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ABSTRACTS OF TALKS GIVEN AT THE 4TH INTERNATIONAL CONFERENCE ON STOCHASTIC METHODS*

(*Translated by A. R. Alimov*)

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The Fourth International Conference on Stochastic Methods (ICSM-4) was held June 2–9, 2019 at Divnomorskoe (near the town of Gelendzhik) at the Raduga sports and fitness center of the Don State Technical University, where the previous conference (ICSM-3) took place in 2018. ICSM-4 was organized by the Steklov Mathematical Institute of Russian Academy of Sciences (Department of Theory of Probability and Mathematical Statistics); Moscow State University (Department of Probability Theory); National Committee of the Bernoulli Society of Mathematical Statistics, Probability Theory, Combinatorics, and Applications; Peoples' Friendship University of Russia; and the Don State Technical University (Department of Higher Mathematics). The conference chairman was A. N. Shiryaev, a member of the Russian Academy of Sciences, who chaired the previous three conferences and also headed the Organizing Committee and the Program Committee.

Many leading scientists from Russia, France, Germany, Portugal, and Bulgaria took part in ICSM-4. Russian participants came from Voronezh, Zernograd, Kaluga, Maikop, Nizhni Novgorod, Rostov-on-Don, Samara, St. Petersburg, Taganrog, Ufa, and Khabarovsk. Approximately one-quarter of the talks were given by postgraduate and undergraduate students. Twenty-one talks were given at joint sessions, and 44 talks were presented at parallel sessions.

The Local Organizing Committee headed by I. V. Pavlov successfully managed the logistics of the conference. The participants recognized the success of the conference.

A. N. Shiryaev, I. V. Pavlov

The abstracts of the talks and presentations given at the conference are provided below.

V. I. Afanasyev (Moscow, Russia)**. Functional limit theorems for decomposable branching processes with two particle types.**

Consider a branching Galton–Watson process with particles of two types. Suppose that a particle of the first type generates descendants of both types (in the same quantities) and that a particle of the second type generates descendants of only its own type.

Let $\varphi(\,\cdot\,)$ and $\psi(\,\cdot\,)$ be generating functions of nonnegative integer random variables (r.v.'s) ξ and η. Suppose that the maximum step of distribution of the r.v. ξ is 1. We also suppose that $\mathbf{E}\xi=1$, $\mathbf{D}\xi:=\sigma_1^2\in(0,\infty)$ and $\mathbf{E}\eta=1$, $\mathbf{D}\eta:=2b_2\in(0,\infty)$.

We introduce generating functions for the progeny of particles of the first and second types, respectively, of the branching process under consideration: for $s_1,s_2\geqslant 0$,

$$f_1(s_1,s_2)=\varphi(s_1s_2),\qquad f_2(s_1,s_2)=\psi(s_2).$$

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We denote by ξ_n and η_n, respectively, the number of particles of the first and second types in the nth generation of the branching process under consideration. It is assumed that $\xi_0 = 1$ and $\eta_0 = 0$. We set $\Sigma_2 = \sum_{n=1}^{\infty} \eta_n$.

Consider the following random processes: $\{l_0^+(t),\, t \geqslant 0\}$ is the local time of a Brownian excursion, and $\{Y(t),\, t \geqslant 0\}$ is the Feller diffusion. We denote $S = \int_0^\infty Y(b_2 t)\, dt$ and introduce the probability densities for $y > 0$:

$$p_1(y) = \frac{\mathbf{P}^{(1)}(\sqrt[4]{S} > y)}{\mathbf{E}^{(1)} \sqrt[4]{S}}, \qquad p_2(y) = \frac{2}{\mathbf{E}^{(1)} \sqrt[4]{S}} \, \frac{\mathbf{P}^{(1)}(\sqrt[4]{S} > y^{-1/2})}{y^{3/2}}$$

(the upper index in both $\mathbf{P}$ and $\mathbf{E}$ means that $Y(0) = 1$). The main results are as follows (see [1], [2]).

THEOREM 1. *As* $N \to \infty$,

$$\left\{ \frac{\xi_{\lfloor t \sqrt[4]{N} \rfloor}}{\sqrt[4]{N}},\, t \geqslant 0 \;\middle|\; \Sigma_2 > N \right\} \xrightarrow{D} \left\{ \frac{\sigma_1}{2\nu} l_0^+ \left(\frac{\sigma_1}{2} t\nu \right),\, t \geqslant 0 \right\},$$

where ν *is an r.v. with probability density* p_1 *and independent of the process* $\{l_0^+(t),$ $t \geqslant 0\}$, *and* $\xrightarrow{D}$ *means the convergence in distribution in* $D[0,\infty)$.

THEOREM 2. *As* $N \to \infty$,

$$\left\{ \frac{\eta_{\lfloor t \sqrt{N} \rfloor}}{\sqrt{N}},\, t > 0 \;\middle|\; \Sigma_2 > N \right\} \xrightarrow{D} \{ Y(b_2 t),\, t > 0 \mid S > 1 \};$$

moreover, the r.v. $Y(0)$ *has probability density* p_2, *and* $\xrightarrow{D}$ *means convergence in distribution in* $D(0,\infty)$.

REFERENCES

[1] V. I. AFANASYEV, *On a decomposable branching process with two types of particles*, Proc. Steklov Inst. Math., 294 (2016), pp. 1–12.

[2] V. I. AFANASYEV, *A functional limit theorem for decomposable branching processes with two particle types*, Math. Notes, 103 (2018), pp. 337–347.

E. V. Alymova, O. E. Kudryavtsev (Rostov-on-Don, Russia). **Neural networks usage for financial time series prediction.** [1]

A Long Short-Term Memory (LSTM) neural network [1] is built to predict the BTC/USD currency pair behavior. This network is based on a logistic distribution in a way that its cumulative distribution function serves as the activation function in the network.

The network was trained on the trading history data consisting of one-minute log-returns of basic trading indicators. The target indicator (increase/decrease) is calculated by the logarithm of the ratio of the closing price at the fifth minute to the opening price at the first minute.

For the above LSTM, we constructed a confusion matrix and evaluated the McNemar's test parameters [2] using the formula $\chi^2 = (\mathrm{FP} - \mathrm{FN})^2/(\mathrm{FP} + \mathrm{FN})$, where FP and FN are the numbers of price growth/fall false predictions.

[1] Supported by the Russian Foundation for Basic Research (grant 18-01-00910).

PROPOSITION. *By McNemar's test ($\chi^2 = 2.3 < \chi^2_{\rm cr} = 3.8$) at a significance level of 5%, one cannot reject the hypothesis that there is no significant difference between the predictive model and the behavior of the time series of BTC/USD prices.*

REFERENCES

[1] T. FISCHER AND C. KRAUSS, *Deep learning with long short-term memory networks for financial market predictions*, European J. Oper. Res., 270 (2018), pp. 654–669.

[2] T. G. DIETTERICH, *Approximate statistical tests for comparing supervised classification learning algorithms*, Neural Comput., 10 (1998), pp. 1895–1923.

Yu. I. Belopolskaya (St. Petersburg Department of Steklov Mathematical Institute of Russian Academy of Sciences, Russia). **Probabilistic interpretation of the MHD–Burgers system as a system of nonlinear forward Kolmogorov equations.** [2]

We derive closed systems of stochastic equations associated with systems of parabolic equations with cross-diffusion and interpret them as systems of forward Kolmogorov equations [1], [2], [3]. We also put forward formulas of the Feynman–Kac type to construct probabilistic representations of mild solutions of the forward Cauchy problem for such systems. We develop a general theory and demonstrate its results in constructing a probabilistic interpretation of the Cauchy problem solution for one of the simplest magneto–hydrodynamics systems, namely, the MHD–Burgers system

$$\frac{\partial v}{\partial t} + v\cdot\nabla v = \frac{\sigma^2}{2}\Delta v + (\nabla\times B)\times B, \quad v(0,y) = v_0(y)\in \mathbf{R}^3, \tag{1}$$

$$\frac{\partial B}{\partial t} = \frac{\mu^2}{2}\Delta B + \nabla\times(v\times B), \quad B(0,y) = B_0(y)\in\mathbf{R}^3, \qquad y\in\mathbf{R}^3, \quad t\in[0,T]. \tag{2}$$

Here v is the velocity of the conducting fluid, and B is the magnetic field intensity. Let $w_m(t)$ be independent Wiener processes, and let ξ_{0m} be r.v.'s that are independent of $w_m(t)$ and have distributions $u_{0m}(dy)$; also let $u_{0m}(dy) = u_{0m}(y)\,dy$ where $u_{0m} = v_{0m}$ for $m = 1,2,3$ and $u_{0m} = B_{0m}$ for $m = 4,5,6$.

We define the stochastic processes $\xi_m(t)$, $\eta_m(t)$ by

$$\xi_m(t) = \xi_{0m} + \sigma w_m(t), \quad m = 1,2,3, \qquad \xi_m(t) = \xi_{0m} + \mu w_m(t), \quad m = 4,5,6, \tag{3}$$

$$\eta_m(t) = 1 + \int_0^t c_m(u(\theta,\xi_m(\theta)), \nabla u(\theta,\xi_m(\theta)))\eta_m(\theta)\,d\theta, \tag{4}$$

where $u = (u_1,\dots,u_6)$, $u_m = v_m$ for $m = 1,2,3$, and $u_m = B_m$ for $m = 4,5,6$,

$$\int_{\mathbf{R}^d} h_m(y)\,u_m(t,dy) = \mathbf{E}[h_m(\xi_m(t))\eta_m(t)], \qquad m = 1,\dots,6, \tag{5}$$

and the coefficients $c_m\colon \mathbf{R}^6\times(\mathbf{R}^6\otimes\mathbf{R}^3)\to\mathbf{R}$ can be found from (1), (2). By the Riesz's theorem, (5) can be replaced by

$$u_m(t,y) = \int_{\mathbf{R}^d} p_m(0,x,t,y)u_{0m}(x)\,dx + \int_0^t\int_{\mathbf{R}^d} \widetilde{c}_m^{\,u}(t,z)p_m(\theta,z,t,y)u_m(\theta,z)\,dz\,d\theta, \tag{6}$$

where $p_m(0,x,t,y)$ are the densities of transition probabilities for the processes $\xi_m(t)$,

$$\widetilde{c}_m^{\,u}(t,z) = c_m(u(t,z),\nabla u(t,z)) \quad\text{and}\quad u_m(t,dy) = u_m(t,y)\,dy.$$

[2]Supported by the Russian Science Foundation (grant 17-11-01136).

THEOREM. *Let* $u_{0m} \in W^{1,1}(\mathbf{R}^3)$. *Then there exists a unique solution of the stochastic system* (3), (4), (6) *on* $[0,T]$. *The functions* $u_m \in L^1([0,T], W^{1,1}(\mathbf{R}^d)) \cap L^1([0,T], L^\infty(\mathbf{R}^d))$ *define a mild solution of the Cauchy problem* (1), (2).

A similar result in the one-dimensional case was obtained in [4].

REFERENCES

[1] YA. BELOPOLSKAYA, *Stochastic models for nonlinear cross-diffusion systems*, in Statistics and Simulation, Springer Proc. Math. Stat. 231, Springer, Cham, 2018, pp. 145–159.

[2] YA. I. BELOPOLSKAYA, *Stochastic interpretation of quasilinear parabolic systems with cross diffusion*, Theory Probab. Appl., 61 (2017), pp. 208–234.

[3] YA. I. BELOPOLSKAYA, *Probabilistic counterparts of nonlinear parabolic partial differential equation systems*, in Modern Stochastics and Applications, Springer Optim. Appl. 90, Springer, Cham, 2014, pp. 71–94.

[4] YA. I. BELOPOLSKAYA AND A. O. STEPANOVA, *Stochastic interpretation of a Burgers-MHD system*, Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov (POMI), 466 (2017), Probability and Statistics 26, pp. 7–29 (in Russian).

Yu. V. Belova (Rostov-on-Don, Russia)**, A. V. Nikitina, A. A. Filina** (Taganrog, Russia)**. Statistical processing of field data for the study of nutrient pollution of a shallow water by river flows when modeling its ecological state.** [3]

Deterministic partial differential equations are used in the study of biogenic contamination of a shallow water body by river flows. Their parameters and functions (for example, the phytoplankton productivity and the coefficient of turbulent transfer of pollutants in the vertical direction) are determined by probabilistic models.

THEOREM. *Let* $q_i(x,y,z,t)$, $R_i \in C^2(D_t) \cap C(\overline{D}_t)$, $D_t = G \times (0 < t < T_0)$, $R_i = p_i(q_j)q_i + \overline{R}_i$, $i \neq j$; $\mu_i = \text{const} > 0$; $U, \nu_i(z) \in C^1(\overline{G})$; $q_i^0 \in C(\overline{G})$, $i = 1, \dots, 10$. *Assume that*

$$\max_G\{\mu_i, \nu_i\} - \frac{1}{\lambda_0} \max_G\{|p_i|\} > 0 \quad \textit{and} \quad 2\mu_i\left(\frac{1}{L_x^2} + \frac{1}{L_y^2}\right) + \frac{2\nu_i}{L_z^2} \geqslant \varphi_i$$

for all $i = 1, \dots, 10$ (*the functions* φ_i *are determined by pollutant sources*), *where*

$$\lambda_0 = \pi^2\left(\frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2}\right)$$

and L_x, L_y, L_z *are the maximum dimensions of the computational domain* G. *Then the model problem describing the process of production–destruction in a shallow water body* [1] *has a unique solution.*

In statistical processing of the field data, the following values are calculated: the asymmetry coefficient C_s, the kurtosis coefficient C_e, the variance D, the standard deviation σ, the coefficient of variation C_ν, the ratio C_s/C_ν, the autocorrelation coefficient, and the Neumann ratio. Anderson's test is used to determine the significance of autocorrelation relationships. The significance of the autocorrelation coefficient for the nutrients under consideration is tested from the Neumann ratio with a given level of significance. The statistical analysis shows heterogeneity of field observations in the samples, and hence they should be subdivided into seasons and months.

[3]This work was performed under R&D theme no. 2.6905.2017/BCh within the framework of the state contract with the Russian Ministry for Education and Science.

REFERENCES

[1] A. V. Nikitina, L. Kravchenko, I. Semenov, Yu. Belova, and A. Semenyakina, *Modeling of production and destruction processes in coastal systems on a supercomputer*, MATEC Web Conf., 226 (2018), 04025.

L. A. Bordag (Zittau, Germany)**. Optimization problem for a portfolio with an illiquid asset in the case of an exponential utility function.**

We study an optimization problem for a portfolio with an illiquid asset, a risky asset, and a risk-free asset within the framework of continuous time. Problems of such type lead to a three-dimensional nonlinear Hamilton–Jacobi–Bellman (HJB) equation on the value function $V(l,h,t)$. In this framework we suppose that the illiquid asset is sold in an exogenous random moment of time T with a prescribed liquidation time distribution with a survival function $\overline{\Phi}(T)$. The HJB equation in the case of a negative exponential utility function $U^{\mathrm{EXPn}}(c) = -e^{-ac}$ after a formal maximization procedure takes the form

$$\begin{aligned} V_t(l,h,t) &+ \frac{1}{2}\eta^2 h^2 V_{hh}(l,h,t) + (rl+\delta h)V_l(l,h,t) + (\mu-\delta)hV_h(l,h,t) \\ &- \frac{(\alpha-r)^2 V_l^2(l,h,t) + 2(\alpha-r)\eta\rho h V_l(l,h,t)V_{lh}(l,h,t) + \eta^2\rho^2\sigma^2h^2{V_{lh}}^2(l,h,t)}{2\sigma^2 V_{ll}(l,h,t)} \\ &+ \frac{1}{a}V_l(l,h,t)\ln V_l(l,h,t) - \frac{1}{a}\big(1+\ln\overline{\Phi}(t)\big)V_l(l,h,t) \\ &- \frac{\ln a}{a}V_l(l,h,t) = 0, \qquad V\to 0, \quad t\to\infty. \end{aligned} \tag{1}$$

Earlier we studied similar optimization problems with HARA and logarithmic utility functions in [1], [2]. We study the optimization problem with negative and positive exponential utility functions (EXPn and EXPp), which are economically equivalent. It is well known that both the logarithmical (LOG) and EXPn utility functions are connected with the HARA utility: in the first case the parameter of the HARA utility is going to zero, and in the second case, to infinity. In [1], [2] devoted to the optimization problem with a general HARA and LOG utility functions, we proved that also the corresponding analytical and Lie algebraic structures are connected by the same limiting procedure. We show that the case of the EXPn utility function differs from the HARA utility function and has its own special Lie algebraic structure which is not connected to the HARA case by the limiting procedure. We carry out the Lie group analysis of PDEs for EXPn and EXPp utility functions and hence are able to obtain the admitted symmetry algebras.

