, \qquad J(u(\cdot)) = \int_0^T f_0(t,x(t),u(t))\,dt + g_0(T,x(T))$$

under the constraints

$$dx(t) = \sigma(t,x(t))\,dW(t) + b(t,X(t),u(t))\,dt, \quad x(0) = x_0, \tag{1}$$

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using the maximum principle, is called the stochastic maximum principle; here $W(t)$ is a Wienier process, and the first term in the right-hand side of (1) is a stochastic Itô integral.

In parallel with this problem, we also consider the appropriate pathwise anticipative control problem of minimizing the pathwise cost functional $J(u(\cdot))$ under the same constraints (1); this problem is called the pathwise problem of the maximum principle.

We study the problem of constructing nonanticipative controls $u(\cdot)$ in both problems.

(a) It is shown that the solutions of both problems are reduced to those of similar problems such that the cost functional is modified by a newly introduced additional term (the Lagrange multiplier $\Lambda(\cdot)$); this idea seems to date back to Davis (see [1]).

(b) It is proved that the boundary-value problem of the stochastic maximum principle can be obtained from the pathwise problem by imposing the "nonbias" condition on the Lagrange multiplier.

(c) It turns out that, using the machinery of symmetric integrals, the pathwise maximum principle can be extended to the case when, instead of the Wienier process $W(t)$, one takes an arbitrary random process with continuous realizations $v(t)$, and in equation (1) the first term on the right is a symmetric integral with respect to the process $v(t)$. Moreover, the boundary-value problem of the maximum principle retains its form. However, in this case, the choice of the Lagrange multiplier is in general nonunique, which calls for the refinement of the statement of the problem itself.

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A. V. Nikitina (Rostov-on-Don, Russia), **A. A. Semenyakina** (Taganrog, Russia). **Modeling of phytoplankton production and destruction processes in shallow water based on the stochastic approach.** [22]

The talk covers the water pollution process by biogenic elements entering with river runoff, as well as a result of natural and industrial challenges; this process is considered probabilistic [1]. The stochasticity is conditioned by many factors, including anthropogenic, climatic, biological, and morphological ones, that define the pollution and phytoplankton concentrations in the control range.

The model of the production-destruction processes of phytoplankton reads as

$$\frac{\partial q_i}{\partial t} + \operatorname{div}(\mathbf{U}, q_i) = \operatorname{div}(\mathbf{k}_i \operatorname{grad} q_i) + R_i, \qquad \mathbf{k}_i = \{\mu_i, \mu_i, \nu_i\}, \quad i = 1, \dots, 10,$$

where q_i is the concentration of the ith component; $\mathbf{u}$ is the velocity vector of the water flow, $\mathbf{u} = \{u, v, w\}$; $\mathbf{U}$ is the matter convective transport velocity, $\mathbf{U} = \{U, V, W\}$; $\mathbf{U} = \mathbf{u} + \mathbf{u}_{0i}$, $\mathbf{u}_{0i}$ stands for the sedimentation velocity of the ith component; R_i is the chemical-biological source, where index i corresponds to the type of substance: 1–3 are substance concentrations from algae *Chlorella vulgaris, Aphanizomenon flos-aquae*,

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and *Sceletonema costatum*, respectively, 4 is PO_4, 5 is POP, 6 is DOP, 7 is NO_3, 8 is NO_2, 9 is NH_4, 10 is Si (here PO_4 are phosphates, POP is suspended organic phosphorus, DOP is dissolved organic phosphorus, NH_4 is ammonium, NO_2 are nitrites, NO_3 are nitrates, and Si is dissolved inorganic silicon); and μ_i, ν_i are diffusion coefficients in the horizontal and vertical directions.

The system is augmented with initial and boundary conditions. The development of stochastic models of mass transfer velocities, which is based on the Mitcherlich conjecture on the simultaneous effect of factors on the mass transfer velocity, includes the development of models for organic matter production velocity in water and destruction by the phyto-, zoo- and bacterial plankton.

An algorithm to verify the adequacy of the developed probabilistic models of observation is derived. Verification of the convergence of the actual (measured) and calculated (simulated) values is carried out on the basis of the randomness criterion: $\delta = D_\Delta / D$, where D and D_Δ are, respectively, the variance of a number of actual values of the parameter and its random component caused by the impact of random elements. If $\delta < 0.7$, then the value is accepted as satisfactory. The agreement of the calculated value with the actual one is considered satisfactory if the difference does not exceed 0.7σ in absolute value, where σ is the standard deviation of the original actual series.

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I. V. Pavlov, I. V. Tsvetkova (Rostov-on-Don, Russia). **Ranking of variables in order of their smallness when solving systems of inequalities for finding weakly interpolating martingale measures.** [23]

Consider a measurable space $(\Omega, \mathscr{F})$ and a filtration $\mathbf{F} = (\mathscr{F}_k)_{k=0}^1$ on Ω such that $\mathscr{F}_0 = \{\Omega, \varnothing\}$, $\mathscr{F}_1 = \sigma(B_1, B_2, \dots)$, where $\{B_1, B_2, \dots\} \subset \mathscr{F}$ is a decomposition of Ω. For a process $Z = (Z_k, \mathscr{F}_k)_{k=0}^1$, we denote $a := Z_0$, $b_i := Z_1|_{B_i}$, $i \in \mathbf{N} = 1, 2, \dots$. Let $\mathscr{P}(Z, \mathbf{F})$ be the set of probability measures P on $(\Omega, \mathscr{F})$ such that $p_i := P(B_i) > 0$, $i \in \mathbf{N}$, and the process $Z = (Z_k, \mathscr{F}_k, P)_{k=0}^1$ is a martingale. We assume that the sequence $\{b_i\}_{i=1}^\infty$ contains r $(3 < r < \infty)$ different values (for example, $b_1 < b_2 < \dots < b_r$), and m_i is the order of the value b_i, $1 \leqslant i \leqslant r$, $1 \leqslant m_i \leqslant \infty$. An essentially different case, where $\{b_i\}_{i=1}^\infty$ contains a countable number of different values, was considered by A. G. Danekyants and N. V. Neumerzhitskaia in the abstracts for ICSM-3. They also gave the definition of a weakly interpolating measure from $\mathscr{P}(Z, \mathbf{F})$; this definition is equivalent to the following weakened noncoincidence barycenter condition (WNBC). A martingale measure $P = (p_1, p_2, \dots)$ is said to belong to WNBC(Z) if $b_i \neq \sum_{j \in J} b_j p_j \;/\; \sum_{j \in J} p_j$ for any $i \in N$ and any index set $J \subset N$ that does not contain i and is such that the complement $\overline{J} = N \setminus J$ is finite. We are interested in the case where WNBC(Z) $\neq \varnothing$ without the assumption that numbers $\{b_i\}_{i=1}^r$ are rational (if all these numbers are rational, then WNBC(Z) $\neq \varnothing$; see [1]). Earlier it was proved that (1) WNBC(Z) $\neq \varnothing$ if $r = 3$, at least two of the numbers m_1, m_2, m_3 are infinite, and either $b_1 < a < b_2$ or $b_2 < a < b_3$ (see [1]); (2) WNBC(Z) $\neq \varnothing$ for $r = 4$, $m_1 = m_2 = m_3 = m_4 = \infty$, $b_1 < a < b_2$ (see [2]).