Theorem. *The HJB equation* (1) *admits the four-dimensional Lie algebra* $L_4^{\mathrm{EXPn}} = \langle \mathbf{U}_1, \mathbf{U}_2, \mathbf{U}_3, \mathbf{U}_4\rangle$, *where*

$$\mathbf{U}_1 = \frac{1}{ar}\frac{\partial}{\partial l} - V\frac{\partial}{\partial V}, \qquad \mathbf{U}_2 = \frac{\partial}{\partial V},$$
$$\mathbf{U}_3 = -\frac{1}{ar}\left(e^{rt}\int e^{-rt}\,d\ln\overline{\Phi}(t)\right)\frac{\partial}{\partial l} + \frac{1}{r}\frac{\partial}{\partial t}, \qquad \mathbf{U}_4 = e^{rt}\frac{\partial}{\partial l},$$

with the following nontrivial commutation relations: $[\mathbf{U}_1,\mathbf{U}_2] = \mathbf{U}_2$, $[\mathbf{U}_3,\mathbf{U}_4] = \mathbf{U}_4$. *Except for finite dimensional Lie algebra* L_4^{EXPn}, (1) *admits also an infinite dimensional algebra* $L_\infty = \langle\psi(h,t)\partial/\partial V\rangle$, *where the function* $\psi(h,t)$ *is any solution of the linear parabolic PDE* $\psi_t(h,t) + (1/2)\eta^2h^2\psi_{hh}(h,t) + (\mu-\delta)h\psi_h(h,t) = 0$.

We prove explicitly that both optimization problems with EXPn and EXPp utility functions are connected by a one-to-one analytical substitution and are identical to the economical, analytical, and Lie algebraic points of view. We provide a complete set of nonequivalent group invariant reductions of the three-dimensional PDEs corresponding to the optimization problem with the EXPn and EXPp utility functions for two-dimensional PDEs in accordance with an optimal system of subalgebras of the admitted Lie algebra. We prove that in one case the invariant reduction is consistent with the boundary condition. The two-dimensional PDE is more convenient for applications of numerical methods than the original three-dimensional PDE. Because of the uniqueness of the solution of the HJB equation we can use the reduced two-dimensional PDE to study the properties of the optimal solution and the investment-consumption strategies.

REFERENCES

[1] L. A. Bordag and I. P. Yamshchikov, *Optimization problem for a portfolio with an illiquid asset: Lie group analysis*, J. Math. Anal. Appl., 453 (2017), pp. 668–699.

[2] L. A. Bordag, I. P. Yamshchikov, and D. Zhelezov, *Portfolio optimization in the case of an asset with a given liquidation time distribution*, Int. J. Eng. Math. Model., 2 (2015), pp. 31–50.

A. E. Chistyakov, S. V. Protsenko (Rostov-on-Don, Russia)**. Accuracy of the solution of a wave problem with perturbed initial conditions.** [4]

During expeditionary research of the Azov Sea, data were obtained on the pulsations of the velocities of the water flow at some points of water bodies with the help of the ADCP (Acoustic Doppler Current Profiler) WHS600 Sentinel probe [1]. The correlation coefficients of the initial data of the velocity vector components with normal and log-normal distributions are found. The results of field measurements are used as initial conditions for modeling wave processes. Below we give theoretical estimates for the error of the numerical solution of the wave problem.

Theorem. *Let* $|k| < 2$, $k = 2 - \lambda_i \tau^2/(1 + \lambda_i \sigma \tau^2)$, *where* τ *is the time step*; σ *is the weight of the scheme*; λ_i *are the eigenvalues of the operator* Λ; $\Lambda c = \sum_{j=1}^{r} (\mu c_{\overline{x}_j})_{x_j}$; $(\,\cdot\,)_{\overline{x}_j}$ *and* $(\,\cdot\,)_{x_j}$ *are, respectively, the left- and right-hand difference derivatives in the spatial direction* x_j; *and* r *is the dimension of the space. Then the error of numerical solution of the wave problem with perturbed initial conditions*

$$c''_{tt} = \operatorname{div}(\mu \operatorname{grad} c),$$
$$c\big|_{x=0} = 0, \quad c\big|_{x=l} = 0, \quad c\big|_{t=0} = c_0, \quad c'_t\big|_{t=0} = c_1, \qquad 0 \leqslant x \leqslant l, \quad t \geqslant 0,$$

is as follows:

$$\psi_i^{n+1} = C^n \psi_i^0 + \xi^n;$$

here ψ_i^0 *is the error in the initial conditions*,

$$C^n = \cos(n\varphi) + \frac{k}{\sqrt{4-k^2}} \sin(n\varphi), \qquad \cos\varphi = \frac{k}{2},$$

n *is the layer number in the time variable, and* ξ^n *is the error accumulated during transitions between temporary layers, which is estimated as*

$$|\xi^n| \leqslant \frac{2n}{\sqrt{4-k^2}} \big|k - 2\cos\big(\sqrt{\lambda_i}\,\tau\big)\big| \sqrt{(\alpha_{i,0})^2 + \frac{1}{\lambda_i}(\alpha_{i,1})^2},$$

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where $c = \sum_i \alpha_i X_i$ *and* X_i *are the eigenvectors of the operator* Λ.

Remark. In the above, we considered Dirichlet boundary conditions (rigid boundary); the result is similar in the case of Neumann boundary conditions (soft boundary).

REFERENCES

[1] V. A. Gushchin, A. I. Sukhinov, A. V. Nikitina, A. E. Chistyakov, and A. A. Semenyakina, *A model of transport and transformation of biogenic elements in the coastal system and its numerical implementation*, Comput. Math. Math. Phys., 58 (2018), pp. 1316–1333.

E. G. Chub, V. A. Pogorelov (Rostov-on-Don, Russia)**. On some solution of an optimal control problem for nonlinear stochastic systems based on the use of information criteria.** [5]

Theorem. *Let an object be described by the nonlinear differential equation* $\dot{Y} = F_1(Y,t)+F_2(Y,t)V+U$, *where* Y *describes the dynamics of the object*; F_1, F_2 *are known nonlinear functions, which are Lipschitz continuous for all* Y, t, *and* N *times differentiable on the time interval* (t_0, t); V *is a normalized white Gaussian noise*; U *is the desired control minimizing the functional* $J = \int_D \Phi_1[\rho]\, dY + \int_{t_0}^{t} \int_D \Phi_2[U]\, dY\, dt$ *where* D *is the region of the state space determined by the optimal control*; *and* $\rho(Y,t)$ *is the probability density of the process* Y *as described by the Fokker–Planck–Kolmogorov* (*FPK*) *equation.*

If the distribution density $\rho(Y,t)$ admits Gaussian approximation and the Shannon criterion is used as Φ_1, and if $\Phi_2[U] = U^2$, then the control has the explicit representation $U = (1/2)\partial\rho/\partial Y$, and the distribution parameters are determined from the system

$$m'(t) = q(m(t)), \quad D'(t) = -2q'(m(t))D(t) + b(m(t)),$$

where $m(t)$ is the expectation and $D(t)$ is the variance of the distribution,

$$q(Y,t) = F_1(Y,t) + \frac{1}{2}F_2\frac{\partial F_2(Y,t)}{\partial Y}, \quad b(Y,t) = F_2^2(Y,t).$$

The above control can be easily implemented on modern computers.

A. G. Danekyants, N. V. Neumerzhitskaia (Rostov-on-Don, Russia)**. On rational and irrational interpolating martingale measures.** [6]

This report continues the studies of [1], [2] and is concerned with the existence of nondegenerate weakly interpolating martingale measures (n.d.w.i.m.m.) of a one-step market with discounted stock price $Z = (Z_k, \mathscr{F}_k)_{k=0}^1$ (the definition of an n.d.w.i.m.m. can be found in [1], [2]). The process Z is defined on a countable outcome space $\Omega = \{\omega_1, \omega_2, \dots\}$; $\mathscr{F}_0 = \{\Omega, \varnothing\}$; $\mathscr{F}_1$ is the set of all subsets of Ω; $a := Z_0$, $b_i := Z_1(\omega_i)$, $i = 1, 2, \dots$; and $\mathbf{b} := (b_1, b_2, \dots)$. It is assumed that the market in question is arbitrage-free; that is, it admits martingale measures $P = (p_1, p_2, \dots)$ on $(\Omega, \mathscr{F}_1)$ such that $p_i = P(\omega_i) > 0$, $i = 1, 2, \dots$, and the process $Z = (Z_k, \mathscr{F}_k, P)_{k=0}^1$ is a martingale.

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A nonzero sequence $\mathbf{r} = (r_1, r_2, \dots)$ is called finite if its components are rational and just a finite number of them are nonzero. Given a sequence of real numbers $\mathbf{d} = (d_1, d_2, \dots)$, we denote by $\mathscr{L}(\mathbf{d})$ the set of numbers of the form $\sum r_i d_i$, where $\mathbf{r}$ runs over all finite sequences.

THEOREM. 1. *Suppose that a is an irrational number and a sequence $\mathbf{b}$ contains an infinite number of different rational numbers. If $a \notin \mathscr{L}(\mathbf{b})$, then the market under consideration admits an n.d.w.i.m.m.*

2. *Let a be a rational number, and let a sequence $\mathbf{b}$ consist of rational numbers. If a martingale measure $P = (p_1, p_2, \dots)$ is such that $\mathscr{L}(P)$ consists only of irrational numbers, then P is an n.d.w.i.m.m.*

The assertion of the theorem extends one result from [2]. The next example illustrates assertion 2.

Example. Let $(d_1, d_2, \dots)$ be an arbitrary positive sequence such that $\mathscr{L}(\mathbf{d})$ consists only of irrational numbers (for example, if a number t is transcendental, then the sequence $(t, t^2, t^3 \dots)$ has this property). We find a sequence $(c_1, c_2, \dots)$ of positive rational numbers such that $\sum c_i d_i = 1$ and put $p_i = c_i d_i$. Let a be an arbitrary rational number. We find a sequence of rational numbers $\mathbf{b}$ such that $\sum b_i p_i = a$. Then $P = (p_1, p_2, \dots)$ is an n.d.w.i.m.m.

REFERENCES

[1] I. PAVLOV, *New family of one-step processes admitting special interpolating martingale measures*, Global Stochast. Anal., 5 (2018), pp. 111–119.

[2] A. G. DANEKYANTS AND N. V. NEUMERZHITSKAIA, *Generalization of a result on the existence of weakly interpolating martingale measures*, abstract, Third International Conference on Stochastic Methods, Theory Probab. Appl., 64 (2019), pp. 134–135.

D. V. Dimitrov (Moscow, Russia). **The Kullback–Leibler divergence estimation via k-nearest neighbor statistics and applications.** [7]

Consider i.i.d. random vectors $\{X_i, Y_i, i \in \mathbb{N}\}$ such that $\mathrm{law}(X_1) = \mathrm{law}(X)$ and $\mathrm{law}(Y_1) = \mathrm{law}(Y)$, where X and Y are random vectors taking values in $\mathbb{R}^d$, $d \geqslant 1$, $i \in \mathbb{N}$. Assume that X and Y have densities $p = d\mathbf{P}_X/d\mu$ and $q = d\mathbf{P}_Y/d\mu$ with respect to the Lebesgue measure μ in $\mathbb{R}^d$. We are interested in statistical estimation of the Kullback–Leibler divergence $D(\mathbf{P}_X \,\|\, \mathbf{P}_Y) := \int_{\mathbf{R}^d} p(x) \ln(p(x)/q(x))\, \mu(dx)$ constructed by means of two observations $\mathbf{X}_n := \{X_1, \dots, X_n\}$ and $\mathbf{Y}_m := \{Y_1, \dots, Y_m\}$, $n, m \in \mathbf{N}$. The relevant estimates $\widehat{D}_{n,m}(k,l)$ (see [1]) involve the specified k-nearest neighbor statistics. This method extends the one developed for $k = 1$ in [2] for analyzing the Shannon differential entropy estimates. In [1], wide conditions were proposed for densities p and q to ensure asymptotic unbiasedness and L^2-consistency of the estimates. Now we formulate the following new result concerning the mixtures of densities $p(x) := \sum_{i=1}^{I} \alpha_i p_i(x)$ and $q(x) := \sum_{j=1}^{J} \beta_j q_j(x)$ where $0 < \alpha_i < 1$, $i \in \{1, \dots, I\}$, $\sum_{i=1}^{I} \alpha_j = 1$, $0 < \beta_j < 1$, $j \in \{1, \dots, J\}$, $\sum_{j=1}^{J} \beta_j = 1$.

THEOREM. *Let densities p_i and q_j be such that, for some $\varepsilon, R > 0$ and $N \in \mathbb{N}$, the functionals $K_{p_i,q_j}(2,N)$, $Q_{p_i,q_j}(\varepsilon,R)$, $T_{p_i,q_j}(\varepsilon,R)$, $K_{p_{i_1},p_{i_2}}(2,N)$, $Q_{p_{i_1},p_{i_2}}(\varepsilon,R)$, and*

[7] This work was supported by the Lomonosov Moscow State University under the grant "Modern Problems of Fundamental Mathematics and Mechanics."

$T_{p_{i_1},p_{i_2}}(\varepsilon, R)$ *are finite for all* $i, i_1, i_2 \in \{1, \dots, I\}$, $j \in \{1, \dots, J\}$. *Then, for any fixed* $k, l \in \mathbb{N}$, *the estimates* $\widehat{D}_{n,m}(k,l)$ *are* L^2*-consistent, i.e.*,

$$\lim_{n,m\to\infty} \mathbf{E}\big(\widehat{D}_{n,m}(k,l) - D(\mathbf{P}_X \,\|\, \mathbf{P}_Y)\big)^2 = 0.$$

The functionals K_{f_1,f_2}, Q_{f_1,f_2}, and T_{f_1,f_2} for densities f_1, f_2 were defined in [1]. As a corollary, we obtain that the estimates of the Kullback–Leibler divergence between any two Gaussian mixtures in $\mathbb{R}^d$ (with components that have nondegenerate covariance matrices) are L^2-consistent.

An application of the above estimates is provided for detecting inhomogeneities within a fiber material filling a parallelepiped $U \subset \mathbb{R}^3$. In contrast to [3], which depends on estimates of the Shannon differential entropy, we use here the scan-statistics invoking $\widehat{D}_{n,m}(k,l)$ for a family of certain parallelepipeds belonging to U. We also discuss simulation results.

REFERENCES

[1] A. Bulinski and D. Dimitrov, *Statistical Estimation of the Kullback–Leibler Divergence*, preprint, arXiv:1907.00196, 2019.
[2] A. Bulinski and D. Dimitrov, *Statistical estimation of the Shannon entropy*, Acta Math. Sin. (Engl. Ser.), 35 (2019), pp. 17–46.
[3] P. Alonso Ruiz and E. Spodarev, *Entropy-based inhomogeneity detection in fiber materials*, Methodol. Comput. Appl. Probab., 20 (2018), pp. 1223–1239.

E. E. Dyakonova, V. A. Vatutin (Steklov Mathematical Institute of Russian Academy of Sciences, Moscow, Russia). **Evolution of a weakly subcritical branching process in random environment: Population size at the initial stage.**[8]

Let $\mathscr{Z} := \{Z_n, n = 0, 1, \dots\}$ be a branching process in random environment (BPRE) specified by a sequence of random independent and identically distributed (i.i.d.) generation functions $\{f_n(s),\ n = 1, 2, \dots\}$ (see [1] for a detailed treatment of properties of such processes). We set

$$X_n = \ln f_n'(1), \qquad \eta_n = \frac{f_n''(1)}{(f_n'(1))^2}.$$

A BPRE is called *weakly subcritical* if $\mathbb{E}[X_1] < 0$ and there is a number $0 < \beta < 1$ such that $\mathbb{E}[X_1 e^{\beta X_1}] = 0$.

Theorem 1. *Let* $\mathscr{Z}$ *be a weakly subcritical BPRE such that* $\mathbb{E}[X_1^2 e^{\beta X_1}] < \infty$ *and* $\mathbf{E}[(\ln^+ \eta_1)^{2+\varepsilon}] < \infty$ *for some* $\varepsilon > 0$. *If* $r = r(n) \to \infty$ *in such a way that* $r = o(n)$, *then, as* $n \to \infty$,

$$\mathscr{L}\left(\frac{1}{\sqrt{r}} \ln Z_r \,\middle|\, Z_n > 0\right) \to \mathscr{L}(B_1),$$

where $\{B_t,\ t \geqslant 0\}$ *is a Brownian motion conditioned to stay nonnegative for all* $t \geqslant 0$ (*see* [2] *for the respective definition*).

This result complements Theorem 1 in [3], which deals with the case $r \sim tn$ for $t \in (0,1)$. An analogue of this result for a critical BPRE was established in [4].

[8]Supported by the Russian Science Foundation (grant 19-01-00111).

REFERENCES

[1] G. Kersting and V. Vatutin, *Discrete Time Branching Processes in Random Environment*, Math. Stat. Ser., ISTE, London; John Wiley & Sons, Hoboken, NJ, 2017.
[2] R. T. Durrett, D. L. Iglehart, and D. R. Miller, *Weak convergence to Brownian meander and Brownian excursion*, Ann. Probab., 5 (1977), pp. 117–129.
[3] V. I. Afanasyev, *A functional limit theorem for the logarithm of a moderately subcritical branching process in a random environment*, Discrete Math. Appl., 8 (1998), pp. 421–438.
[4] V. Vatutin and E. Dyakonova, *Path to survival for the critical branching processes in a random environment*, J. Appl. Probab., 54 (2017), pp. 588–602.

M. L. Esquível (FCT NOVA & CMA UNL, Portugal). **On a stochastic model for a cooperative banking scheme for microcredit.** [9]

For microcredit modeling purposes, we consider

$$S_t = P_t - R_t = \Big(\sum_{i=1}^{m}\sum_{j=1}^{N_t^i} X_{i,j}\Big) - \Big(\sum_{i=1}^{m}\sum_{k=1}^{M_t^i} Y_{i,k}\Big), \tag{1}$$

a model (with zero interest rate) for a *collective vault* of m *vault* owners such that, for each owner $i \in \{1, 2, \dots, m\}$, the r.v. $X_{i,j}$ (respectively, $Y_{i,k}$) represents the payments (respectively, the loans granted) with common nontrivial moment generating function φ_i (respectively, ψ_i), and $(N_t^i)_{t\geqslant 0}$ (respectively, $(M_t^i)_{t\geqslant 0}$) is the counting Poisson process for the payment times (respectively, the granted loans times) such that $N_t^i \frown \mathscr{P}(\nu_i)$ (respectively, $M_t^i \frown \mathscr{P}(\mu_i)$) and all r.v.'s are independent. As a consequence of the optional sampling theorem for continuous martingales, we have the following result ensuring the stability of the *collective vault*, meaning the solvency of the microbank [1].

Theorem. *Let the first ruin time after time $\delta > 0$ be given by*

$$\tau_\delta := \inf\{t > \delta \colon S_t < 0\}.$$

Then the cumulant generating function of S_1 is given by

$$g(u) := \ln(\mathbf{E}[e^{uS_1}]) = \sum_{i=1}^{m}\big(\nu_i(\varphi_i(u) - 1) + \mu_i(\psi_i(-u) - 1)\big).$$

Moreover, if either $\mathbf{E}[P_1] > \mathbf{E}[R_1]$ (which is a condition at time $t = 1$) or

$$\forall\, i = 1, \dots, m \quad \forall\, t > 0 \quad \mathbf{E}\Big[\sum_{k=1}^{M_t^i} Y_{i,k}\Big] < \mathbf{E}\Big[\sum_{j=1}^{N_t^i} X_{i,j}\Big],$$

then there exists $u_a < 0$ with $g(u_a) < 0$ such that

$$\mathbf{P}[\tau_\delta < +\infty] \leqslant e^{\delta g(u_a)}.$$

As an example, in the case where $X_{i,j} \frown \chi^2_{k_i}(\lambda_{1,i})$ and $Y_{i,k} \frown \chi^2_{l_i}(\lambda_{2,i})$ and with the parameters given in the following table, the conditions of the theorem are

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verified and we can determine $u_a^\star < 0$ such that $g(u_a^\star) = \min_{u_a<0}(g(u_a))$ which gives $u_a^\star = -0.0790191$ with $g(u_a^\star) = -6.40092$. Hence $\mathbf{P}[\tau_1 < +\infty] \approx 0.00166003$ and also $\mathbf{P}[\tau_2 < +\infty] \approx 2.75568 \cdot 10^{-6}$.

i	$\lambda_{1,i}$	κ_i	ν_i	$\lambda_{2,i}$	l_i	μ_i	$\mu_i(\lambda_{2,i} + l_i)$	$\nu_i(\lambda_{1,i} + \kappa_i)$
1	1	4	10	6	3	1.1	9.9	50
2	1.8	5	12	7	3	0.9	9.0	81.6
3	2.3	4	9	7	3	1.2	12.0	56.7

The practical management of a *collective vault* requires a set of conditioning lending rules that destroy the independence [2], [3]. An example deserving further study is obtained by replacing at each time t the receivables R_t by $\min(R_t, (1+\alpha)S_t^+)$, which amounts to granting the requested loan R_t only if $R_t \leqslant (1+\alpha)\max(S_t, 0)$.

REFERENCES

[1] M. L. Esquível, P. P. Mota, and J. P. Pina, *On a Stochastic Model for a Cooperative Banking Scheme for Microcredit*, preprint, 2019.

[2] H. Dong and Z. Liu, *The ruin problem in a renewal risk model with two-sided jumps*, Math. Comput. Model., 57 (2013), pp. 800–811.

[3] J. J. Rebello and K. K. Thampi, *Some ruin theory components of two sided jump problems under renewal risk process*, Int. Math. Forum, 12 (2017), pp. 311–325.

M. A. Fedotkin, A. V. Zorine (Nizhni Novgorod, Russia)**. Stochastic models for adaptive control processes for conflicting flows of nonhomogeneous customers.**

In this talk, we give a survey of some methods for mathematical and simulation modeling and analysis of control processes for flows of inhomogeneous customers, which were developed at the Lobachevsky State University of Nizhni Novgorod. The methods are based on the notion of an *abstract stochastic control system* introduced by Lyapunov and Yablonsky [1]. Under this approach, the following principles should be followed: discreteness of the control system operation acts, and nonlocality in the control system blocks description. A control systems consists of (1) a block for input flows formation; (2) the input flows $\Pi_1, \Pi_2, \dots, \Pi_{\widehat{m}}$, $1 \leqslant \widehat{m} < \infty$, and saturation flows $\Pi_1^{\mathrm{sat}}, \Pi_2^{\mathrm{sat}}, \dots, \Pi_m^{\mathrm{sat}}$, $1 \leqslant m < \infty$; (3) the queues $O_1, O_2, \dots, O_m$; (4) the block for service strategy mechanism; (5) the server; (6) the flow control algorithm; and (7) the output flows $\Pi_1^{\mathrm{out}}, \Pi_2^{\mathrm{out}}, \dots, \Pi_m^{\mathrm{out}}$. Let an increasing sequence $\{\tau_i, i = 0, 1, \dots\}$ define a scale of observation instants. On the time interval $(\tau_i, \tau_{i+1}]$, the r.v.'s are defined in terms of χ_i (the state of the block for input flows formation), the vector η_i of arrival counts from the input flows, the vector ξ_i that counts the customers in the saturation flows, and the vector $\overline{\xi}_i$ of exiting customer counts. The server state variable Γ_i and the vector κ_i of queue sizes correspond to the time instant τ_i. Now the random vector-valued sequence

$$\{(\tau_i, \chi_i, \eta_i, \xi_i, \Gamma_i, \kappa_i, \overline{\xi}_i),\ i = 0, 1, 2, \dots\},$$

is a mathematical model for the control process for conflicting flows.

Theorem. *The vector-valued sequence*

$$\{(\chi_i, \Gamma_i, \kappa_i, \overline{\xi}_{i-1}),\ i = 0, 1, 2, \dots\} \tag{1}$$

is a countable controlled Markov chain under the constraints from [2], [3], [4], [5] *on the conditional probability distributions of the vector sequence* $\{(\tau_i, \chi_i, \eta_i, \xi_i),\ i =$

$0, 1, 2, \dots\}$ *and functional relations between* Γ_{i+1}, κ_{i+1}, $\bar{\xi}_i$ *on the one hand, and* Γ_i, κ_i, η_i, ξ_i *on the other hand, as in* [2], [3], [4], [5].

Here, a control policy appears in the form of the recurrence relation $\Gamma_{i+1} = u(\Gamma_i, \kappa_i, \eta_i)$. Necessary and sufficient conditions for the existence of a stationary distribution for sequence (1) were given in [2], [3], [4], [5].

The proposed approach allows one to find constraints on the conflicting flow control process parameters such that a stationary regime exists. Moreover, it becomes possible to determine quasi-optimal parameters subject to minimization of the mean sojourn time of an arbitrary customer in the system by means of computer-aided simulation.

REFERENCES

[1] A. A. Lyapunov and S. V. Yablonskii, *Theoretical problems of cybernetics*, in Problems of Cybernetics 9, Fizmatgiz, Moscow, 1963, pp. 5–22.
[2] A. V. Zorin and M. A. Fedotkin, *Optimization of control of doubly stochastic nonordinary flows in time-sharing systems*, Autom. Remote Control, 66 (2005), pp. 1115–1124.
[3] E. V. Kudryavtsev and M. A. Fedotkin, *Analysis of a discrete model of an adaptive control system for conflicting nonhomogeneous flows*, Moscow Univ. Comput. Math. Cybernet., 43 (2019), pp. 17–24.
[4] M. Rachinskaya and M. Fedotkin, *Stationarity conditions for the control systems that provide service to the conflicting batch Poisson flows*, in Analytical and Computational Methods in Probability Theory (ACMPT 2017), Lecture Notes in Comput. Sci. 10684, Springer, Cham, 2017, pp. 43–53.
[5] E. V. Proidakova and M. A. Fedotkin, *Control of output flows in the system with cyclic servicing and readjustments*, Autom. Remote Control, 69 (2008), pp. 993–1002.

A. A. Filina (Taganrog, Russia)**, A. V. Nikitina** (Taganrog, Russia)**, T. V. Lyashchenko** (Taganrog, Russia)**, L. V. Kravchenko** (Zernograd, Russia)**, A. I. Zabalueva** (Taganrog, Russia)**. Mathematical modeling of microbiological destruction of oil pollution in a coastal system based on the stochastic approach.** [10]

The system of deterministic equations

$$(P_i)'_t + \operatorname{div}(\mathbf{u}P_i) = \mu_i \Delta P_i + \varphi_i, \qquad i \in 1, \dots, 4, \tag{1}$$

that describes the oil biodegradation processes in coastal systems on the basis of combined methods of mathematical modeling using the stochastic approach with its probabilistic submodels, takes into account the simultaneous influence of external factors including salinity, temperature, and illumination on the mass transfer rate. Here P_i is the concentration of the ith component: 1—oil; 2—biogenic matter; 3, 4—green algae (*Chlorella vulgaris Beijer*) and its metabolite, respectively; $\mathbf{u}$ is the water flow velocity vector; μ_i are diffusion coefficients; and φ_i is a chemical–biological source [1].

Theorem. *Let the second equation of* (1) *with due account of the fluctuation of environment read as* $\dot{P}_2 = (\alpha - \beta + y(t))P_2$, $m(t) = P_2^0 e^{(\alpha-\beta)t}$, $\sigma^2(t) = P_2^0 e^{2(\alpha-\beta)t}(e^{\sigma^2 t} - 1)$, *where* α, β *are, respectively, the phytoplankton growth rate and mortality*; $\delta = \alpha - \beta$; P_2^0 *is the concentration* P_2 *at the initial time*; *and* $m(t)$, $\sigma^2(t)$ *are, respectively, the expectation and variance of the fluctuation* $y(t)$. *Assume that* $\delta < \sigma^2$. *Then the probability of degeneration of the Chlorella vulgaris Beijer population in time*

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increases and tends in the limit to one—the population is probabilistically unstable; i.e., sufficiently long action of disturbances most likely leads to death. If $\delta > \sigma^2$, *the probability of degeneration decreases, and as* $t \to \infty$, *it approach zero, and the population is stable in this case.*

The adequacy of the proposed probabilistic observational models is checked by using an algorithm taking into account the variances of some actual values of the parameter and of its component caused by the influence of randomness elements.

REFERENCES

[1] V. A. Gushchin, A. I. Sukhinov, A. V. Nikitina, A. E. Chistyakov, and A. A. Semenyakina, *A model of transport and transformation of biogenic elements in the coastal system and its numerical implementation*, Comput. Math. Math. Phys., 58 (2018), pp. 1316–1333.

Yu. E. Gliklikh (Voronezh, Russia). **Stochastic equations with current velocities and osmotic velocities (mean derivatives).** [11]

Necessary preliminaries about mean derivatives and, in particular, about current velocities (symmetric mean derivatives D_{S}), osmotic velocities (antisymmetric mean derivatives D_{A}), and quadratic mean derivatives D_2 can be found in [1].

Assume that the diffusion coefficient (i.e., the field of symmetric matrices $(a^{ij}(x))$) is smooth, autonomous, and positive definite. Since all matrices $(\alpha^{ij}(x))$ are nondegenerate and the field is smooth, there exists a smooth field of converse symmetric and positive definite matrices (α_{ij}). Hence this field can be used as a new Riemannian metric $\alpha(\,\cdot\,,\cdot\,) = \alpha_{ij}\,dx^i \otimes dx^j$ on $\mathbb{R}^n$. The volume form of the metric $\alpha(\,\cdot\,,\cdot\,)$ is $\Lambda_\alpha = \sqrt{\det(\alpha_{ij}(x))}\; dx^1 \wedge \cdots \wedge dx^n$.

By $\rho(t,x)$ we denote the density of probabilistic distribution of a random element $\xi(t)$ with respect to the volume form $dt \wedge \Lambda_\alpha$ on $\mathbf{R} \times \mathbf{R}^n$; i.e.,

$$\int_0^T \mathbf{E}(f(t,\xi(t)))\,dt = \int_0^T \left(\int_\Omega f(t,\xi(t))\,d\mathbf{P}\right) dt = \int_{\mathbf{R}\times\mathbf{R}^n} f(t,x)\rho(t,x)\,dt \wedge \Lambda_\alpha$$

for every continuous bounded function $f\colon [0,T] \times \mathbb{R}^n \to \mathbb{R}$.

Let a Borel vector field $v(t,x)$ and a Borel field of symmetric positive semidefinite matrices $\alpha(t,x)$ be given on $\mathbb{R}^n$. The system

$$\begin{cases} D_S\xi(t) = v(t,\xi(t)), \\ D_2\xi(t) = \alpha(t,\xi(t)), \end{cases} \tag{1}$$

where the equalities are fulfilled a.s., is called the first-order equation with current velocities.

Throughout this note, we suppose that the fields v and α are smooth and all matrices $\alpha(x)$ are autonomous and positive definite. If, in addition, v, α, and the partial derivatives of the coefficients of (a^{ij}) satisfy the Itô inequality, and the density ρ of the initial value is smooth and nowhere vanishes, then (1) has a solution (see [2]).