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The result of the present study is that $\mathrm{WNBC}(Z) \neq \varnothing$ for $3 < r < \infty$, $m_1 < \infty$, $\ldots, m_{r-2} < \infty$, $m_{r-1} = m_r = \infty$, $b_1 < a < b_2$. The underlying idea is that in a special system of inequalities that gives measures from $\mathrm{WNBC}(Z)$, the unknown variables are subdivided by the degree of smallness into three groups, which allows one to solve this system.

In conclusion, we note that measures from $\mathrm{WNBC}(Z)$ are used to transform incomplete financial markets into complete ones.

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I. V. Pavlov, S. I. Uglich (Rostov-on-Don, Russia)**. Investigation of maximum points of the objective function of a quasi-linear system with priorities and the minimax problem.** [24]

Consider the functions $F_j(x) = \bigl(\sum_{i=1}^n a_{ij}x_i + b_j\bigr) I\bigl\{\sum_{i=1}^n a_{ij}x_i + b_j > 0\bigr\}$, $j = 1, 2, \ldots, k$, where I_A is the indicator of a set A. Let $\alpha_j = \alpha_j(\omega)$ be an arbitrary r.v. defined on a probability space $(\Omega, \mathscr{F}, \mathbf{P})$ and assuming values from the interval $(0; 1]$.

The first part of the talk is dedicated to the study of the maximum of the objective function $F = \prod_{j=1}^k \mathbf{E} u_j^{\alpha_j}$, where $\mathbf{E}$ denotes the expectation with respect to probability $\mathbf{P}$, and all $u_j > 0$. It follows from [1] that the stationary points in the positivity domain of the function F exist only under the condition that for vectors of the system $\mathbf{a}_j = (a_{1j}, \ldots, a_{nj})$, $j = 1, \ldots, k$, the representation $\mathbf{a}_k = -\sum_{i=1}^{k-1} c_i \mathbf{a}_i$ holds, where all c_i are strictly positive numbers. In this case, $u_k = -\sum_{i=1}^{k-1} c_i u_i + \sum_{i=1}^{k-1} c_i b_i + b_k > 0$. According to [1], if in addition $c_k := \sum_{i=1}^{k-1} c_i b_i + b_k > 0$, then the objective function F has in the domain $F > 0$ a unique stationary point $x_{\max}(c_1, \ldots, c_{k-1})$, which is the maximum point of the function F.

In the second part of the talk, given a fixed $\mathbf{a}_j$, $j = 1, \ldots, k-1$, we vary the model vector of the coefficients of $\mathbf{a}_k$ under the condition $b_i > 0$, $i = 1, \ldots, k$, taking various strictly positive values of the parameters $(c_1, \ldots, c_{k-1})$. As a result, we get the functions $x_{\max}(c_1, \ldots, c_{k-1})$ and $F_{\max} = F(x_{\max}(c_1, \ldots, c_{k-1}))$. We write down a system of equations that can yield stationary points of the function $F_{\max}$, which are often the minimum points. In conclusion, numerical illustrations with minimax are given.

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E. A. Pchelintsev, S. S. Perelevskiy (Tomsk, Russia). **Estimation of the drift coefficient in diffusion processes.** [25]

Let the stochastic differential equation $dy_t = S(y_t)\,dt + dw_t$, $0 \leqslant t \leqslant T$, be defined on a probability space $(\Omega, \mathscr{F}, \mathbf{P})$, where $(w_t)_{t\geqslant 0}$ is a scalar standard Wiener process, the initial value y_0 is a given constant, and $S(\cdot)$ is an unknown drift function from the functional class $\Sigma_{L,N}$, which was defined in [1]. The problem is to estimate the function $S(x)$, $x \in [a,b]$, from observations of the process $(y_t)_{0\leqslant t\leqslant T}$ and to obtain a sharp nonasymptotic bound for the quadratic risk. An asymptotically efficient model selection procedure based on weighted LSE $\widehat{S}$ was proposed in [1] for estimation of the function S. In the present talk, a model selection procedure is proposed based on improved estimates S^*; this procedure outperforms the estimate from [1] in the mean square accuracy.

THEOREM. *The estimate S^* outperforms the estimate $\widehat{S}$ in the mean square accuracy; i.e.,* $\sup_{S\in\Sigma_{L,N}}\big(\mathbf{E}_S\|S^* - S\|^2 - \mathbf{E}_S\|\widehat{S} - S\|^2\big) < 0$, *where $\|\cdot\|$ is the $L_2[a,b]$-norm.*

For an improvement of the nonasymptotic estimation quality, we use special shrinkage estimates from [2], [3]. A sharp nonasymptotic oracle inequality is proposed for a quadratic risk of the proposed estimate.

This is a joint work with S. Pirogov, A. Vladimirov, A. Yambartsev, and G. Schütz.

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E. A. Pechersky (Moscow, Russia). **Large deviations for a class of Markov processes.** [26]

The main object of this investigation is continuous-time Markov processes taking their values in $\mathcal{N} \times \mathbf{Z}_+^d$, $\mathcal{N} = \{0, 1, \dots, N\}$ and $d \in \mathbf{N}$. Let $\zeta = (\xi, \eta_1, \dots, \eta_d)$ be such a process. The transition probabilities are defined by the generator $\mathbf{L}$ acting on functions $g\colon \mathcal{N} \times \mathbf{Z}_+^d \to \mathbf{R}$. The generator $\mathbf{L}$ depends on positive real numbers $\lambda, \mu_1, \dots, \mu_d$ and triplets (k_i, r_i, s_i), $i = 1, \dots, d$, of integers; here we assume that all the triplets are different for different i and, moreover, $k_i \geqslant r_i \geqslant s_i \geqslant 1$. Then for the generator of ζ we have

$$\begin{aligned}\mathbf{L}g(m,\ell_1,\dots,\ell_d) &= \lambda(N-m)[g(m+1,\ell_1,\dots,\ell_d) - g(m,\ell_1,\dots,\ell_d)]\\ &\quad + \sum_{i=1}^d \mu_i\pi_i(m)[g(m-s_i,\ell_1,\dots,\ell_i+s_i,\dots,\ell_d) - g(m,\ell_1,\dots,\ell_d)], \qquad (1)\end{aligned}$$

where $\pi_i(m) = \binom{m}{r_i}\binom{N-m}{k_i-r_i}$.

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We are concerned with the mean field theory.

We study large deviations of ζ on the path level. The process ζ is considered on the time-interval $[0,T]$. Our main interest is the behavior of certain functionals of the process paths as $N\to\infty$. In particular, we study the function

$$\widehat{x}(t)=\lim_{N\to\infty}\mathbf{E}\biggl(\frac{1}{N}\xi(t)\;\bigg|\;\frac{1}{N}\sum_{i=1}^{d}\eta_i(T)>B\biggr).$$

THEOREM. *There exists $i_0\in\{1,\dots,d\}$ independent of the parameters defining the process ζ such that*

$$\lim_{B\to\infty}\hat{x}(t)\equiv\frac{r_{i_0}}{k_{i_0}+s_{i_0}} \tag{2}$$

on the open interval $(0,T)$.

A preliminary result in this direction was published in [1], where we studied the case $d=1$ and the triple $(1,1,1)$. In this case, the limit in (2) is equal to $1/2$.