Lemma 1. *Let* $\rho(t,x)$, $v(t,x)$, $\alpha(x)$, *and* Λ_α *be the same as above, and let* (1) *be solvable. Then the flow of the vector field* $(1, v(t,x))$ *on* $\mathbb{R}\times\mathbb{R}^n$ *preserves the volume form* $\rho(t,x)dt \wedge \Lambda_\alpha$; *i.e., the Lie derivative* $L_{(1,v(t,x))}\rho(t,x)\,dt \wedge \Lambda_\alpha$ *vanishes.*

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The system

$$\begin{cases} D_{\mathrm{A}}\xi(t) = u(t,\xi(t)), \\ D_2\xi(t) = \alpha(\xi(t)), \end{cases} \tag{2}$$

where $u(t,x)$ is a Borel vector field and α is as above, is called the first-order differential equation with osmotic velocity.

By using properties of the osmotic velocity and the quadratic derivative, and employing Stokes' theorem, we can find $\rho(t,x)$ that can be the density of the solution of (2). We set $p(t,x) = \log \rho(t,x)$.

Condition 1. For all $t \in [0,T]$, the integral $\int_{\mathbf{R}\times\mathbf{R}^n} e^{p(t,x)}\,dt \wedge \Lambda_\alpha$ is finite; i.e., it is equal to a finite constant $C_{(t)}$ that is C^∞-smooth in t.

THEOREM 1. *If Condition* 1 *is satisfied, then, under the above hypotheses,* (2) *has a solution for a certain initial value with smooth and nowhere vanishing density that depends on the right-hand side. This solution is not unique.*

The proof uses Lemma 1 for finding the current velocity of the solution.

REFERENCES

[1] YU. E. GLIKLIKH, *Global and Stochastic Analysis with Applications to Mathematical Physics*, Theoret. Math. Phys., Springer-Verlag London, London, 2011.

[2] S. V. AZARINA AND YU. E. GLIKLIKH, *On the solvability of nonautonomous stochastic differential equations with current velocities*, Math. Notes, 100 (2016), pp. 3–10.

A. A. Gushchin (Steklov Mathematical Institute of Russian Academy of Sciences, Moscow, Russia)**. The joint law of the maximum and terminal value of a max-continuous local submartingale.** [12]

In 1993, Rogers [1] described the class of all possible joint laws of the terminal value of a random process and its maximum for two families of processes: uniformly integrable martingales and a.s. convergent continuous local martingales vanishing at 0. It turns out that in the second case, it is possible to naturally extend the family of processes with preservation of the corresponding class of joint laws. Namely, we consider the totality $\mathscr{X}$ of all a.s. convergent right-continuous local submartingales $X = (X_t)_{t\geqslant 0}$, $X_0 = 0$, such that their running maximum $\overline{X}_t := \sup_{s\leqslant t} X_s$ is continuous (such processes are sometimes called max-continuous). In our talk, the following questions are discussed: (1) description of the class of joint laws $\mathrm{Law}(X_\infty, \overline{X}_\infty)$ of the terminal value X_∞ of the process and its global maximum $\overline{X}_\infty$, as X runs over $\mathscr{X}$; (2) for every measure μ from this class, find one or another "simple" representative X from $\mathscr{X}$ with $\mathrm{Law}(X_\infty, \overline{X}_\infty) = \mu$. From Rogers' theorem it follows that there always exists a continuous local martingale with this property. We offer an alternative proof of this fact. The second question is interesting because, as we prove, whether the process X from $\mathscr{X}$ is a closed submartingale, a closed supermartingale, or a uniformly integrable martingale depends only on the joint law $\mathrm{Law}(X_\infty, \overline{X}_\infty)$.

PROPOSITION 1. *For a process X in $\mathscr{X}$, we define a change of time by $C_s := \inf\{t\colon \overline{X}_t > s\}$. Then the time-changed process $Y := X \circ C := (X_{C_s})_{s\geqslant 0}$ is a* max-*continuous submartingale with respect to the filtration $(\mathscr{F}_{C_s})_{s\geqslant 0}$ and is expressed as*

$$Y_s = s\mathbf{1}_{\{s<\overline{X}_\infty\}} + X_\infty \mathbf{1}_{\{s\geqslant \overline{X}_\infty\}}.$$

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In particular, $Y_\infty = X_\infty$ *and* $\overline{Y}_\infty = \overline{X}_\infty$.

PROPOSITION 2. *Let* W *and* L *be r.v.'s on some probability space* $(\Omega, \mathscr{F}, \mathbf{P})$, $W \geqslant \max\{L, 0\}$, *and let the function*

$$s \rightsquigarrow \mathbf{E}[s\mathbf{1}_{\{s<W\}} + L\mathbf{1}_{\{s\geqslant W\}}], \qquad s \geqslant 0,$$

vanish at 0 *and be monotone increasing in* s. *We define* $\mathscr{F}_s$ *as the* σ*-algebra of subsets from* $\mathscr{F}$ *having an intersection with the set* $\{W > s\}$ *that either is empty or coincides with* $\{W > s\}$. *We also set*

$$Y_s = s\mathbf{1}_{\{s<W\}} + L\mathbf{1}_{\{s\geqslant W\}}. \tag{1}$$

Then $Y = (Y_s)_{s\geqslant 0}$ *is an* $(\mathscr{F}_s)$*-submartingale.*

Propositions 1 and 2 provide solutions to the above two questions. Given a probability measure $\mu = \mu(dx, dy)$ on $\mathbb{R}^2$ supported in the set $\{(x, y) \colon y \geqslant \max\{x, 0\}\}$, a necessary and sufficient condition that there exists a process X in $\mathscr{X}$ such that $\mu = \operatorname{Law}(X_\infty, \overline{X}_\infty)$ is that the function

$$s \rightsquigarrow \int [s\mathbf{1}_{\{y>s\}} + x\mathbf{1}_{\{y\leqslant s\}}]\, \mu(dx, dy), \qquad s \geqslant 0,$$

vanishes at $s = 0$ and is monotone nondecreasing in s. For each such measure μ, we can construct a "simple" submartingale Y of form (1) such that $\operatorname{Law}(Y_\infty, \overline{Y}_\infty) = \mu$. If we now embed the submartingale Y into a Brownian motion by a change of time consisting of minimal stopping times using the Monroe theorem or its generalizations, then we can define in some way a continuous martingale X and justify that $\operatorname{Law}(X_\infty, \overline{X}_\infty) = \mu$.

REFERENCES

[1] L. C. G. ROGERS, *The joint law of the maximum and terminal value of a martingale*, Probab. Theory Related Fields, 95 (1993), pp. 451–466.

Yu. Kabanov (Université Bourgogne Franche-Comté, Lomonosov Moscow State University, Federal Research Center "Informatics and Control")**. Ruin probabilities with risky investments.**

Let $X = X^u$ be a generalized Ornnstein–Uhlenbeck process; that is, X is a solution of the linear stochastic equation

$$dX_t = X_{t-}\, dR_t + dP_t, \qquad X_0 = u > 0,$$

where R and P are independent Lévy processes, $\Delta R > -1$, and the process P is not a subordinator (that is, it is not increasing). Let $\tau^u := \inf\{t : X^u_t \leqslant 0\}$ be the ruin time, and let $\Psi(u) := \mathbf{P}(\tau^u < \infty)$ be the ruin probability. This model includes the well-studied models for the reserve of an insurance company, in particular, the Lundberg–Cramér one ($dR_t = 0$) and its version with deterministic investments ($dR_t = r \neq 0$). Special cases of models with investments in a risky asset with price dynamics that is given by a geometric Brownian motion ($dR_t = a\, dt + \sigma\, dW_t$, $\sigma \neq 0$) were studied in [1], [2] using methods of integro-differential equations for the ruin probability as a function of u. It was shown that as $u \to \infty$ the ruin probability behaves as $\Psi(u) \sim Cu^{-\beta}$, $C > 0$, where $\beta := 2a/\sigma^2 - 1 > 0$. The general model of the above type was introduced by Paulsen in 1973 and studied in the series of his papers, where it was shown that the ruin probability $\Psi(u) \sim Cu^{-\beta}$, where β is the root of the equation $H(\beta) = 0$ (which is assumed to exist). Here H is the cumulant generating function of the increment of the log price process $V_t = \ln \mathscr{E}_t(R)$ of the risky asset, that is, $H(p) := \ln \mathbf{E}e^{-qV_1}$. To find the asymptotic as $u \to \infty$, Paulsen used Kesten–Coldie implicit renewal theory (also known as the theory of random equations or equations in the sense of laws). Unfortunately, until recently, this theory did not provide a direct answer to the question of whether the constant C is strictly positive. The proof of this important property in the given context has led to additional assumptions and cumbersome formulations. Recent progress in the implicit renewal theory allows us to get the following result, which is easy to memorize.

THEOREM. *Suppose that the law of V_1 is not arithmetic, the process P has only upward jumps, and $H(\beta+) < \infty$. If $\int |x|^{\beta} I_{\{x>1\}}\, \Pi_P(dx) < \infty$, where Π_P is the Lévy measure of the process P, then $\Psi(u) \sim Cu^{-\beta}$, $C > 0$.*

REFERENCES

[1] A. Frolova, Yu. Kabanov, and S. Pergamenshchikov, *In the insurance business risky investments are dangerous*, Finance Stoch., 6 (2002), pp. 227–235.

[2] Yu. Kabanov and S. Pergamenshchikov, *In the insurance business risky investments are dangerous: The case of negative risk sums*, Finance Stoch., 20 (2016), pp. 355–379.

[3] Yu. Kabanov and S. Pergamenshchikov, *The Ruin Problem for Lévy-Driven Linear Stochastic Equations with Applications to Actuarial Models with Negative Risk Sums*, preprint, arXiv:1604.06370, 2018.

E. V. Karachanskaya (Khabarovsk, Russia). **Construction of a continuum of invariant converters for an automorphic function defined on an n-dimensional space ($n \geqslant 2$).**

We propose a method for constructing a function which ensures that a given function is automorphic. This theorem is applied to construct a system of Itô diffusion equations with jumps for which a function $u(t,\mathbf{x}) \in \mathcal{C}^{1,1}_{t,\mathbf{x}}$ is a stochastic first integral [1], [2].

THEOREM. *Let $\mathbf{x}\colon [0,T] \to \mathbf{R}^n$, $n \geqslant 2$; $u\colon [0,T] \times \mathbf{R}^n \to \mathbf{R}^1$; and $h\colon [0,T] \times \mathbf{R}^n \times \mathbf{R}^m \to \mathbf{R}^n$, $m \geqslant 1$. Assume that the following conditions are satisfied: (1) $u(t,\mathbf{x}) \in \mathscr{C}^{1,1}_{t,\mathbf{x}}$; (2) $h(t,\mathbf{x},\gamma) \in \mathscr{C}^{1,1,1}_{t,x,\gamma}$; (3) $\alpha(t,\mathbf{x}) \in \mathscr{C}_0(t,\mathbf{x})$, $\alpha(t,\mathbf{x}) \neq 0$ for all t and $\mathbf{x}$; (4) $\mathbf{y}(t,\mathbf{x},\gamma)$ is a solution to the system*

$$\frac{\partial \mathbf{y}(\,\cdot\,,\gamma)}{\partial \gamma} = \det \begin{bmatrix} \vec{e}_1 & \vec{e}_2 & \dots & \vec{e}_n \\ \dfrac{\partial u(t,\mathbf{y}(\,\cdot\,,\gamma))}{\partial y_1} & \dfrac{\partial u(t,\mathbf{y}(\,\cdot\,,\gamma))}{\partial y_2} & \dots & \dfrac{\partial u(t,\mathbf{y}(\,\cdot\,,\gamma))}{\partial y_n} \\ \varphi_{31}(t,\mathbf{y}(\,\cdot\,,\gamma)) & \varphi_{32}(t,\mathbf{y}(\,\cdot\,,\gamma)) & \dots & \varphi_{3n}(t,\mathbf{y}(\,\cdot\,,\gamma)) \\ \dots & \dots & \dots & \dots \\ \varphi_{n1}(t,\mathbf{y}(\,\cdot\,,\gamma)) & \varphi_{n2}(t,\mathbf{y}(\,\cdot\,,\gamma)) & \dots & \varphi_{nn}(t,\mathbf{y}(\,\cdot\,,\gamma)) \end{bmatrix},$$

with the initial condition $\mathbf{y}(t,\mathbf{x},0) = \mathbf{x}$, where $\varphi_{ij}(\,\cdot\,) = \partial\varphi_i(\,\cdot\,)/\partial y_j$. Also let $\{u(t,\mathbf{y}) \cup \{\varphi_i(t,\mathbf{y})\}_{i=3}^n\}$ consist of mutually independent functions. Then the function $h(t,\mathbf{x},\gamma) = \mathbf{y}(t,\mathbf{x},\gamma) - \mathbf{x}$ is an invariant converter ensuring that any function $u(t,\mathbf{x}(t))$ such that $u(t,\mathbf{x}(t) + h(t,\mathbf{x}(t),\gamma)) = u(t,\mathbf{x}(t))$ is automorphic. Moreover, the set of invariant converters for $u(t,\mathbf{x}(t))$ has the cardinality of the continuum.

REFERENCES

[1] E. V. Karachanskaya, *The generalized Itô–Venttsel' formula in the case of a noncentered Poisson measure, a stochastic first integral, and a first integral*, Siberian Adv. Math., 25 (2015), pp. 191–205.

[2] V. A. Doubko, *The invariance problem and construction of algorithm of automorphone transformation set for given function*, in Vidkr. Evol. Syst. (Kiev, 2003), Vol. 2, VNZ VMURoL, Kiev, 2004, pp. 66–68.

V. M. Kocheganov (Nizhni Novgorod, Russia). **Analysis of a tandem of queueing systems with cyclic service with prolongations.**

Consider a tandem of queuing systems. In the first system, the customers are serviced in the class of cyclic algorithms. The serviced high-priority customers are transferred from the first system to the second with random delays and constitute the high-priority input flow of the second system. In the second system, customers are serviced in the class of cyclic algorithms with prolongations. A statement of the problem and mathematical model of construction can be found in [1]. The central object of the mathematical model is a multidimensional denumerable Markov chain $\{(\Gamma_i, \varkappa_{1,i}, \varkappa_{2,i}, \varkappa_{3,i}, \varkappa_{4,i});\ i \geqslant 0\}$. Let $\{\tau_i;\ i = 0, 1, \ldots\}$ be a discrete time scale for which the system is actually observed. Also, let Γ_i be the server state during the time interval $(\tau_{i-1}; \tau_i]$; $\varkappa_{j,i} \in \mathbf{Z}_+$ be the number of customers in the queue of the jth input flow at time τ_i; $\eta_{j,i} \in \mathbb{Z}_+$ be the number of customers arriving in the queue of the jth input flow during the interval $(\tau_i; \tau_{i+1}]$; and $\overline{\xi}_{j,i} \in \mathbb{Z}_+$ be the actual number of serviced customers from the queue of the jth input flow during the interval $(\tau_i; \tau_{i+1}]$, $j \in \{1, 2, 3, 4\}$. Sufficient conditions for the existence of a stationary regime for the Markov chains $\{(\Gamma_i, \varkappa_{3,i});\ i \geqslant 0\}$ and $\{(\Gamma_i, \varkappa_{1,i}, \varkappa_{3,i});\ i \geqslant 0\}$ were obtained in [1]. A simulation model was built, and experiments were performed in [2] to analyze the tandem systems in more details. In this work, we present necessary conditions for the sequence $\{(\Gamma_i, \varkappa_{3,i});\ i \geqslant 0\}$ to be stationary.

Theorem. *A necessary condition for a Markov chain $\{(\Gamma_i, \varkappa_{3,i});\ i \geqslant 0\}$ to have a stationary distribution is that*

$$\max_{k=1,\ldots,d} \frac{\sum_{r=1}^{n_k} \ell(k, r, 3)}{\lambda_3 f_3'(1) \sum_{r=1}^{n_k} T^{(k,r)}} > 1.$$

REFERENCES

[1] V. Kocheganov and A. V. Zorine, *Sufficient condition for primary queues stationary distribution existence in a tandem of queuing systems*, Vestn. TVGU. Ser. Prikl. Matem., no. 2, 2018, pp. 49–74.

[2] A. Zorine and V. Kocheganov, *Statistical analysis and optimization of a tandem of queuing systems under prolongable cyclic service*, Upr. Bol. Syst. (Sbornik Trudov), 78 (2019), pp. 122–148 (in Russian).