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M. V. Platonova, K. S. Ryadovkin (St. Petersburg Department of Steklov Mathematical Institute of Russian Academy of Sciences, St. Petersburg, Russia)**.**
A branching random walk on graphene lattice. [27]

We consider a branching random walk on a graphene lattice with periodic sources of branching and continuous time. The case of a homogeneous branching random walk with periodic sources of branching was considered in [1]. A homogeneous branching random walk with the finite number of branching sources was studied in detail (see [2], [3], and references therein). Consider the set $\Gamma=\{g\in\mathbf{Z}^2\colon g=n_1g_1+n_2g_2,\ n_j\in\mathbf{Z},\ j=1,2\}$, where $g_1=(1,0)$, $g_2=(0,2)$. We assume that the matrix of transition intensities is a periodic matrix with respect to the lattice G; i.e., $a(v,u)=a(u,v)=a(v+g,u+g)$ for each vector $g\in\Gamma$. Given $v_1=(0,0)$, $v_2=(0,1)$, we assume that $a(v_1,v_1)=-3$, $a(v_1,v_2)=1$, $a(v_1,v_2-g_1)=1$, $a(v_1,v_2-g_2)=1$, $a(v_2,v_2)=-3$, and $a(v_1,u)=0$ for the remaining vertices u. Next we assume that a branching source with intensity β_1 is located at the vertices $v=v_1+\Gamma$ and that a branching source with intensity β_2 is located at the vertices $v=v_2+\Gamma$.

Denote by $M(v_j+\gamma_{v_j},v_k+\gamma_{v_k},t)$ the expected value of the number of particles at time t at point $v_k+\gamma_{v_k}$ if at moment $t=0$ there is one particle at point $v_j+\gamma_{v_j}$. We show that, as $t\to\infty$,

$$M(v_j+\gamma_{v_j},v_k+\gamma_{v_k},t)=\frac{e^{\lambda_1(0)t}}{2\pi t}\sqrt{\frac{(\beta_1-\beta_2)^2+36}{48}}\,\frac{\psi_1(v_k,0)\psi_1(v_j,0)}{\|\psi_1(0)\|^2_{\ell_2(\Omega)}}\bigl(1+O(t^{-1})\bigr),$$

where $j,k=1,2$; $\gamma_{v_j},\gamma_{v_k}\in\Gamma$; $\lambda_1(0)$ is the largest eigenvalue of the matrix

$$A(0)=\begin{pmatrix}-3+\beta_1 & 3\\ 3 & -3+\beta_2\end{pmatrix};$$

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and $\psi_1(v_j, 0)$ is the jth component of the normalized eigenfunction of the matrix $A(0)$ corresponding to the eigenvalue $\lambda_1(0)$.

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V. V. Rodochenko, O. E. Kudryavtsev (Rostov-on-Don, Russia). **On using the Laplace transform for evaluation of the Wiener–Hopf factors for option pricing in stochastic volatility models with jumps.** [28]

The machinery of [1] is capable of reducing the problem of pricing barrier options in stochastic volatility models with jumps to a certain sequence of 1D integro-differential equations. The kernels of the equations are defined by Lévy processes X_t obtained from the initial model by using the tree-structured volatility process approximations from [2].

An analytic solution for each of the problems can be expressed in terms of the Laplace–Carson transforms $\phi_q^+(\xi)$ and $\phi_q^-(\xi)$ of the corresponding characteristic functions of supremum and infimum processes $\overline{X}_t$ and $\underline{X}_t$, respectively.

In the present talk, we consider the advantages of the application of the approximate formulas consistent with those from [3] for the Wiener–Hopf factors $\phi_q^{\pm}(\xi)$. In the case of the Heston model, we can compare the explicit formulas for the factors with the approximate ones. Numerical experiments show that our implementation of approximate formulas has an error of at most 1% for a reasonably low number of space points of 2^{10}–2^{12} with reducing the speed of calculations by approximately 1.5 times. The comparison of the option prices obtained demonstrates a similar difference. We conclude that the approximate Wiener–Hopf factorization formulas can be efficiently used for general stochastic volatility models with jumps, including the famous Bates model.

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D. B. Rokhlin (Southern Federal University, Rostov-on-Don, Russia). ***Q*-learning in a stochastic Stackelberg game.** [29]

We consider a game between a leader and a follower, where the players' actions affect the stochastic evolution of the state process x_t, $t \in \mathbb{Z}_+$. The players observe

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their rewards and the system state x_t. Each player does not know the transition kernel of the process x and the reward function of the other player. At each stage of the game the leader is the first to select action a_t. This action is known to the follower before he or she selects b_t. The follower's actions are unknown to the leader (an uniformed leader). Each player tries to maximize his or her discounted reward by applying the Q-learning algorithm [1]. A special feature of the algorithm under consideration is that, when updating his or her Q-function, the follower believes that the action of the leader in the next state is the same as in the current one (a naive follower). Under other assumptions, the Q-learning algorithm for the stochastic Stackelberg game was considered in [2].

Let X be a finite state space. Denote by A and B the sets of admissible actions of the leader and the follower. Assume that the evolution of the system is characterized by the transition kernel $p(y|x,a,b)$. This means that if the system is at a state $x \in X$ and if the leader and follower select $a \in A$, $b \in B$, then the probability of transition to the state $y \in X$ is equal to $p(y|x,a,b)$. At each stage of the game

—the leader observes the system state $x_t \in X$ and selects an action $a_t \in A$;

—the follower observes the system state and the leader's action and selects an action $b_t \in B$;

—the system moves to a state x_{t+1} with probability $p(x_{t+1} \mid x_t, a_t, b_t)$;

—the leader and the follower get the rewards $r_1(x_t, a_t, b_t, x_{t+1})$ and $r_2(x_t, a_t, b_t, x_{t+1})$, respectively.

These rules imply the inequality of players which is typical for a Stackelberg game.

Randomized strategies of players are defined as Boltzmann distribution depending on the Q-functions Q^l, Q^f of the leader and the follower that are updated in the course of learning. So, at each stage of the game the leader and the follower sequentially select their actions $a \in A$, $b \in B$ with probabilities

$$\frac{\exp(Q^l(x,a)/\tau_1)}{\sum_{a' \in A} \exp(Q^l(x,a')/\tau_1)}, \qquad \frac{\exp(Q^f(x,a,b)/\tau_2)}{\sum_{b' \in B} \exp(Q^f(x,a,b')/\tau_2)}.$$

It is shown that the existence of deterministic stationary strategies generating an irreducible Markov chain is sufficient for the convergence of the algorithm. The proof is based on the known results [3] developing the idea of stochastic approximation.

The limiting large time behavior of Q-functions is described in terms of controlled Markov processes, which for clarity are related to a virtual leader and a virtual follower. The distributions of the players' actions converge to the Boltzmann distributions, depending on the limiting Q-functions. We also consider the behavior of these limiting distributions for small "temperature" parameters τ_i.

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V. V. Rykov, D. V. Kozyrev (Moscow, Russia). **On problems of sensitivity of stochastic models.** [30]

Consider a renewable hot double redundant system such that its components fail according to exponential law with intensities α_i and are renewed during random time distributed with general p.d.f. $B_i(x)$ $(i=1,2)$. We set $b_i(x)=B_i'(x)$, $\widetilde{b}_i(s)=\int_0^\infty e^{-sx}b_i(x)\,dx$, $b_i=\int_0^\infty(1-B_i(x))\,dx$, $\rho_i=\alpha_i b_i$ $(i=1,2)$. We give analytic expressions for the reliability function of the system, stationary and quasi-stationary probabilities (q.s.p.'s) of its states $\overline{\pi}_j=\lim_{t\to\infty}\mathbf{P}\{J(t)=j\mid T>t\}$, where T is the system lifetime, as well as their asymptotic insensitivity to the shape of renewal time distributions. Results for q.s.p.'s are also given.