M. A. Kocheganova (Rachinskaya) (Nizhni Novgorod, Russia). **Limit theorems for a multidimensional Markov chain as a model of a queueing system controlled with a threshold-priority algorithm with prolongations.**

Consider a controlled queueing system with $m \geqslant 2$ conflicting input flows. The first flow has the highest priority of customers, while the flow with the number m has the largest intensity. This problem was posed in [1]. We construct a model as a multidimensional Markov sequence $\{(\Gamma_i, \varkappa_{1,i}, \varkappa_{m,i}, \xi'_{1,i-1}, \xi'_{m,i-1}),\ i = 0,1,2,\dots\}$ with the recurrent relations

$$\Gamma_{i+1} = u(\Gamma_i, \varkappa_{1,i}, \eta_{1,i}), \qquad \varkappa_{j,i+1} = \max\{0, \varkappa_{j,i} + \eta_{j,i} - \xi_{j,i}\},$$
$$\xi'_{j,i} = \min\{\varkappa_{j,i} + \eta_{j,i}, \xi_{j,i}\}, \qquad j = \overline{1,m}.$$

Here $\{\tau_i;\ i = 0,1,2,\dots\}$ is a time scale. The following r.v.'s and elements are also introduced: Γ_i is the random state of the server on the time interval $[\tau_i, \tau_{i+1})$, $\eta_{j,i}$ is the number of arrivals for the jth flow during the period $[\tau_i, \tau_{i+1})$, $\xi_{j,i}$ and $\xi'_{j,i}$ are the maximal and real numbers of served customers for the jth flow in the period $[\tau_i, \tau_{i+1})$, and $\varkappa_{j,i}$ is the number of waiting customers of the jth flow at the moment τ_i. The function $u(\,\cdot\,,\cdot\,,\cdot\,)$ describes the control algorithm with prolongations, threshold priority, and feedback based on the number of waiting customers in a high-priority flow. An ergodic theorem for the Markov chain is proved. Necessary and sufficient conditions for the Markov chain to be stationary are also proved. For example, the following result holds.

THEOREM. *A stationary mode for the first flow exists if and only if* $\lambda_1 T^*(2s_1 + q_1 + 1) - l_1 < 0$. *Here* λ_1, q_1, s_1 *are input flow parameters*, T^* *is the duration of the minimum cycle of the control algorithm*, *and* l_1 *is the service capacity for the first flow.*

REFERENCES

[1] M. A. RACHINSKAYA AND M. A. FEDOTKIN, *Investigation of the stationary mode existence in a system of conflict service of non-homogeneous demands*, Vestn. Tomsk. Gos. Univ. Mat. Mekh., 51 (2018), pp. 33–47.

A. A. Kozhevin (Moscow, Russia). **An information theory-based approach to feature selection.** [13]

The talk topics are the estimators of conditional entropy [1] and mutual information in the mixed model [2]. We present a new result for a feature selection procedure based on an information-theoretic approach.

A mixed model was described in [1]. Let $\zeta_n = \{(X^i, Y^i)\}_{i=1}^n$ be a sample of i.i.d. observations, $(X^1, Y^1) \sim (X, Y)$, where $X = (X_1, \dots, X_d)$ is an absolutely continuous random vector in $\mathbb{R}^d$, and Y is an r.v. with values in a finite set M. The set of indices $S = \{s_1, \dots, s_m\} \subset \{1, \dots, d\}$ ($s_i \neq s_j$ for $i \neq j$) and the set of factors X_S, where $u_L = (u_{l_1}, \dots, u_{l_m})$ for $u = (u_1, \dots, u_d)$ and $L = \{l_1, \dots, l_m\}$, are called relevant if $f_{Y|X}(y \mid X) = f_{Y|X_S}(y \mid X_S)$ a.s. for each $y \in M$. Let $Q_m = \{\{l_1, \dots, l_m\} \subset \{1, \dots, d\} \colon l_i \neq l_j,\ i \neq j\}$. For each $L \in Q_m$, we define the sample $\zeta_{n,L} = \{(X_L^i, Y^i)\}_{i=1}^n$ and estimate the mutual information $I(X_L; Y)$ for each sample $\zeta_{n,L}$ by the method proposed in [2]. The resulting estimates are denoted by $\hat{I}_{n,k,L}$, $\widehat{I}_{n,k,L}$, where $k \in \{1, \dots, n-1\}$ is the method parameter.

We set $\widehat{S}_{n,k} = \operatorname{argmax}_{L \in Q_m} \widehat{I}_{n,k,L}$. If the maximum $\widehat{I}_{n,k,L}$ is reached on several sets Q_m, then we can take as $\widehat{S}_{n,k}$ the first of such sets taken in the lexicographical order. The following new result for selection of a significant factor holds.

[13] This work is supported by the Lomonosov Moscow State University under the grant "Modern Problems of Fundamental Mathematics and Mechanics."

THEOREM. *Let m be known, and let a relevant set of factors of length m be unique. Assume that the density $f_X(\cdot)$ is positive and that for each $L \subset \{1,\dots,d\}$ and each $y \in M$, the density $f_{X_L,Y}(\cdot, y)$ is C_0-constricted ($C_0 > 0$). Next, suppose that for some $\varepsilon > 0$,*

$$\mathbf{E}|\ln f_{X_L}(X_L)|^{2+\varepsilon} < \infty.$$

Then $\mathbf{P}(\widehat{S}_{n,k} = S) \to 1$ as $n \to \infty$ for any $\alpha \in (0,1)$ and $k \propto n^\alpha$.

REFERENCES

[1] A. BULINSKI AND A. KOZHEVIN, *Statistical estimation of conditional Shannon entropy*, ESAIM Probab. Stat., 23 (2019), pp. 350–386.

[2] A. BULINSKI AND A. KOZHEVIN, *Statistical estimation of mutual information for mixed model*, Methodology and Computing in Appl. Probab., to appear.

N. P. Krasii, I. V. Pavlov (Rostov-on-Don, Russia)**. Generalization of the model with priorities.**[14]

We consider the function

$$F(u_1, u_2, \dots, u_k) = \prod_{j=1}^{k} \mathbf{E}^{\mathbf{P}} f_j(u_j, \cdot) \tag{1}$$

on a probability space $(\Omega, \mathscr{F}, \mathbf{P})$.

THEOREM. *Let functions $f_j(u_j, \omega)$, $j = 1, \dots, k$, satisfy the following conditions*:

(1) *$f_j(u_j, \omega)$ is defined and measurable on $[0,\infty) \times \Omega$ and continuous on $[0,\infty)$ for $\mathbf{P}$-almost all $\omega \in \Omega$, and $f_j(0,\omega) = 0$;*

(2) *$f_j(u_j, \omega)$ is twice continuously differentiable on $(0,\infty)$ for $\mathbf{P}$-almost all $\omega \in \Omega$; additionally, the first and second derivatives are bounded on sets of the form $K \times \Omega$, where K is a compact subset of $(0,\infty)$;*

(3) *for $\mathbf{P}$-almost all $\omega \in \Omega$ and all $u_j \in (0,\infty)$, the function $f_j(u_j,\omega)$ is positive, its first derivative is positive, and the second derivative is negative*;

(4) *$u_k = -\sum_{j=1}^{k-1} c_j u_j + c_k$, where $c_j > 0$, $j = 1, 2, \dots, k$.*

Then, in the domain $\{u_j > 0,\ j = 1, 2, \dots, k-1,\ \sum_{j=1}^{k-1} c_j u_j < c_k\}$, function (1) *has a unique stationary point that is a local* (*and also global*) *maximum point.*

If we put $f_j(u_j, \omega) = u_j^{\alpha_j(\omega)}$ where $\alpha_j(\omega)$ is an r.v. (priority), $\mathbf{P}(\alpha_j = 0) = 0$, and $\mathbf{P}(0 < \alpha_j < 1) > 0$, then (1) coincides with the function obtained in the optimization problem for a quasi-linear model with independent priorities α_j (see [1]). The presented theorem generalizes Theorem 1 in [1].

REFERENCES

[1] I. V. PAVLOV AND S. I. UGLICH, *Optimization of complex systems of quasilinear type with several independent priorities*, Vestn. RGUPS, no. 3(67), 2017, pp. 140–145 (in Russian).

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E. V. Kudryavtsev (Nizhni Novgorod, Russia). **Limit theorems for flow control systems in a class of closed-loop algorithms.**

We consider limit properties of a sequence $\{(\Gamma_i, \kappa_i);\, i = 0, 1, 2, \dots\}$ where $\Gamma_i \in \{\Gamma^{(1)}, \dots, \Gamma^{(8)}\}$ and $\kappa_i = (\kappa_{1,i}, \kappa_{2,i}) \in \{0, 1, 2, \dots\} \times \{0, 1, 2, \dots\} = X \times X$. The components of the vector sequence $\{(\Gamma_i, \kappa_i); i = 0, 1, 2, \dots\}$ satisfy the recurrence relations $\Gamma_{i+1} = u(\Gamma_i, \kappa_i, \eta_i')$ and $\kappa_{i+1} = v(\Gamma_i, \kappa_i, \eta_i, \xi_i)$. In [1], [2], random vectors $\eta_i = (\eta_{1,i}, \eta_{2,i}) \in X \times X$, $\xi_i = (\xi_{1,i}, \xi_{2,i}) \in X \times X$, a random element $\eta_i' \in \{(0,0), (0,1), (1,0)\}$, and their distributions were defined. The sequence $\{(\Gamma_i, \kappa_i);\, i \geqslant 0\}$ was shown to have the Markov property, the classification of its states was given, and the conditions for existence of a stationary mode were derived.

THEOREM. *A necessary condition for the existence of the limit distribution of the Markov sequence* $\{(\Gamma_i, \kappa_i); i \geqslant 0\}$ *is that* $\theta_1\lambda_1 M_1/\mu_{1,2} + \theta_2\lambda_2 M_2/\mu_{2,2} < 1$, *where* λ_1, λ_2, M_1, M_2, θ_1, θ_2, $\mu_{1,2}$, $\mu_{2,2}$ *are distribution parameters for* η_i *and* ξ_i.

Note that the sequence $\{(\Gamma_i, \kappa_i); i \geqslant 0\}$ is a mathematical model of a control system for conflict flows of nonhomogeneous requests.

REFERENCES

[1] E. V. KUDRYAVTSEV AND M. A. FEDOTKIN, *Analysis of a discrete model of an adaptive control system for conflicting nonhomogeneous flows*, Moscow Univ. Comput. Math. Cybernet., 43 (2019), pp. 17–24.

[2] E. V. KUDRYAVTSEV AND M. A. FEDOTKIN, *Research of the mathematical model of adaptive control of conflict flows of inhomogeneous demands*, Vestnik TVGU. Ser. Prikl. Matem., no. 1, 2019, pp. 23–37.

O. E. Kudryavtsev (Rostov-on-Don, Russia). **On approaches to pricing European and American lookback options.** [15]

A lookback option is an exotic derivative, whose payoff depends on the extrema of a certain underlying asset price reached within the option lifetime. Lookback options pricing under various processes were extensively studied. The most efficient approaches are deterministic methods involving integral transforms machinery (see, e.g., [1], [4]).

Practically, the trader needs to price lookback options during the whole time period up to the expiration date rather than at the initial time only. Let $S_t = e^{X_t}$ be the price of an underlying asset driven by an exponential Lévy model. Then the time-t prices of European floating and fixed strike lookback puts, with the expiration date T conditional on $X_t = x$ and the extrema of the underlying asset price observed prior to the current time t, can be expressed as

$$V_{\text{fl}}(t,x,y) = \mathbf{E}^x\big[e^{-r(T-t)}(e^{\max\{\overline{X}_T, y\}} - e^{X_T})\big],$$
$$V_{\text{fx}}(t,x,y) = \mathbf{E}^x\big[e^{-r(T-t)}(K - e^{\min\{\underline{X}_T, y\}})_+\big],$$

where K is a strike price, r is the riskless rate, and $\overline{X}_t$ and $\underline{X}_t$ are supremum and infimum processes.

Both types of options could be computed by direct implementation of the generalized Monte Carlo method based on the Wiener–Hopf factorization developed in [3]. Numerical experiments show that the Monte Carlo methods are sufficiently fast and

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accurate for lookback options under Lévy models in comparison to the deterministic methods from [1], [4].

The second part of this talk deals with finite difference methods for pricing American lookbacks in the Black–Scholes framework. Unlike European lookback options, American lookback options cannot be priced by closed-form formulae, even in the Black–Scholes model (see [2]), and require the use of numerical methods.

Let $U_{\mathrm{fl}}(t,x,y)$ be the price function of an American floating strike lookback put on a dividend paying stock with the maturity T conditional on $X_t = x$ and $\overline{X}_t = y$.

THEOREM 1. *Let q be a continuous dividend rate. Then the price function $U_{\mathrm{fl}}(t,x,y)$ can be represented as*

$$U_{\mathrm{fl}}(t,x,y) = e^y - e^x + e^y F(t, x-y), \qquad x \leqslant y,$$

where $F(t,x)$ is a function nondecreasing in x such that $F(T,x) = 0$, and the variational inequality

$$\begin{gathered}\max\bigl(-F, \partial_t F + 0.5\sigma^2 \partial_x^2 F + (r - 0.5\sigma^2 - q)\partial_x F \\ - rF - r + qe^x\bigr) = 0, \qquad t < T, \quad x < 0, \\ 1 + F(t,0) - \frac{\partial F}{\partial x}(t,0) = 0, \qquad t < T,\end{gathered}$$

is satisfied.

We apply the Wiener–Hopf method to prove the theorem and efficiently solve the problem by an iterative finite difference scheme.

REFERENCES

[1] S. Boyarchenko and S. Levendorskiĭ, *Efficient Laplace inversion, Wiener–Hopf factorization and pricing lookbacks*, Int. J. Theor. Appl. Finance, 16 (2013), 1350011.
[2] M. Dai and Y. K. Kwok, *American options with lookback payoff*, SIAM J. Appl. Math., 66 (2005), pp. 206–227.
[3] O. E. Kudryavtsev, *Approximate Wiener–Hopf factorization and Monte Carlo methods for Lévy processes*, Theory Probab. Appl., 64 (2019), pp. 186–208.
[4] O. Kudryavtsev and S. Levendorskii, *Efficient Pricing Options with Barrier and Lookback Features under Lévy Processes*, working paper, 2011.

D. F. Kuznetsov (St. Petersburg, Russia). **Strong approximation of iterated Itô and Stratonovich stochastic integrals.**

This work continues the research started in [1] on development of efficient methods of mean-square approximation of the iterated Itô and Stratonovich stochastic integrals, which can be applied for numerical solution of Itô stochastic differential equations.