THEOREM 1. *The q.s.p.'s of the model under consideration have the form*

$$\overline{\pi}_0=\left[1+\frac{\alpha_1}{\alpha_2-\gamma}\big(1-\widetilde{b}_1(\alpha_2-\gamma)\big)+\frac{\alpha_2}{\alpha_1-\gamma}\big(1-\widetilde{b}_2(\alpha_1-\gamma)\big)\right]^{-1},$$

$$\overline{\pi}_i=\alpha_i\frac{(1-\widetilde{b}_i(\alpha_{i^*}-\gamma))(\alpha_i-\gamma)}{(\alpha_i-\gamma)(\alpha_{i^*}-\gamma)+\alpha_i\phi_i(-\gamma)+\alpha_{i^*}\phi_{i^*}(-\gamma)}\qquad(i=1,2),$$

where $i^*=2$ *for* $i=1$, *and conversely*, γ *is a root of the equation* $\phi_1(s)+\phi_2(s)=-s$, *and* $\phi_i(s)=\alpha_i(1-\widetilde{b}_i(s+\alpha_{i*}))$ $(i=1,2)$.

The result demonstrates the clear sensitivity of the system q.s.p.'s to the shapes of the renewal time distributions. However, this sensitivity becomes negligible for rare failures of the system components.

THEOREM 2. *Under the rare system component failures, when* $\max\{\alpha_i,\, i=1,2\}\to 0$, *its q.s.p.'s become asymptotically insensitive to the shape of their component renewal time distributions and have the form*

$$\pi_0\approx\frac{1}{1+\rho_1+\rho_2},\qquad \pi_i\approx\frac{\rho_i}{1+\rho_1+\rho_2}\quad(i=1,2).$$

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N. A. Saifutdinova (Don State Technical University, Rostov-on-Don, Russia). **On the randomization of the elasticity coefficients in the resources allocation problem.**

This talk presents a model similar to that considered in [1]. Consider the function $F=f_1+f_2$, where $f_1=x^{\alpha_1}y^{1-\alpha_1}$, $f_2=(1-x)^{\alpha_2}(1-y)^{1-\alpha_2}$ describe the performance of two economic objects (we assume that $0\leqslant\alpha_1\leqslant1$ and $0\leqslant\alpha_2\leqslant1$ are some constants). The problem of maximization of the function F on the set $D=\{(x,y)\colon 0\leqslant x\leqslant1,\ 0\leqslant y\leqslant1\}$ is solved; i.e., we find an optimal allocation of resources x,y leading to the maximum economic effect. In [2] it was shown that for $\alpha_1\to0$ and $\alpha_2\to1$ (or, in the symmetrical setting, $\alpha_1\to1$ and $\alpha_2\to0$) it is possible to obtain values of F close to 2. Further, we assume that the indicators α_1 and α_2 are

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the result of various expert recommendations on the choice of production technology, which leads us to consider α_1 and α_2 as random variables. Consider the case when these two random variables are dependent or, more precisely, $\alpha_2 = 1 - \alpha_1$. We set $\alpha_1 = \alpha$. In the case when α is a random variable with uniform distribution on $[0; 1]$, it turns out that $\mathbf{E}F \leqslant 1$. The values of $\mathbf{E}F$ close to 2 can be achieved if as $\varepsilon \to 0$ we consider the random variable of α with uniform distribution on the interval $[0; \varepsilon]$ (or on the interval $[1 - \varepsilon; 1]$).

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V. V. Shamraeva (Moscow, Russia)**. Some models of the financial market with an infinite number of buyers of shares.** [31]

We consider an arbitrage-free and incomplete (B, S)-market defined on the set $\{\Omega, \mathbf{F}\}$, where $\Omega = \{\omega_i\}_{i=1}^{\infty}$, $\mathbf{F} = (\mathscr{F}_0, \mathscr{F}_1)$ is a one-step filtration ($\mathscr{F}_0 = \{\Omega, \varnothing\}$, and $\mathscr{F}_1$ is the σ-algebra of all subsets of Ω). By $Z = (Z_n, \mathscr{F}_n)_{n=0}^{1}$ we denote an $\mathbf{F}$-adapted random process, which we think of as the discounted value of shares ($Z_0 = a$, $Z_1(\omega_i) = b_i$, $b_i > 0$, $i = 1, 2, \dots$). We say that a measure P satisfies the *noncoincidence barycenter condition* (NBC) if the series $\sum_{i=1}^{\infty} b_i p_i$ converges absolutely and $\sum_I b_i p_i / \sum_I p_i \neq \sum_J b_j p_j / \sum_J p_j$ for any $I, J \subset \mathbf{N}$ such that $I \cap J = \varnothing$ and $|I| \leqslant |J|$. The set of nondegenerate martingale measures P of the original (B, S)-market is denoted by $\mathscr{P}(Z, \mathbf{F})$; by NBC$(Z)$ we denote the class of martingale measures satisfying NBC.

Lemma 1. *Let* $b_1 < b_2 < b_3 < \cdots$. *If*

$$(b_i - b_{i-1}) \min_{1 \leqslant j \leqslant i-1} p_j > \sum_{j=i+1}^{\infty} b_j p_j \quad \forall\, i \geqslant 2,$$

then the measure P *satisfies NBC.*

Note that the inequality from Lemma 1 for $i = 2$ and $P \in \mathscr{P}(Z, \mathbf{F})$ implies the inequality $a < b_2$. For such measures, the theorem on the nonemptiness of NBC(Z) from [1] also holds minor refinements. The result can be instrumental in updating the algorithms for calculating share prices in the market, interpolating the initial (B, S)-market, and evaluating fair prices of financial liability and components of the hedging portfolio [2].

Lemma 2. *Let* $\widehat{P} = \{P \in \mathscr{P}(Z, \mathbf{F})\colon b_i = \delta b_{i-1}\ \forall\, i \geqslant 2;\ p_i = \frac{1}{\delta+1} p_{i-1}\ \forall\, i \geqslant 3; \delta > 0\}$. *Then* $P \in \widehat{P}$ *does not satisfy NBC.*

Note that if $p_2 \geqslant 1/(\delta + 1)$, then $a > b_2$ under the hypotheses of Lemma 2.

Topical here is the consideration of arbitrage-free complete markets, i.e., markets such that $P \in$ NBC(Z).

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L. K. Shiryaeva (Samara, Russia)**. On properties of Grubbs statistics in the case of normal sample with outlier.**

Consider the Grubbs statistics

$$T_{n,(1)} = \frac{\overline{X} - \min\{X_i\}}{S} \quad \text{and} \quad T_n^{(1)} = \frac{\max\{X_i\} - \overline{X}}{S},$$

as calculated for a normal sample of size n (see [1]). Assume that in the sample $\{X_i\}_{i=1}^n$ there is one abnormal observation (outlier) and its number is unknown. We believe that the outlier differs from other observations by the shift α and scale parameters $\nu > 0$. We denote

$$G_{n,(1)}(t;\alpha,\nu) = \mathbf{P}(T_{n,(1)} < t), \quad G_n^{(1)}(t;\alpha,\nu) = \mathbf{P}(T_n^{(1)} < t),$$
$$\Upsilon_n(x,y;\alpha,\nu) = \mathbf{P}(T_{n,(1)} < x, T_n^{(1)} < y).$$

Recursive relationships for description of marginal functions of Grubbs statistics and the function of their joint distribution were found in [2]. An algorithm for computing such distribution functions is constructed. The effect of the parameters n, α, and ν on numerical characteristics of Grubbs statistics is studied. To investigate the strength of interdependence between Grubbs statistics, an algorithm for calculating estimates of the Spearman and Kendal rank correlation coefficients, as well as the Pearson linear correlation coefficient, is developed. Statistical modeling show that the force of interdependence between the marginals decreases with increasing n but increases with increasing ν and $|\alpha|$. The following theorem describes properties of the Grubbs distribution functions.

Theorem. *Let*

$$\Sigma_n = \left[\frac{1}{\sqrt{n}} \leqslant x \leqslant \frac{n-1}{\sqrt{n}};\, \theta_n(x) \leqslant y \leqslant \frac{n-1}{\sqrt{n}}\right], \qquad n > 2,$$

and

$$\theta_n(x) = \frac{x}{n-1} + \sqrt{n-2}\sqrt{1 - \frac{nx^2}{(n-1)^2}}.$$

Then, for all $(x,y) \in \Sigma_n$,

$$\Upsilon_n(x,y;\alpha,\nu) = G_{n,(1)}(x;\alpha,\nu) + G_n^{(1)}(y;\alpha,\nu) - 1.$$

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A. A. Shishkova (Tomsk, Russia)**. Hedging problem for the Asian option.**[32]

Consider a standard Black–Scholes model with several risky assets. We assume that the time horizon is $T = 1$. The riskless asset is a constant over the entire time interval ($B = 1$), and the risky assets with price processes $(S_i(t))_{1\leqslant i\leqslant d}$ are driven by the system of SDEs

$$dS_i(t) = \sigma_i S_i(t)\, dW_i(t), \qquad 0 \leqslant t \leqslant 1, \quad i = 1, \dots, d.$$

The Asian option payoff function is given by

$$f_1 = \biggl(\frac{1}{d}\int_0^1 \sum_{i=1}^{d} S_i(t)\, dt - K\biggr)_+,$$

where K is the strike price. The main result of the present work is the formula for calculating the hedging strategy

$$\gamma_i(t) = G'_{y_i}(t, \xi(t), S(t)), \qquad 0 \leqslant t \leqslant 1, \quad i = 1, \dots, d,$$

where

$$G(t,x,y) = \mathbf{E}\biggl(\frac{1}{d}\biggl(\sum_{i=1}^{d} x_i + \sum_{i=1}^{d} y_i \widetilde{\eta}_i(v)\biggr) - K\biggr)_+,$$

$\xi_i(t) = \int_0^t S_i(v)\, dv$, and $\widetilde{\eta}_i(v) = \int_0^v \exp\{\sigma_i W_i(u) - \sigma_i^2 u/2\}\, du$, $v = 1 - t$. Using a Brownian bridge, we find the densities of random variables $\widetilde{\eta}_i(v)$ and study the analytic properties (differentiability) of the densities obtained. The above problem is solved based on the results presented in [2]. The function $G(t, x, y)$ can be represented by the Itô formula, since we proved the following theorem for this function.

THEOREM 1. *Let $\widetilde{\eta}_i(v) = \int_0^v \exp\{\sigma_i W_i(u) - u\sigma_i^2/2\}\, du$ be random variables, and let $x = (x_1, \dots, x_d)$ and $y = (y_1, \dots, y_d)$ be vectors. Then the function*

$$G(t,x,y) = \mathbf{E}\biggl(\frac{1}{d}\biggl(\sum_{i=1}^{d} x_i + \sum_{i=1}^{d} y_i \widetilde{\eta}_i(t)\biggr) - K\biggr)_+$$

has continuous derivatives $\partial G/\partial t$, $\partial G/\partial x_i$, $\partial G/\partial y_i$, $\partial^2 G/\partial y_i^2$.

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S. G. Shorokhov (Moscow, Russia)**. On option prices in some local volatility models.**

We study the European option pricing problem in local volatility models when the asset dynamics is described by the stochastic differential equation $dS_t = rS_t\,dt + \sigma(S_t,t)S_t\,dW_t$, $S(t_0) = S_0 > 0$, where the volatility σ is a function of the asset price S_t and time t. Basically, the European option pricing problem reduces to determination of the transition probability density function via an initial value problem with delta function for the Fokker–Planck partial differential equation and consequent calculation of option prices satisfying the Black–Scholes–Merton partial differential equation with corresponding boundary conditions. From the analytic formula for the European call option price one can recover the volatility function by Dupire's formula [1].

We outline both well-known local volatility models [2] and give some new local volatility models related to the nonlinear partial differential equation for the volatility function from [3]. Application of local volatility models in derivative pricing and assessing market [4] and credit [5] risks is discussed.

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N. V. Smorodina (St. Petersburg Department of Steklov Mathematical Institute of Russian Academy of Sciences, St. Petersburg, Russia)**. Approximation of an evolution operator by mathematical expectations of functionals of Poisson random fields.** [33]

Consider an operator $H = -(1/2)d^2/dx^2 + V(x)$ on the domain $W_2^2(\mathbf{R})$. We assume that a potential V is real-valued and bounded, which implies that the operator H is self-adjoint. In this case, the family of operators e^{-itH} is a group of unitary operators in $L_2(\mathbf{R})$. The operator e^{-itH} maps a function $\varphi \in W_2^2(\mathbf{R})$ into the solution $u(t,x)$ of the Cauchy problem for the Schrödinger equation $i\partial u/\partial t = Hu$ with initial function $u(0,x) = \varphi(x)$ (for more details, see [1]). It is well known that for the heat equation $\partial u/\partial t = -Hu$ the solution of the Cauchy problem with the initial function $u(0,x) = \varphi(x)$ admits a probabilistic representation in the form of an expectation of a Wiener process functional (the Feynmann–Kac formula). Namely,

$$u(t,x) = e^{-tH}\varphi(x) = \mathbf{E}\biggl[\varphi(x+w(t))\exp\biggl\{-\int_0^t V(x+w(\tau))\,d\tau\biggr\}\biggr],$$

where $w(t)$ is a standard Wiener process. The last formula means that one can simulate the evolution of the initial function φ under the heat semigroup e^{-tH} by statistical methods. To this end, it suffices to be able to generate the trajectories of the Wiener process.

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In the present talk, a similar approach is developed for the operator e^{-itH}. Namely, we construct a family Q_ε^t of operators in $L_2(\mathbf{R})$, which depend on the additional parameter $\varepsilon > 0$ and possess the following properties:

(1) for each $\varepsilon > 0$, the family Q_ε^t is a semigroup; i.e., $Q_\varepsilon^{t+s} = Q_\varepsilon^t Q_\varepsilon^s$;

(2) the operator norm of the operator Q_ε^t is not greater than 1;

(3) the operator Q_ε^t is defined as the expectation of a Poisson point field functional;

(4) as $\varepsilon \to 0$ the operators Q_ε^t approximate the operator e^{-itH} in strong operator topology; i.e., $\|Q_\varepsilon^t \varphi - e^{-itH}\varphi\|_2 \to 0$ for any $\varphi \in L_2(\mathbf{R})$ for $\varepsilon \to 0$.