THEOREM. *Let $\psi_1(\tau),\dots,\psi_k(\tau)$ be continuous functions on $[t,T]$, and let $\phi_j(\tau)$ be a complete orthonormal polynomial or trigonometric basis for $L_2([t,T])$, $i_1,\dots,i_k = 0,1,\dots,m$. Then $I^k_{T,t} = \mathrm{l.i.m.}_{p_1,\dots,p_k\to\infty} I^{p_1\dots p_k}_{T,t}$ $(k \in \mathbf{N})$, $J^k_{T,t} = \mathrm{l.i.m.}_{p\to\infty} J^{k,p}_{T,t}$ $(k \leqslant 5)$, and, moreover,*

$$\mathbf{E}(I^k_{T,t} - I^{p_1\dots p_k}_{T,t})^2 \leqslant k! \left(\|K\|^2 - \sum_{j_1=0}^{p_1} \cdots \sum_{j_k=0}^{p_k} C^2_{j_k\dots j_1} \right)$$

for all $T-t\in(0,1)$, *where*

$$I_{T,t}^{k}=\int_t^T \psi_k(t_k)\cdots\int_t^{t_2}\psi_1(t_1)d\mathbf{W}_{t_1}^{(i_1)}\cdots d\mathbf{W}_{t_k}^{(i_k)},$$

$$J_{T,t}^{k}=\int_t^T\cdots\int_t^{t_2}\circ d\mathbf{W}_{t_1}^{(i_1)}\cdots\circ d\mathbf{W}_{t_k}^{(i_k)},$$

$$I_{T,t}^{p_1\ldots p_k}=\sum_{j_1=0}^{p_1}\cdots\sum_{j_k=0}^{p_k}C_{j_k\ldots j_1}\Biggl(\prod_{\ell=1}^{k}\zeta_{j_\ell}^{(i_\ell)}-\underset{N\to\infty}{\text{l.i.m.}}\sum_{(\ell_1,\ldots,\ell_k)\in G_k}\prod_{s=1}^{k}\phi_{j_s}(\tau_{\ell_s})\Delta\mathbf{W}_{\tau_{\ell_s}}^{(i_s)}\Biggr),$$

$$J_{T,t}^{k,p}=\sum_{j_1,\ldots,j_k=0}^{p}C_{j_k\ldots j_1}\prod_{\ell=1}^{k}\zeta_{j_\ell}^{(i_\ell)},$$

$$C_{j_k\ldots j_1}=\int_{[t,T]^k}K(t_1,\ldots,t_k)\prod_{\ell=1}^{k}\phi_{j_\ell}(t_\ell)\,dt_1\cdots dt_k;$$

$\|\cdot\|$ *is the* $L_2([t,T]^k)$*-norm*; d *and* $\circ d$ *are the Itô and Stratonovich differentials, respectively*; $K(t_1,\ldots,t_k)=I\{t_1<\cdots<t_k\}\psi_1(t_1)\ldots\psi_k(t_k)$, $\mathbf{W}_\tau^{(i)}$ $(i=1,\ldots,m)$ *are independent standard Wiener processes*; $\mathbf{W}_\tau^{(0)}=\tau$, $\Delta\mathbf{W}_{\tau_j}^{(i)}=\mathbf{W}_{\tau_{j+1}}^{(i)}-\mathbf{W}_{\tau_j}^{(i)}$, $\zeta_j^{(i)}=\int_t^T\phi_j(\tau)\,d\mathbf{W}_\tau^{(i)}$ $(i\neq 0)$ *are i.i.d.* $N(0,1)$*-r.v.'s*; $t=\tau_0<\cdots<\tau_N=T$, $\max_{0\leqslant j\leqslant N-1}(\tau_{j+1}-\tau_j)\to 0$ *as* $N\to\infty$; $H_k=\{(\ell_1,\ldots,\ell_k)\colon \ell_1,\ldots,\ell_k=0,1,\ldots,N-1\}$; *and* $L_k=\{(\ell_1,\ldots,\ell_k)\colon \ell_1,\ldots,\ell_k=0,1,\ldots,N-1;\ \ell_g\neq\ell_r\ (g\neq r);\ g,r=1,\ldots,k\}$, $G_k=H_k\setminus L_k$.

REFERENCES

[1] D. F. Kuznetsov, *Stochastic Differential Equations: Theory and Practic of Numerical Solution*, 6th ed., Electr. J. "Differential Equations and Control Processes," no. 4, SPbSU, St. Petersburg, 2018 (in Russian); also available online from https://diffjournal.spbu.ru/EN/numbers/2018.4/article.2.1.html.

K. S. Kuznetsov (St. Petersburg, Russia)**. Weighted average price management of manufacturer realization on commodity exchanges with predetermined volume of sales.**

Theorem. *We assume that* $t\in\mathbf{N}_0$ *and* Δt *correspond to the unit time interval between trading days*; *i.e.*, $t=0,1,\ldots,T$. *The observed price* $\widetilde{x}_t$ *on a commodity exchange is a realization of a stochastic process* (*see* [1], [2]), *which follows the stochastic differential equation*

$$dx_t=c_tx_t\,dt+\sigma x_t\,dW_t,$$

where c_t *is the coefficient*, W_t *is a standard Wiener process*, *and* σ *is the volatility coefficient* (*a constant*). *We also assume that the quantity of commodity units* $\widetilde{a}_t$ *sold at a certain time interval* $[0,t]$ *is given by*

$$\widetilde{a}_t=A\widetilde{x}_t+B+\widetilde{x}_tA\frac{1}{\sigma^2T}e^{\sigma^2(T-t)}-\widetilde{x}_tA\frac{1}{\sigma^2T}-\widetilde{x}_tA\frac{T-t}{T},$$

where

$$A=\frac{a_{\max}-a_{\min}}{x_{\max}-x_{\min}},\quad B=-\frac{a_{\max}-a_{\min}}{x_{\max}-x_{\min}}x_{\min}+a_{\min}$$

and $a_{\max}$, $x_{\max}$, $a_{\min}$, $x_{\min}$ *are constants such that* $a_{\max} > a_{\min}$, $x_{\max} > x_{\min}$, $a_{\max} > 0$, $x_{\max} > 0$, $a_{\min} > 0$, $x_{\min} > 0$. *Assume that the market price decreases* $(\widetilde{x}_0 > \widetilde{x}_1 > \cdots > \widetilde{x}_T)$ *on an a priori given time interval* $[0,T]$. *Then the weighted average price of manufacturer sales increases* $\widetilde{x}_0^{\mathrm{av}} < \widetilde{x}_1^{\mathrm{av}} < \cdots < \widetilde{x}_T^{\mathrm{av}}$. *Otherwise, if the market price increases* $(\widetilde{x}_0 < \widetilde{x}_1 < \cdots < \widetilde{x}_T)$ *on an a priori given time interval* $[0,T]$ *and if the inequality*

$$\frac{\widetilde{x}_i}{\widetilde{x}_{i-1}} > \left(\frac{1}{\sigma^2 T} e^{\sigma^2(T-(i-1))} - \frac{1}{\sigma^2 T} + \frac{i-1}{T}\right)\left(\frac{1}{\sigma^2 T} e^{\sigma^2(T-i)} - \frac{1}{\sigma^2 T} + \frac{i}{T}\right)^{-1}$$

holds for all $i = 1, \ldots, T$, *then the weighted average price of manufacturer sales will increase*: $\widetilde{x}_0^{\mathrm{av}} < \widetilde{x}_1^{\mathrm{av}} < \cdots < \widetilde{x}_T^{\mathrm{av}}$.

REFERENCES

[1] S. A. Vavilov and K. S. Kuznetsov, *Weighted average price management of manufacturer sales on commodity exchanges*, Int. J. Financ. Eng., 5 (2018), 1850029.

[2] S. A. Vavilov and K. S. Kuznetsov, *A stochastic control model for the average price of manufacturer sales on commodity exchanges*, Autom. Remote Control, 80 (2019), pp. 1098–1108.

E. Lépinette (Dauphine University, Paris). **Conditional cores and conditional convex hulls of random sets.**

In [1], Lépinette and Molchanov defined two nonlinear operations with random (not necessarily closed) sets in Banach space: the conditional core and the conditional convex hull. While the first is sublinear, the second is superlinear (in the reverse set inclusion ordering). Furthermore, we introduce the generalized conditional expectation of random closed sets and show that it is sandwiched between the conditional core and the conditional convex hull. The results rely on measurability properties of not necessarily closed random sets considered from the point of view of the families of their selections. Furthermore, we develop analytical tools suitable to handle random convex (not necessarily compact) sets in Banach spaces; these tools are based on considering support functions as functions of random arguments. Our research is motivated by applications to assessing multivariate risks in mathematical finance.

Let $\mathbf{X}$ be a separable (real) Banach space with norm $\|\cdot\|$ and the Borel σ-algebra $\mathscr{B}(\mathbf{X})$ generated by its strong topology. Fix a complete probability space $(\Omega, \mathscr{F}, \mathbf{P})$. Let $\mathscr{H}$ be a sub-σ-algebra of $\mathscr{F}$, which may coincide with $\mathscr{F}$. An *$\mathscr{H}$-measurable random set* (for short, random set) is a set-valued function $\omega \mapsto X(\omega) \subseteq \mathbf{X}$ from Ω to the family of all subsets of $\mathbf{X}$ such that its graph

$$\operatorname{Graph} X = \{(\omega, x) \in \Omega \times \mathbf{X} \colon x \in X(\omega)\} \tag{1}$$

belongs to the product σ-algebra $\mathscr{H} \otimes \mathscr{B}(\mathbf{X})$.

Definition 1. *An $\mathscr{H}$-measurable random element ξ such that $\xi(\omega) \in X(\omega)$ for almost all $\omega \in \Omega$ is said to be an $\mathscr{H}$-measurable* selection (*selection for short*) *of* X; $\mathrm{L}^0(X, \mathscr{H})$ *denotes the family of all $\mathscr{H}$-measurable selections of X.*

Definition 2. *Let X be any set-valued mapping. The conditional core* $\mathrm{m}(X \mid \mathscr{H})$ *of X (also called $\mathscr{H}$-core) is the largest $\mathscr{H}$-measurable random set X' such that $X' \subseteq X$ a.s.*

LEMMA. *If* $\mathrm{m}(X \mid \mathscr{H})$ *exists and is a.s. nonempty, then*

$$\mathrm{L}^0(X, \mathscr{H}) = \mathrm{L}^0(\mathrm{m}(X \mid \mathscr{H}), \mathscr{H}); \tag{2}$$

in particular, $\mathrm{m}(X \mid \mathscr{H})$ *is a.s. nonempty if and only if* $\mathrm{L}^0(X, \mathscr{H}) \neq \varnothing$.

The following theorem is one of our main results. The general case is an open problem.

THEOREM. *If* X *is a random closed set, then* $\mathrm{m}(X \mid \mathscr{H})$ *exists and is a random closed set, which is a.s. convex (respectively, is a cone) if* X *is a.s. convex (respectively, is a cone).*

This concept of conditional core naturally appears in geometrical financial models in the presence of transaction costs [2]. Indeed, for such models in discrete time, the dynamics of a portfolio process V is $V_t - V_{t-1} \in -G_t$, where G is a random closed set [3] adapted to the filtration $(\mathscr{F}_t)_{t=0,\dots,T}$ of consideration. This is equivalent to $V_{t-1} \in G_t + V_t$, i.e., $V_{t-1} \in \mathrm{m}(G_t \mid \mathscr{F}_{t-1})$.

The conditional core is related to another concept, the conditional convex hull $\mathrm{M}(X \mid \mathscr{H})$; see [1]. In particular, they are "dual" of each other when X is a random cone. More precisely, if X is a convex cone, we have $\mathrm{m}(X \mid \mathscr{H}) = \mathrm{M}(X^* \mid \mathscr{H})^*$, where $*$ designates the positive dual.

REFERENCES

[1] E. LÉPINETTE AND I. MOLCHANOV, *Conditional cores and conditional convex hulls of random sets*, J. Math. Anal. Appl., 478 (2019), pp. 368–392.
[2] E. LEPINETTE AND I. MOLCHANOV, *Risk Arbitrage and Hedging to Acceptability*, preprint, arXiv:1605.07884, 2016.
[3] E. LEPINETTE AND T. TRAN, *General financial market model defined by a liquidation value process*, Stochastics, 88 (2016), pp. 437–459.

A. A. Lykov, V. A. Malyshev, M. V. Melikian (Moscow, Russia)**. Stability problems for infinite linear chains of oscillators.**

We consider an infinite number of point particles $\cdots < x_k < x_{k+1} < \cdots$, $k \in \mathbf{Z}$, on $\mathbf{R}$ (an infinite chain of oscillators) with formal Hamiltonian

$$H = \sum_k \frac{v_k^2}{2} + \frac{\omega_0^2}{2} \sum_k (x_k - ka)^2 + \frac{\omega_1^2}{2} \sum_k (x_{k+1} - x_k - a)^2, \qquad a > 0,$$
$$y = \{y_k(t) = x_k(t) - ka\}, \qquad v(t) = \{\dot{y}_k = \dot{x}_k\}, \qquad M(t) = \sup_{k\in\mathbf{Z}} |y_k(t)|.$$

We present some results on the stability (in l_∞) of a fixed point (a zero energy point) under various perturbations. (For different results on such chains, see [1], [2].)

THEOREM 1. *Let* $y(0), v(0) \in l_2(\mathbf{Z})$. *Then*
(1) *if* $\omega_0 > 0$, *then* $\sup_{t\geqslant 0} M(t) < \infty$;
(2) *if* $\omega_0 = 0$, *then, for all* $t \geqslant 0$,

$$M(t) \leqslant \frac{2}{\sqrt{\omega_1}} \|v(0)\|_2 \sqrt{t} + \|y(0)\|_2;$$

however, for any $\delta > 1/2$, *there exist initial conditions* $y(0) = 0$, $v(0) \in l_2(\mathbf{Z})$ *such that* $\lim_{t\to\infty} (y_0(t)/\sqrt{t}) \ln^\delta t = \Gamma(\delta) > 0$ *(here* Γ *is the gamma function).*

THEOREM 2. *Let* $\omega_0 = 0$ *and* $v(0) = 0$. *Then*

(1) *if* $y(0) \in l_\infty(\mathbf{Z})$, *then, for all* $t \geqslant 0$,

$$M(t) \leqslant (c\sqrt{t} + 2)M(0)$$

with some constant $c \geqslant 0$;

(2) *if* $y_k(0)$, $k \in \mathbf{Z}$, *are i.i.d. r.v.'s bounded in* k *with probability* 1 (*i.e.*, $\sup_k |y_k(0)| < \infty$ *a.s.*), *then, for all* $n \in \mathbf{Z}$,

$$\mathbf{P}\Big(\limsup_{t\to\infty} y_n(t) = +\infty\Big) = \mathbf{P}\Big(\liminf_{t\to\infty} y_n(t) = -\infty\Big) = 1.$$

REFERENCES

[1] A. A. LYKOV, V. A. MALYSHEV, AND M. V. MELIKIAN, *Phase diagram for one-way traffic flow with local control*, Phys. A, 486 (2017), pp. 849–866.

[2] A. A. LYKOV AND V. A. MALYSHEV, *From the N-body problem to Euler equations*, Russ. J. Math. Phys., 24 (2017), pp. 79–95.

A. V. Makarova, V. A. Gorlov (Voronezh, Russia)**. Stochastic differential inclusions with forward mean derivatives and special right-hand side.** [16]

A stochastic differential inclusion with forward mean derivatives and a set-valued lower semicontinuous decomposable right-hand side in $\mathbf{R}^n$ is shown to have a solution.

THEOREM (see [1]). *Let* $\mathbf{a}$ *be a set-valued lower semicontinuous field in* $\mathbf{R}^n$ *with closed decomposable images, and let* $\boldsymbol{\alpha}$ *be a set-valued positive definite lower semicontinuous field with closed decomposable images and such that*

$$\|\operatorname{tr}\alpha(t,x)\| < K(1+\|x\|)^2, \tag{1}$$

$$\|a(t,x)\| < K(1+\|x\|) \tag{2}$$

for all $\alpha(t,x) \in \boldsymbol{\alpha}$, $a \in \mathbf{a}$, *and some* $K > 0$.

Then, for the initial condition $\xi(0) = \xi_0$, *the inclusion*

$$\begin{cases} D\xi(t) \in \mathbf{a}(t,\xi(t)), \\ D_2\xi(t) \in \boldsymbol{\alpha}(t,\xi(t)) \end{cases} \tag{3}$$

has a solution for all $t \in [0,T]$.

REFERENCES

[1] A. V. MAKAROVA, *Stochastic inclusions with forward mean derivatives having decomposable right-hand sides*, Vestnik YuUrGU. Ser. Mat. Model. Progr., 12 (2019), pp. 143–149.

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G. V. Martynov (Institute for Information Transmission Problems of the Russian Academy of Sciences (Kharkevich Institute), Moscow, Russia)**. New Cramér–von Mises multivariate uniformity test for large dimensions.**

Here we consider the problem of testing the hypothesis of uniformity of a distribution in the cube $[0,1]^m$, $m \geqslant 2$. To solve this problem, the following statistic is proposed:

$$\begin{aligned}\widetilde{\omega}_n^2 &= n\int_{[0,1]^m} \big(\widetilde{F}_n(t_1,\dots,t_m) - \widetilde{F}(t_1,\dots,t_m)\big)^2\,dt_1\cdots dt_m \\ &= \int_{[0,1]^m} \widetilde{\xi}_n^2(t_1,\dots,t_m)\,dt_1\cdots dt_m.\end{aligned}$$

Here, $\widetilde{F}(t_1,\dots,t_m) = t_1^{r_1}\cdots t_m^{r_m}$ is the generalized function of the uniform distribution on $[0,1]^m$. We assume that the constants $r_1,\dots,r_m$ are greater than -1. The corresponding generalized empirical distribution function is $\widetilde{F}_n(t_1,\dots,t_m) = (1/n)\sum_{i=1}^n \prod_{j=1}^m I\{T_{i,j} < t_j^{r_j}\}$, where $T_i = (T_{i,1},\dots,T_{i,m})$, $i = 1,\dots,n$, are n observations of the random m-vector T having the uniform distribution on $[0,1]^m$ under H_0. We denote by $\widetilde{\xi}_n(t_1,\dots,t_m)$ a generalized empirical process.