As in the case of the heat conduction equation, the evolution of the wave function can be statistically modeled under this approach by generating realizations of a random point field. It is also worth mentioning that the square of the wave function modulus is always a density of a probability distribution. The evolution of the wave function generates the evolution of the probability distribution density, which is usually called a "quantum random walk." The approach proposed here gives a theoretic possibility of simulating the "quantum random walk" by classical statistical machinery. A particular case of the above construction (for $V = 0$) can be found in [2].

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D. A. Suchkova (Ufa, Russia)**. Construction of the solution of the stochastic long wave equation (BBM) with white noise dispersion.**

The deterministic BBM (Benjamin–Bona–Mahony) equation

$$u_t + u_x + uu_x - u_{xxt} = 0, \tag{1}$$

as an approximation for the description of unidirectional propagation of waves with small wave-amplitude and large wavelength in nonlinear dispersive systems, has several advantages in comparison with the well-known Korteweg–de Vries equation [1]. In particular, the phase velocitiy and group velocity corresponding to (1) are bounded for all wave numbers; moreover, both velocities approach zero for large wave numbers.

The stochastic BBM equation (the regularized long wave equation with white noise dispersion)

$$du_t - du_{xx} + u_x * dW + uu_x\,dt = 0, \qquad u(s) = u_s, \tag{2}$$

is a more adequate model of particular physical systems which are stochastic in nature. The introduction of white noise in the dispersion term justifies this observation numerically [2]. The existence and uniqueness of the solution of problem (2) were proved previously in [2] for a certain class of functions.

It is shown that in order to find a solution to problem (2) it is sufficient to know the solution of the original problem (1); in this case, the solution to (2) is a determinate function of a Wiener process [3].

Theorem (on the structure of the solution). *Let $\phi(x,t)$ be a function of the deterministic BBM equation* (1)*. Then the function $u = -\phi(W(t) + x, t) - 1$ is a solution of problem* (2)*.*

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Using this method, the numerical simulation of the solution is performed.

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A. I. Sukhinov (Rostov-on-Don, Russia)**, V. V. Sidoryakina** (Taganrog, Russia)**. Combined stochastic models of sediment transport and multicomponent suspension of coastal systems.** [34]

This work is devoted to the construction of a joint model of sediment [1] and suspension [2] transport in a coastal zone with due account of the stochastic nature of wind waves [3], which are the main factor controlling the flow, and therefore the movement, of bottom sediments and suspended matter in a coastal zone. The correctness of the model is investigated, and the convergence of the approximating chain of problems to the solution of the initial nonlinear problem in norm of the Sobolev space L_1 with velocity O (τ) is proved, where τ is the time step. The model takes into account many physically significant factors such as the complicated bottom relief, the porosity of bottom sediments, the size and density of particles, its components, the effect of gravity, etc., and it also requires the diffusion coefficient and the tangential stress value near the bottom surface. The diffusion coefficient depends mainly on the frequency of the wind waves described, in general, by the semi-empirical distribution function $S_\xi(\omega)$ (ω is the wave frequency), which has high- and low-frequency components [3]. The energy E is proportional to the square of the amplitude of the wave and can be determined from the frequency spectrum as its average value. The energy value E in a given frequency interval is found from formula $\Delta E = \rho_\omega g \int_\omega^{\omega+\Delta\omega} S_\xi(\omega)\, d\omega$, where ρ_ω is the density of water, and g is the acceleration of gravity. As the value of wind wave frequency, it is proposed to use its expectation; if experimental data for wave frequency distribution are available, the average value is used. Determination of the velocity distribution of the water medium is carried out numerically on the basis of a 3D model of hydrodynamics. The input data are the distribution of the heights of the wind waves, which is described near the boundary of the bottom influence zone by a function close to the Rayleigh distribution. It is proposed to use the expected value for the heights of wind waves as input data for the 3D model of wave hydrodynamics.

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A. N. Tikhomirov (Syktyvkar, Russia)**. Local limit theorems for random matrices.** [35]

In this talk we survey the latest results on local limit theorems for various ensembles of random matrices, including Wigner matrices, sample covariance matrices, Ginibre–Girko matrices, and their products.

Significant progress in this area was achieved in the last decade, largely due to the works of a group led by H.-T. Yau and L. Erdös; see the papers [1], [2], [3], [4]. The main problem is the study of the behavior of the Stieltjes transform of the empirical spectral distribution (ESD) of Hermitian random matrices of large order on the complex plane near the real axis if the distance to the real axis is inversely proportional to the size n of the matrix up to a logarithmic factor. Estimates of the closeness of the Stieltjes transform of the ESD of a random matrix to the Stieltjes transform of the corresponding limit distribution (the semicircle law in the case of Wigner matrices, and the Marchenko–Pastur distribution law in the case of sample matrices) are of order $(nv)^{-1}\log^{\beta} n$, where v is the distance to the real axis in the complex plane, and β is a quantity that depends on n but grows no faster than $\log\log n$. Estimates of this kind can be used to provide information about the local behavior of the spectrum of a random matrix, i.e., about the distribution of the eigenvalues in a small neighborhood of a fixed point; give the limiting distribution for so-called spacings, i.e., the distances between neighborhood eigenvalues; estimate the closeness of eigenvalues to the corresponding quantiles of the limiting distribution (the rigidity of the spectrum), etc. In joint works of the author with F. Götze, A. Naumov, and D. Timushev, the main emphasis was placed on developing methods suitable for obtaining estimates of the order $O((nv)^{-l}\log^{\beta} n)$ under minimal moment assumptions and with optimal order β. Corresponding results are given in the papers [5], [6], [7], [8].

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M. S. Tikhov (University of Nizhni Novgorod, Nizhni Novgorod, Russia). **Fourier method for recursive estimation of distribution function in dose-effect relationship.**

Let $\mathscr{U}^{(n)} = \{(U_1, W_1), \dots, (U_n, W_n)\}$ be independent and identically distributed pairs of random variables, where U_i is interpreted as a "dose" and $W_j = I(X_j < U_j)$ is an indicator of the event $(X_j < U_j)$ (the "effect"); the random variables $\{X_j\}_{j=1}^n$ have a common distribution function $F(x) = \mathbf{P}(X_j < x)$ with density $f(x)$. The distribution function $G(u) = \mathbf{P}(U_j < u)$ has density $g(u)$ with respect to a Lebesgue measure λ on the real line $\mathbf{R}$. It is required, from the sample $\mathscr{U}^{(n)}$, to estimate the unknown distribution function $F(x)$ or its quantiles; the distribution function $G(u)$ is also unknown. This model is interpreted as a "dose-effect" relationship [1], [2], [3].

As an estimate of the distribution function $F(x)$, we take

$$\widehat{F}_n(x) = \frac{S_{2,n}(x)}{S_{1,n}(x)},$$

where

$$S_{2,n}(x) = \frac{1}{n}\sum_{j=1}^{n} W_j K_{b_j}(x, U_j), \qquad S_{1,n}(x) = \frac{1}{n}\sum_{j=1}^{n} K_{b_j}(x, U_j),$$

$K_b(x,u) = K((x-u)/b)/b$, $K(x)$ is a kernel function (finite symmetric density), and $\{b_j\}_{j=1}^n$ is a sequence of smoothing parameters.

The proof depends on recursive representations of the estimates $\widehat{F}_n(x)$ and $\widehat{g}_n(x)$,

$$\widehat{g}_n(x) = S_{1,n}(x) = \widehat{g}_{n-1}(x) + n^{-1}[K_{b_n}(x, U_n) - \widehat{g}_{n-1}(x)];$$
$$\widehat{F}_n(x) = \widehat{F}_{n-1}(x) + \gamma_n[W_n - \widehat{F}_{n-1}(x)], \qquad \gamma_n = \gamma_n(x) = (\widehat{g}_n(x)\, n)^{-1} K_{b_n}(x, U_n).$$

Let $B_n = \sum_{j=1}^n b_j^{-1}$. Under some regularity conditions on $g(x)$ and $f(x)$, and under the conditions (see [4, pp. 184, 185])

$$(\mathbf{D1}) \quad b_n \to 0, \quad n^2 B_n^{-1} \to \infty, \qquad (\mathbf{D2}) \quad \min_{1\leqslant j\leqslant n} b_j = o(B_n)$$

as $n \to \infty$, the sequence $nB_n^{-1/2}(\widehat{F}_n(x_0) - \mathbf{E}(\widehat{F}_n(x_0)))$ is asymptotically normal $N(0, F(x_0)(1-F(x_0))\|K\|^2/g(x_0))$.

We also consider the estimation problem of the distribution function $F(x)$ in the convolution model using the Fourier method, and the problem of estimation of distributions using the theory of reproduced kernels.

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V. V. Ulyanov (Moscow, Russia). **Nonasymptotic bounds for the closeness of Gaussian measures of balls.** [36]

The talk provides an overview of the tight nonasymptotic bounds for the Kolmogorov distance between the probabilities of two Gaussian elements to hit a ball of a Hilbert space. The key property of these bounds is that they are dimension-free and depend on the nuclear (Schatten-one) norm of the difference between the covariance operators of the Gaussian elements, and on the norm of the mean shift. The bounds obtained significantly improve the bound based on Pinsker's inequality via the Kullback–Leibler divergence. We also establish anticoncentration bounds for the squared norm of a noncentered Gaussian element in a Hilbert space. A number of examples motivating our results and their applications to statistical inference and to high-dimensional CLT (see, e.g., [1]) are given. The statements and proofs of the results mentioned in the talk can be found in [2], [3], [4], [5], [6]; see also the author's pages on MathNet.ru and ResearchGate.net. Here we give two principal results from [4] and [5].

Let H be a real separable Hilbert space with norm $\|\cdot\|$.

THEOREM 1. *Let ξ and η be Gaussian elements in H with zero mean and covariance operators Σ_ξ and Σ_η, respectively. Let $\lambda_{1\xi} \geqslant \lambda_{2\xi} \geqslant \cdots$ and $\lambda_{1\eta} \geqslant \lambda_{2\eta} \geqslant \cdots$ be the eigenvalues of Σ_ξ and Σ_η, respectively. Then there exists an absolute constant C such that*

$$\sup_{x>0}\bigl|\mathbf{P}(\|\xi\| \leqslant x) - \mathbf{P}(\|\eta\| \leqslant x)\bigr| \leqslant C\bigl((\Lambda_{1\xi}\Lambda_{2\xi})^{-1/2} + (\Lambda_{1\eta}\Lambda_{2\eta})^{-1/2}\bigr)\sum_{i=1}^{\infty}|\lambda_{i\xi} - \lambda_{i\eta}|$$

for $\Lambda_{k\xi}^2 := \sum_{j=k}^{\infty}\lambda_{j\xi}^2$, $\Lambda_{k\eta}^2 := \sum_{j=k}^{\infty}\lambda_{j\eta}^2$, $k = 1, 2$.

The following estimate for the probability density function $p(x)$ of the random variable $\|\xi\|^2$ plays an important role in the proof of Theorem 1.

LEMMA 1. *Let ξ be a Gaussian element in a separable Hilbert space H with zero mean and a covariance operator Σ_ξ. Then, for some constant c,*

$$\max_{x\geqslant 0} p(x) \leqslant c(\Lambda_{1\xi}\Lambda_{2\xi})^{-1/2}. \tag{1}$$

If the "effective" dimension of Σ_ξ is at least 2, i.e., if $2\lambda_{1\xi}^2 \leqslant \Lambda_{1\xi}^2$, then $\Lambda_{1\xi} \approx \Lambda_{2\xi}$ and the right-hand side of (1) is inversely proportional to the Frobenius norm $\Lambda_{1\xi}$ for Σ_ξ. In particular, in the d-dimensional case $H = \mathbf{R}^d$ for $d \geqslant 2$, if Σ_ξ is close to the identity matrix I, then $\max_{x\geqslant 0} p(x) \leqslant cd^{-1/2}$ by (1), which is consistent with the maximum value of the chi-square distribution density function with d degrees of freedom.

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V. A. Vasiliev (Tomsk, Russia). **Optimal parameter estimation of an autoregression by observations with additive noise.** [37]

Consider the estimation problem of the parameter λ of a scalar autoregressive process $(x_n)_{n\geqslant 0}$ satisfying the equation

$$x_n = \lambda x_{n-1} + \xi_n, \qquad n \geqslant 1, \tag{1}$$

from observations $y_n = x_n + \eta_n$, $n \geqslant 0$. Process (1) is supposed to be stable, i.e., $|\lambda| < 1$. The processes (ξ_n), (η_n), and x_0 are mutually independent; the noises ξ_n and η_n form sequences of i.i.d.r.v., and the variance of the noise in the observations $\mathbf{E}\eta_0^2$ is unknown. We construct estimators λ_n of the parameter λ on the basis of the truncated estimation method [1]. These estimators are optimal in the sense of the criterion

$$R_n = A\mathbf{E}(\lambda_n - \lambda)^2 + n \to \min_n .$$

The parameter A can be interpreted as the cost of the mean square quality of the parameter λ estimator. Asymptotic properties of the optimal sample size and the risk function R_n as $A \to \infty$ are investigated. Optimization problems in the sense of the risk function of a similar structure were first considered in the book [2] and references therein.

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T. A. Volosatova, I. V. Pavlov (Rostov-on-Don, Russia). **Solution of the minimax problem for the objective function of a quasilinear complex system with deterministic priorities.** [38]

This talk, which is concerned with the investigation of models of economic systems with finite number of deterministic priorities, is a continuation of the studies [1] and [2]. In accordance with the notation of [1], we write the objective function of an arbitrator in the form $F(u) = \prod_{j=1}^{k-1} u_j^{\alpha_j} \bigl(-\sum_{i=1}^{k-1} c_i u_i + c_k\bigr)^{\alpha_k}$, where $\alpha_i \in (0;1)$, $u_i > 0$, $c_i > 0$, $i = 1,\dots,k$, and $c_k = \sum_{i=1}^{k-1} c_i b_i + b_k$. The objective function has a unique stationary point, which is a local (and global) maximum point (cf. [1, Theorem 1]). The components of this point are as follows: $u_j = \alpha_j c_k / c_j$, $j = 1,\dots,k-1$. The maximal value of the target function reads as

$$F_{\max} = (c_k)^{\sum_{i=1}^k \alpha_i} \prod_{j=1}^{k-1} \biggl(\frac{\alpha_j}{c_j}\biggr)^{\alpha_j} \biggl(-\sum_{i=1}^{k-1} \alpha_i + 1\biggr)^{\alpha_k}.$$

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By varying the values of the coefficients c_i, we obtain various modifications of the mathematical model of the economic system. It is natural to believe that the main goal of the arbitrator is to optimally manage the entire system with the minimum possible cost. In this connection, a new optimization problem arises: to minimize the function $F_{\max}(c)$. The following result holds.