THEOREM. *A generalized empirical process $\widetilde{\xi}_n(t_1,\dots,t_m)$ weakly converges in $L^2([0,1]^m)$ to a Gauss process with the covariance function*

$$\widetilde{K}(t_1,\dots,t_m,\tau_1,\dots,\tau_m) = \prod_{j=1}^m \min(t_j^{r_j},\tau_j^{r_j}) - \prod_{j=1}^m t_j^{r_j j}\tau_j^{r_j}.$$

When $r_j = 1$, $j = 1,\dots,m$, the statistic $\widetilde{\omega}_n^2$ becomes the classical multivariate uniformity statistic ω_n^2 for testing the uniformity of a distribution on $[0,1]^m$. This statistic was considered, in particular, in [1] and [2]. The advantage of the above new statistic is that, with an appropriate choice of the sequence r_j, $j = 1,2,\dots$, its limit distribution converges, as $m \to \infty$, to a certain nonsingular distribution (see [3]). The distribution of the classical statistics in this case degenerates. Although the limit distributions of both types of statistics can be evaluated exactly, the values of the statistics themselves for a given sample are calculated by the Monte Carlo method.

REFERENCES

[1] E. N. KRIVYAKOVA, G. V. MARTYNOV, AND YU. N. TYURIN, *On the distribution of the ω^2 statistics in the multidimensional case*, Theory Probab. Appl., 22 (1978), pp. 406–410.
[2] G. V. MARTYNOV, *Omega-Square Tests*, Nauka, Moscow, 1978 (in Russian).
[3] G. MARTYNOV, *A Cramér–von Mises test for Gaussian processes*, in Mathematical Statistics and Limit Theorems, Springer, Cham, 2015, pp. 209–229.

L. E. Melkumova (Samara, Russia)**. Simplified pair copula construction and conditional quantile reproduciblity.**

Pair copula construction or PCC (see [1] for the definition) is a hierarchical method of constructing multivariate probability distributions using pair copulas, which became widely used in the late 1990s. The PCC is based on the following decomposition of a 3-dimensional conditional distribution using bivariate distributions and a pair copula:

$$F_{12|3}(x_1,x_2 \mid x_3) = C_{12|3}(F_{1|3}(x_1 \mid x_3), F_{2|3}(x_2 \mid x_3); x_3).$$

In the case where $C_{12|3}$ does not depend on x_3, the method is called the simplified PCC, and the assumption that $C_{12|3}$ is independent of x_3 is usually referred to as the simplifying assumption. We prove the following.

THEOREM. *Let a triple of r.v.'s* (X_1, X_2, X_3) *with absolutely continuous d.f.* $F_{123}(x_1, x_2, x_3)$ *and strictly monotone marginal and conditional d.f.'s satisfy the simplifying assumption; that is,* $C_{12|3}$ *is independent of* x_3*. Then the three-dimensional conditional distribution* $F_{1|23}(x_1|x_2, x_3)$ *that corresponds to* $F_{123}(x_1, x_2, x_3)$ *has the reproducibility property for the respective conditional quantiles*

$$q_{1|23}^{(x_1^0,x_2^0,x_3^0)}\big(q_{2|3}^{(x_2^0,x_3^0)}(x_3), x_3\big) = q_{1|3}^{(x_1^0,x_3^0)}(x_3), \tag{1}$$

where $q_{i|\mathbf{j}}^{(\mathbf{x^0})}(\mathbf{x_j})$ *are conditional quantiles going through the point* $\mathbf{x^0}$*. The converse is also true: if* (1) *holds, then the simplifying assumption is satisfied.*

The conditional quantile reproducibility property and its version (the "full" conditional quantile reproducibility) were considered in detail in [2], which also provides some examples of probability distributions with reproducible conditional quantiles and gives a necessary condition for the full reproducibility in terms of a Pfaffian differential equation of a certain form. In this talk, we also discuss the connection between pair copulas corresponding to different pairs of r.v.'s from the (X_1, X_2, X_3) triple.

REFERENCES

[1] J. STÖBER, H. JOE, AND C. CZADO, *Simplified pair copula constructions—limitations and extensions*, J. Multivariate Anal., 119 (2013), pp. 101–118.
[2] S. YA. SHATSKIKH AND L. E. MELKUMOVA, *Reducing the sample size when estimating conditional quantiles*, CEUR Workshop Proceedings, 1638 (2016), pp. 769–781; available online at http://ceur-ws.org/Vol-1638/Paper91.pdf.

F. S. Nasyrov (Ufa, Russia)**. On strong solutions of stochastic differential equations.**

Let $(\Omega, \mathscr{F}, (\mathscr{F}_t)_{t\in[t_0,T]}, \mathbf{P})$ be a filtered probability space, let $W(t)$ be a standard $\mathscr{F}_t$-Wiener process on this space, and let $\mathscr{B}(\mathbf{R}^2)$ be a Borel σ-algebra.

THEOREM. 1. *An ordinary differential equation* $y' = f(t, W(t), y + W(t))$, $t \in [t_0, T]$, $y(t_0) = y_0$, *where* $f(t, v, y) = f(t, v, y, \omega)$ *is a* $\mathscr{P} \times \mathscr{B}(\mathbf{R}^2)$*-measurable random function, has a strong solution if, with probability* 1,

$$|f(t, v, y)| \leqslant n(t), \quad \textit{where } n(t) = n(t, \omega) \textit{ is summable in } t. \tag{1}$$

2. *Let* $X(s)$ *be an arbitrary continuous function. Assume that the following conditions are satisfied:* (a) *the equation* $(\varphi^*)'_v = \sigma(t, v, \varphi^*)$ *has a general solution* $\phi^*(t, v, C(t))$; (b) *the measurable function*

$$f(t, y) = \frac{B(t, X(t), \varphi^*(t, X(t), y)) - (\varphi^*)'_t(t, X(t), y)}{\sigma(t, X(t), \varphi^*(t, X(t), y))}$$

satisfies (1) *with nonrandom function* $n(t)$*. Then the equation with symmetric integral*

$$\xi(t) - \xi(t_0) = \int_{t_0}^{t} \sigma(s, X(s), \xi(s)) * dX(s) + \int_{t_0}^{t} B(s, X(s), \xi(s))\, ds \tag{2}$$

has a solution $\xi(t) = \phi^*(t, X(t), C(t))$.

3. *If the assumptions of the previous assertion are satisfied with probability* 1 *and with a Borel function* $\sigma(t, u, \varphi)$ *and a* $\mathscr{P} \times \mathscr{B}(\mathbf{R}^2)$*-measurable function* $B(t, u, \varphi, \omega)$, *then there exists a strong solution of the Stratonovich equation* (2) *with* $X(s)$ *replaced by* $W(s)$.

4. *Consider the Itô equation*

$$d\xi(t) = \sigma(t, W(t), \xi(t))\, dW(t) + B(t, W(t), \xi(t))\, dt, \qquad \xi(0) = \xi_0. \tag{3}$$

Under the hypotheses of assertion 3, *we assume that the continuous function* $\sigma(t, u, \varphi)$ *has continuous partial derivatives* $\sigma'_u(t, u, \varphi)$ *and* $\sigma'_\varphi(t, u, \varphi)$. *Then there exists a strong solution* $\xi(t) = \phi^*(t, W(t), C(t))$ *of* (3), *and the random function* $\phi(t, W(t)) \equiv \phi^*(t, W(t), C(t))$ *satisfies the relation*

$$\phi'_t(t, W(t)) = -\frac{1}{2}\, \phi''_{uu}(t, W(t)) + b\bigl(t, W(t), \phi(t, W(t))\bigr), \qquad t \in [t_0, T],$$

with probability 1.

V. A. Naumov (Helsinki, Finland)**, Yu. V. Gaidamaka** (Moscow, Russia)**, K. E. Samouylov** (Moscow, Russia)**. Product-form Markovian multiresource loss systems.** [17]

Multiresource loss systems are a generalization of the classical Erlang Loss System in that an arriving customer may require one or more types of limited resources. A loss system with general cumulative distribution function of the requested quantity of a single resource was studied in [1]. Later, queueing systems with random resource requirements were extensively studied. The notion of positive and negative customers (introduced in [2]) has radically increased the scope of applications of queueing theory. In multiresource loss systems, arrivals of negative customers temporarily increase the amount of resources available to positive customers [3]. We study a state-dependent multiresource loss system with positive and negative customers described by a homogeneous Markov jump process $\mathrm{X}(t) = (\boldsymbol{\xi}(t), \boldsymbol{\theta}(t))$, where the vector $\boldsymbol{\xi}(t) = (\xi_1(t), \dots, \xi_K(t))$ represents the number of customers of each class in the system at time t, and $\boldsymbol{\theta}(t) = (\theta_1(t), \dots, \theta_K(t))$ represents the resource quantities allocated to each customer.

THEOREM. *The stationary probability distribution of the process* $X(t)$ *is given by*

$$P_{\mathbf{n}}(\mathbf{x}) = p_0 \Phi_{\mathbf{n}}(\mathbf{x}) \prod_{k=1}^{K} \prod_{j=1}^{n_k} \frac{\lambda_k(j-1)}{\mu_k(j)},$$
$$\Phi_{\mathbf{n}}(\mathbf{x}) = \int_{\mathbf{y} \leqslant \mathbf{x},\, \mathbf{y}\mathbf{U} \leqslant \mathbf{R}} F_1(d\mathbf{y}_{1,1}) \cdots F_K(d\mathbf{y}_{K,n_K}),$$

where $F_k(\mathbf{x})$ *is the cumulative distribution function of resource quantities requested by a customer of type* k.

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REFERENCES

[1] E. L. Romm and V. V. Skitovich, *On certain generalization of problem of Erlang*, Autom. Remote Control, 32 (1971), pp. 1000–1003.

[2] E. Gelenbe, *Product-form queuing networks with negative and positive customers*, J. Appl. Probab., 28 (1991), pp. 656–663.

[3] V. Naumov and K. Samouylov, *Analysis of multi-resource loss system with state-dependent arrival and service rates*, Probab. Engrg. Inform. Sci., 31 (2017), pp. 413–419.

I. V. Pavlov, I. V. Tsvetkova (Rostov-on-Don, Russia)**. Interpolating deflators and interpolating martingale measures.** [18]

It is well known that on nonarbitrage financial (B,S)-markets with a fixed physical probability Q, there is a one-to-one correspondence between martingale measures (m.m.) equivalent to Q and martingale (relative to Q) deflators (m.d.). If a (B,S)-market is defined on no more than a countable probabilistic space $(\Omega,\mathcal{F},Q)$, then for construction of hedge portfolios it turns out to be useful to consider the "most fair" m.m.'s, which we called interpolating m.m.'s [1], [2]. We also designate the corresponding deflators as interpolating deflators. Let us clarify this definition in a one-step model.

Let $(\mathscr{F}_k)_{k=0}^1$ be a filtration on Ω such that $\mathscr{F}_0=\{\Omega,\varnothing\}$, and let $\mathscr{F}_1$ be the set of all subsets of a countable Ω with strictly positive Q-probability (except for $\varnothing$). Let $Z=(Z_k,\mathscr{F}_k,Q)_{k=0}^1$ be a discounted stock price, and let $H=(H_k,\mathscr{F}_k,Q)_{k=0}^1$ be a strictly positive m.d. with $H_0=1$. We fix some family of interpolating filtrations $\mathbf{F}=\{\mathbf{F}^\alpha\}$ indexed by parameter α, where $\mathbf{F}^\alpha=(\mathscr{F}_n^\alpha)_{n=0}^\infty$ and such that $\mathscr{F}_0^\alpha=\mathscr{F}_0$, $\mathscr{F}_\infty^\alpha=\mathscr{F}_1$ for each α. Consider the following martingale interpolations of the original deflator H: $H_n^\alpha=E^Q[H_1\mid\mathscr{F}_n^\alpha]$, $n=0,1,2,\dots$. On the other hand, let P be an $\mathbf{F}$-interpolating m.m. of the process Z; that is, for each α, the process $Z_n^\alpha=E^P[Z_1\mid\mathscr{F}_n^\alpha]$, $n=0,1,2,\dots$, admits a unique m.m. (namely, only the measure P). If in the original model the measure P corresponds to H (that is, $dP=H_1\,dQ$), then the generalized Bayes formula implies that $H_n^\alpha Z_n^\alpha=E^Q[H_1Z_1\mid\mathscr{F}_n^\alpha]$ for any α; that is, H_n^α is an m.d. of the process Z_n^α. Since P is a unique m.m. of the process Z_n^α, H_n^α is a unique m.d. of this process. This establishes the following.

Proposition. *An m.d. H of the process Z corresponding to an m.m. P is an interpolating m.d. if and only if it has the following uniqueness property*: *for any α, the process H_n^α is a unique m.d. of the process Z_n^α.*

In our talk, conditions for the existence of interpolating deflators are given in terms of the parameters of the process Z and properties of the physical measure Q.

REFERENCES

[1] I. V. Pavlov, V. V. Shamrayeva, and I. V. Tsvetkova, *On the existence of martingale measures satisfying the weakened condition of noncoincidence of barycenters in the case of countable probability space*, Theory Probab. Appl., 61 (2017), pp. 167–175.

[2] I. V. Pavlov, *New family of one-step processes admitting special interpolating martingale measures*, Global Stochastic Anal., 5 (2018), pp. 111–119.

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A. Yu. Perevaryukha (Saint-Petersburg, Russia). **Population model of pest with stochastic transition to the outbreak phase.** [19]

We propose a modification of the computational model [1] for a *Cardiaspina albitextura* psyllid pest outbreak scenario in an Australia forest after an accidental release of population state out of the control interval Ω_s. Survival of generations $R = N(T)$ beginning with $N(0) = \lambda S$, $S \in \Omega_S$, is described on the interval $t \in [0, \dots, \xi, \omega, \dots, T]$ in different mortality rates on different stages of psyllid insect ontogeny by a predictively redefined system

$$\frac{dN}{dt} = \begin{cases} -(\alpha\overline{w}(\xi)N(t) + \overline{\Theta}(N(0))\beta)N(t), & 0 < t < \xi, \\ -\left(\dfrac{\alpha_1 N(\xi)}{w(\omega)} + \beta\right)N(t), & \xi < t < \omega, \\ -(\alpha_2 N(t))N(t - \varsigma), & \omega < t < T, \end{cases} \tag{1}$$

where ς is the lag due to the time delayed action of density regulation, $[0, \xi], [\xi, \omega]$ is the duration of stages, α, β are indicators of mortality rates, $\Theta(N(0)) = [1 + \exp(-\kappa N(0)^2)]$, and $\lim_{N(0)\to\infty} \Theta(N(0)) = 1$ is the threshold reduction in reproduction efficiency for $S < \mathcal{L}$. Let $\mathscr{L} \subset U_1 \in \Omega_S$ be the region of a small group of individuals where reproduction is due to random factors. Also let $\overline{\Theta}(N(0), w) = \Theta(N(0)) \times w(t)$, where $w(t)$ is the index of conditional dimensional development, as defined from the equation $\dot{w}(t) = [G/(N^{2/3} + \sigma)] \times \gamma$, $w(0) = w_0$, and γ is a uniformly distributed r.v. The iteration trajectory $x_{n+1} = \psi(x_n)$, $x_0 < \mathscr{L}$, as obtained from the unimodal dependence $\psi(x) = \bigcup_{N(0)} N(T)$, $N(0) \in \mathbf{Z}_+$, of numerical solutions of the Cauchy problem (1) on the interval $t \in [0, T]$, has properties of a bounded stochastic perturbation. Instead of a threshold point such that $\psi(x_*) = x_* < \max\psi(x)$ $\forall\, x < x_* - \epsilon$, we get some neighborhood $\lim_{n\to\infty} \psi^n(x_*) = U_0$, $U_0 < \epsilon$. The set U_0 forms an interval of probabilistic behavior of the trajectory, which simulates an outbreak situation caused by a small group of insects.