THEOREM 1. *If $b_i > 0$ for all $i = 1, \dots, k$, then the function $F_{\max}(c)$ is a unique point of local* (*and also global*) *minimum*

$$c^* = (c_1^*, \dots, c_{k-1}^*)\colon \quad c_j^* = \frac{\alpha_j b_k}{b_j \alpha_k}, \quad j = 1, \dots, k-1.$$

If $b_i \leqslant 0$ for some $i \in \{1, \dots, k\}$, then the function $F_{\max}(c)$ has no stationary points.

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A. L. Yakymiv (Steklov Mathematical Institute, Moscow, Russia)**. Multivariate regular variation and multiple power series distributions.** [39]

Let $(a(i) \geqslant 0,\ i \in \mathbf{Z}_+^n)$ be a multiple sequence such that the power series

$$B(x) = \sum_{i \in \mathbf{Z}_+^n} a(i) x^i \equiv \sum_{i_1, \dots, i_n \in \mathbf{Z}_+} a(i_1, \dots, i_n) x_1^{i_1} \cdots x_n^{i_n} \in (0, \infty)$$

for $x = (x_1, \dots, x_n) \in [0,1)^n$. Assume that, for $x \in (0,1)^n$, the random vector ξ_x has the power series distribution $B(x)$; i.e., $\mathbf{P}\{\xi_x = i\} = a(i)x^i/B(x)$ for any $i \in \mathbf{Z}_+^n$. Let $b = b(k) = (b_1(k), \dots, b_n(k)) \in (0,\infty)^n$, $k \in \mathbf{N}$, be a sequence such that $b_j = b_j(k) \to \infty$ for any $j = 1, \dots, n$ as $k \to \infty$. We also assume that $B(x)$ is regularly varying as $x \uparrow \mathbf{1} = (1, \dots, 1)$ along $b = b(k)$; i.e.,

$$(1) \qquad \frac{B(\exp(-\lambda/b))}{B(\exp(-\mathbf{1}/b))} \equiv \frac{B(\exp(-\lambda_1/b_1), \dots, \exp(-\lambda_n/b_n))}{B(\exp(-1/b_1), \dots, \exp(-1/b_n))} \to \Psi(\lambda) \in (0, \infty)$$

for any fixed $\lambda = (\lambda_1, \dots, \lambda_n) \in (0,\infty)^n$ as $k \to \infty$. Further, assume that, for any sequence $z_j = z_j(k) > 1$, $z_j = 1 + o(1)$ and for any $j = 1, \dots, n$, either the liminf of the fraction $a(b_1, \dots, b_{j-1}, z_j b_j, b_{j+1}, \dots, b_n)/a(b)$ is not smaller than 1 or the suplim of this fraction is not greater than 1 as $k \to \infty$. Given any fixed $u \in (0,\infty)^n$, we set $x = \exp(-u/b)$. From (1) it follows that the function $\Psi(\lambda)$ is the Laplace transform of some σ-finite measure ν on $\mathbf{R}_+^n$. Let ν be absolutely continuous in $(0,\infty)^n$ with continuous density $\varphi(\,\cdot\,)$. Then, for any compact set $K \subset (0,\infty)^n$,

$$\frac{\mathbf{P}\{\xi_x = [y/(\mathbf{1} - x)]\}}{\prod_{j=1}^n (1 - x_j)} \overset{y \in K}{\rightrightarrows} \frac{\varphi(y/u) \exp(-(y, \mathbf{1}))}{\prod_{j=1}^n u_j \Psi(u)}.$$

[39] This work was carried out in the framework of the state contract with Steklov Mathematical Institute.

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The proof of this result depends substantially on Theorem 3 of [1]. A short survey of different definitions of a multivariate regular variation is given in the talk.

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M. V. Zhitlukhin (Steklov Mathematical institute, Moscow, Russia), **K. A. Borovkov** (The University of Melbourne, Australia). **Estimates for the maximum of a discretely sampled fractional Brownian motion.**

Let $\{B_t^H\}_{t\geqslant 0}$ denote the fractional Brownian motion with Hurst parameter $H \in (0,1]$, which by definition is a continuous Gaussian process with $B_0^H = 0$, zero mean, and covariance function $\mathbf{E}(B_s^H B_t^H) = (s^{2H} + t^{2H} - |t-s|^{2H})/2$, $s,t \geqslant 0$.

We show that the distribution of the maximum $\overline{B^{H,\mathbb{T}}} = \max_{t\in\mathbb{T}} B_t^H$ of the fractional Brownian motion with Hurst parameter $H \to 0$ over an n-point set $\mathbb{T} \subset [0,1]$ can be approximated by the normal law with mean $\sqrt{\ln n}$ and variance $1/2$ provided that $n\to\infty$ slowly enough, and the points in $\mathbb{T}$ are not too close to each other.

The main results are as follows. Let $H_k \in (0,1]$ be such that $H_k \to 0$ as $k\to\infty$, and let $\mathbb{T}_k = \{t_{k,i}\}_{i=1}^{n_k}$ be a sequence of finite subsets of $(0,1]$, $t_{k,1} < \cdots < t_{k,n_k}$, such that $n_k\to\infty$, $\delta_k := \min_{1\leqslant i\leqslant n_k}(t_i - t_{i-1}) \to 0$, where $t_0 = 0$.

Denote by $\preccurlyeq$ the stochastic order relation for random variables, i.e., $\xi \preccurlyeq \eta$ if and only if $\mathbf{P}(\xi\leqslant x) \geqslant \mathbf{P}(\eta \leqslant x)$ for any $x\in\mathbf{R}$. Let $o_{\mathbf{P}}(1)$ stand for a sequence of random variables converging to zero in probability.

Theorem. (i) *If* $H_k(\ln n_k)^{1/2} \to 0$ *and* $H_k \ln(n_k\delta_k) \to 0$ *as* $k\to\infty$, *then*

$$\overline{B^{H_k,\mathbb{T}_k}} \preccurlyeq \sqrt{\ln n_k} + \frac{\zeta_0}{\sqrt{2}} + o_{\mathbf{P}}(1).$$

(ii) *If* $H_k(\ln n_k)^2 \to 0$ *and* $H_k \ln \delta_k \to 0$ *as* $k\to\infty$, *then*

$$\overline{B^{H_k,\mathbb{T}_k}} \succcurlyeq \sqrt{\ln n_k} + \frac{\zeta_0}{\sqrt{2}} + o_{\mathbf{P}}(1).$$

In particular, under the assumptions of assertion (ii),

$$\overline{B^{H_k,\mathbb{T}_k}} - \sqrt{\ln n_k} \xrightarrow{\mathrm{d}} \frac{Z}{\sqrt{2}} \quad \text{as} \quad k\to\infty,$$

where Z has standard normal distribution. Note that the conditions $H_k\ln(n_k\delta_k)\to 0$ and $H_k\ln\delta_k \to 0$ are automatically met in the case of uniform grids $\mathbb{T}_k$. For the proof the reader is referred to [3]. Other closely related results can be found in [1], [2].

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