THEOREM 1. *The probability of the event* $\{x_0 < x_*,\ \psi^k(x_0) > \max\psi(x)$ *for* $k < \infty\}$ *is positive.*

THEOREM 2. *For all* x_0 *and* $\psi^n(x_0)$, *there exists a stable* ω*-limit set* $\psi^p(x_i) = \psi^{p+2}(x_i)$, $x_i > \max\psi(x)$.

Model (1) combines stochastic and deterministic behavior in two ranges that do not have a smooth boundary for allowable insect pest values Ω_S.

REFERENCES

[1] A. Yu. Perevaryukha, *Modeling the phenomenon of collapse of the exploited fish population with stochastic uncertainty*, abstract, Second International Conference on Stochastic Methods, Theory Probab. Appl., 62 (2018), pp. 659–660.

[19] This work was supported by the budget of the St. Petersburg Institute for Informatics and Automation of Russian Academy of Sciences (grant AAAA-A16-116051250009-8).

M. V. Platonova, S. V. Tsykin (St. Petersburg Department of Steklov Mathematical Institute of Russian Academy of Sciences, Russia)**. On probabilistic approximations of the solution to the Cauchy problem for higher-order Schrödinger type equations.** [20]

Consider the Cauchy problem for the higher-order Schrödinger type equation

$$i\frac{\partial u}{\partial t} = \frac{(-1)^m}{(2m)!}\,\frac{\partial^{2m} u}{\partial x^{2m}}, \quad u(0,x) = \varphi(x), \qquad m \in \mathbf{N}.$$

A probabilistic method of approximation of the solution $u(t,x)$ to the Cauchy problem for the Schrödinger equation ($m=1$) by expectations of functionals of stochastic processes was proposed in [1]. We extend this approach to the case $m \geqslant 2$.

Let $\nu(dt,dx)$ be a Poisson random measure on $(0,\infty)\times(0,\infty)$ with intensity

$$\mathbf{E}\,\nu(dt,dx) = \frac{dt\,dx}{x^{2m+1}}.$$

Given $\varepsilon>0$, we define the r.v. $\xi_\varepsilon(t) = \int_0^t \int_\varepsilon^{e\varepsilon} x\,\nu(ds,dx)$ and consider the function

$$u_\varepsilon(t,x) = \mathbf{E}\big[(\varphi_- * h_\varepsilon)(x-\sigma\xi_\varepsilon(t)) + (\varphi_+ * h_\varepsilon)(x+\sigma\xi_\varepsilon(t))\big],$$

where

$$\varphi_+(x) = \frac{1}{2\pi}\int_{-\infty}^0 e^{-ipx}\widehat{\varphi}(p)\,dp, \quad \varphi_-(x) = \frac{1}{2\pi}\int_0^\infty e^{-ipx}\widehat{\varphi}(p)\,dp,$$

and the function $h_\varepsilon(x)$ is defined by its Fourier transform

$$\widehat{h}_\varepsilon(p) = \exp\biggl(-t\int_\varepsilon^{e\varepsilon}\biggl(\sum_{j=1}^{2m-1}\frac{(i|p|\sigma x)^j}{j!} + \frac{(i|p|\sigma x)^{2m+1}}{(2m+1)!}\biggr)\frac{dx}{x^{2m+1}}\biggr).$$

THEOREM. *For any function $\varphi \in W_2^{2m+2}(\mathbf{R})$ and any $t \geqslant 0$, there exists a constant $C>0$ such that*

$$\|u(t,x) - u_\varepsilon(t,x)\|_{L_2(\mathbf{R})} \leqslant Ct\varepsilon^2\|\varphi\|_{W_2^{2m+2}(\mathbf{R})}.$$

REFERENCES

[1] I. A. Ibragimov, N. V. Smorodina, and M. M. Faddeev, *On a limit theorem related to probabilistic representation of solution to the Cauchy problem for the Schrödinger equation*, J. Math. Sci. (N.Y.), 229 (2018), pp. 702–713.

E. L. Presman (Moscow, Russia)**, Sh. K. Formanov** (Tashkent, Uzbekistan)**. On one modification of the Lindeberg and Rotar' conditions.**

We consider a triangular array scheme for the sums of independent (within each row) r.v.'s with finite variances and zero expectations. Without loss of generality we assume that the sum of variances within each row is 1. Lindeberg introduced a characteristic, which associates with each $\varepsilon>0$ the sequence $\{L_n(\varepsilon)\}_{n=1}^\infty$ of sums of the variances of ε-tails of distributions of summands. It is well known (see, for example, [1, Chap. III, section 4]) that the Lindeberg condition ($\lim_{n\to\infty} L_n(\varepsilon)=0$

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$\forall\varepsilon > 0$) is sufficient for the normal convergence of the sequence of corresponding sums; the Lindeberg condition is also necessary in the case of uniform infinite smallness of the summands.

We associate with each $\alpha > 0$ the sequence of sums of absolute moments of order $2+\alpha$ for the distributions of the summands truncated at the unit level; its sum with the Lindeberg characteristic corresponding to $\varepsilon = 1$ is called the α-characteristic and denoted by $\{L_n^\alpha\}_{n=1}^\infty$.

THEOREM. (a) *If* $\lim_{n\to\infty} L_n^\alpha = 0$ *for some* $\alpha > 0$, *then the Lindeberg condition holds.*

(b) *If the Lindeberg condition holds, then* $\lim_{n\to\infty} L_n^\alpha = 0$ *for all* $\alpha > 0$.

Thus, when checking the normal convergence, rather than checking the convergence to zero of the Lindeberg characteristic *for any* $\varepsilon > 0$, it suffices to check that there *exists* an $\alpha > 0$ such that the α-characteristic converges to zero.

Rotar' (see [2] or [1, section III.5]) considered an analogue of the Lindeberg characteristic and showed that the convergence of his characteristic to zero for any $\varepsilon > 0$ is a necessary and sufficient condition for normal convergence without the assumption of uniform infinite smallness of the summands. We give the corresponding modification also for the Rotar' characteristic.

These results can be found in [3] in English and [4] in Russian; however, there are some inaccuracies in the formulation and proof of Lemma 2 in [3], which were corrected in the Russian version [4].

REFERENCES

[1] A. N. SHIRYAEV, *Probability*—1, 3rd ed., Grad. Texts in Math. 95, Springer, New York, 2016.
[2] V. I. ROTAR', *An extension of the Lindeberg–Feller theorem*, Math. Notes, 18 (1975), pp. 660–663.
[3] E. L. PRESMAN AND SH. K. FORMANOV, *On Lindeberg–Feller limit theorem*, Dokl. Math., 99 (2019), pp. 204–207.
[4] E. L. PRESMAN AND SH. K. FORMANOV, *On Lindeberg–Feller limit theorem*, Doklady RAN, Ser. Matem., 485 (3), (2019), pp. 16–19 (in Russian).

I. V. Rodionov (Trapeznikov Institute of Control Sciences of Russian Academy of Sciences, Moscow, Russia). **Parameter estimation of distribution tails.** [21]

We focus on the problem of estimation of distribution tail parameters. The problem of tail estimation is central in the statistics of extremes of independent observations. The generally accepted approach to estimate the distribution tail in this theory is semiparametric and based on Pickands–Balkema–de Haan's theorem (see [1], [2]), which reduces this problem to that of estimation of the extreme value index (see [3] for details). This approach works well for distributions with power-law tails that often appear in finance and insurance. However, one cannot distinguish between the distribution tails with exponential rate of decrease using this approach [4]. Moreover, the conditions of Pickands–Balkema–de Haan's theorem are not satisfied for a large class of distributions, in particular, for distributions with logarithmic tails. This calls for a general method of tail estimation that is not based on the above theorem and such that it can be applied to most practical distributions.

Let $\mathbf{X} = (X_1, \dots, X_n)$ be i.i.d. r.v.'s with continuous distribution function F. Let $X_{(1)} \leqslant \dots \leqslant X_{(n)}$ be the nth order statistics corresponding to $\mathbf{X}$. Evidently, only the

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largest order statistics can be used in the problem of tail estimation. Assume that F belongs to the parametric class of continuous distribution tails $\mathscr{F} = \{F_\theta,\, \theta \in \Theta\}$, $\Theta \subset \mathbf{R}$. (For selection of an appropriate parametric class in this problem, see [4], [5].) Consider the statistic

$$R_{k,n}(\theta) = \ln(1 - F_\theta(X_{(n-k)})) - \frac{1}{k} \sum_{i=n-k+1}^{n} \ln(1 - F_\theta(X_{(i)}))$$

and the estimator $\widehat{\theta}_{k,n} = \arg\{\theta\colon R_{k,n}(\theta) = 1\}$ of the parameter θ based on $R_{k,n}(\theta)$.

THEOREM 1. *Let a parametric family $\mathscr{F}$ be ordered w.r.t. θ (for definition, see [6]). Then the solution of the equation $R_{k,n}(\theta) = 1$ w.r.t. θ is unique a.s. and the estimator $\widehat{\theta}_{k,n}$ is consistent as $k \to \infty$, $k/n \to 0$, $n \to \infty$.*

In this talk, we discuss the properties of the proposed method, viz., the unique solvability of the equation $R_{k,n}(\theta) = 1$, the consistency and asymptotic normality of $\widehat{\theta}_{k,n}$, and also modification of this method for Weibull-tail and log-Weibull-tail index estimation.

REFERENCES

[1] A. A. Balkema and L. de Haan, *Residual life time at great age*, Ann. Probab., 2 (1974), pp. 792–804.

[2] J. Pickands, III, *Statistical inference using extreme order statistics*, Ann. Statist., 3 (1975), pp. 119–131.

[3] L. de Haan and A. Ferreira, *Extreme Value Theory. An Introduction*, Springer Ser. Oper. Res. Financ. Eng., Springer, New York, 2006.

[4] I. V. Rodionov, *On discrimination between classes of distribution tails*, Probl. Inf. Transm., 54 (2018), pp. 124–138.

[5] I. V. Rodionov, *A discrimination test for tails of Weibull-type distributions*, Theory Probab. Appl., 63 (2018), pp. 327–335.

[6] I. Rodionov, *On parametric estimation of distribution tails*, in 4th ISNPS (Salerno, 2018), to appear.

V. V. Rodochenko, O. E. Kudryavtsev (Rostov-on-Don, Russia). **On nonparametric calibration scheme for CGMY model on cryptocurrency markets via Gaussian process regression.** [22]

Consider a set of cryptocurrency market data observations $(X, y) = \{(x_i, y_i),\, i = 1, \dots, n\}$, where each x_i is an input vector (BTC/USD rate history and probabilities of crossing a set of barriers by log returns of the rate), and y_i is the corresponding output (CGMY model parameters calibrated using x_i (see, e.g., [2])). To find the relation between inputs and outputs, we use the Gaussian process regression (GPR) approach described in [2] and assume that $y_i = f(x_i) + \varepsilon_i$, where $f(x)$ is a Gaussian process, and a set of i.i.d. variables $\varepsilon_i \sim N(0, \sigma^2)$, $\sigma^2 \geqslant 0$, represents the noise in the data. Given $X^* \in X$, we construct a covariance matrix $K(X^*, X^*)$ (by choosing two hyperparameters of a squared exponential kernel function), define $f \sim N(0, K(X^*, X^*))$, and make an initial guess about f to generate $(X, f) = \{(x_i, f_i),\, i = 1, \dots, n\}$. To train the algorithm, we invert $K(X^*, X^*)$ and calculate a posteriori $f(x)$ using the scheme presented in [2]. Now, for $X^{**} \in X \setminus X^*$, we compare (X^{**}, f) with (X^{**}, y). The trained GPR method offers calibration by several folds faster than the one offered by the scheme from [1].

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PROPOSITION. *Two populations of* 50 *CGMY process samples with* $t = 1/365$ *and parameters obtained via the scheme from* [1] *and by the trained GPR method, respectively, are statistically indistinguishable by the Wilcoxon signed-rank test at a significance level of* 5%.

REFERENCES

[1] O. KUDRYAVTSEV AND S. GRECHKO, *Statistical methods for calibrating models of cryptocurrencies prices*, Accounting and Statistics, 4 (2018), pp. 67–76.

[2] J. DE SPIEGELEER, D. B. MADAN, S. REYNERS, AND W. SCHOUTENS, *Machine learning for quantitative finance: Fast derivative pricing, hedging and fitting*, Quant. Finance, 18 (2018), pp. 1635–1643, https://doi.org/10.2139/ssrn.3191050.

D. B. Rokhlin (Southern Federal University, Rostov-on-Don, Russia). **Analysis of some incentive mechanisms in multiagent systems.** [23]

We consider three problems related to resource management in multiagent systems. In each of them the aim is to construct an incentive function prompting the agents to follow the strategies that are optimal for the system manager. We discuss in more detail the allocation of the network link capacities between a large number of users.

Consider a network with m links and N users. Each user i transmits packets over a fixed set of links. The network structure is determined by the routing matrix $R = (R_i^j) \in \mathbf{R}^{m\times N}$. Its columns $R_i \neq 0$, $i = 1, \dots, N$, are binary m-dimensional vectors such that $R_i^j = 1$ if the link j is utilized by the ith user, and $R_i^j = 0$ otherwise. The link capacities are described by a vector $b \in \mathbb{R}^m$ with positive components. The users evaluate the network quality via the utility functions $u_i(x^i)$, which depend on the transmission rates $x^i \in \mathbb{R}_+$. An optimal resource allocation corresponds to an optimal solution $x^* \in \mathbb{R}_+^N$ of the network utility maximization (NUM) problem

$$u(x) = \sum_{i=1}^N u_i(x^i) \to \max, \qquad Rx = \sum_{i=1}^N R_i x^i \leqslant b, \qquad x \in \mathbf{R}_+^N;$$

this problem was formulated in [2].

For given link prices $\overline{\lambda} \in \mathbf{R}_+^m$, the users select optimal transmission rates $\overline{x}_i$ maximizing the difference between the utility and price of $\overline{x}_i$: $\overline{x}^{\,i} \in \arg\max_{x^i \in \mathbf{R}_+} \bigl(u_i(x^i) - x^i \sum_{j=1}^m \overline{\lambda}^j R_i^j\bigr)$. The aim of the management is to stimulate this optimal resource allocation x^* by setting the link prices $\lambda^* \in \mathbb{R}_+^m$.

THEOREM 1. *Let* u_i *be twice continuously differentiable and such that* $u_i(0) = 0$, $0 < u_i'(0) \leqslant B < \infty$, *and let* $-u_i$ *be* $(N\sigma)$*-strongly convex*: $-u_i''(x^i) \geqslant N\sigma$, $x^i \in \mathbf{R}_+$, $\sigma > 0$. *We define* λ_t *by the recurrence relation*

$$\lambda_{t+1} = \Pi_\Lambda\bigl(\lambda_t - \eta_t(b - NR_{\xi_{t+1}}\overline{x}^{\,\xi_{t+1}}(\lambda_t))\bigr), \qquad t \geqslant 1, \quad \lambda_1 \in \Lambda,$$

where Π_Λ *is the projection onto the hypercube* $\Lambda = [0, B]^m$, *and* $\eta_t = K/\sqrt{t}$ *and* ξ_t *are independent r.v.'s uniformly distributed on* $\{1, \dots, N\}$. *Then, for* $\overline{\lambda}_T = (1/T)\sum_{t=1}^T \lambda_t$,

$$\mathbf{E}\biggl(\sum_{i=1}^N R_i^j \overline{x}^i(\overline{\lambda}_T) - b^j\biggr)^+ \leqslant \sqrt{\frac{2D}{\sigma}}\,\frac{1}{T^{1/4}}, \qquad u(x^*) - \mathbf{E}u(\overline{x}(\overline{\lambda}_T)) \leqslant B\sqrt{\frac{2D}{\sigma}}\,\frac{1}{T^{1/4}},$$

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where $D = mB^2/(2K) + KL^2$, $y^+ = \max\{y, 0\}$.

The proposed algorithm does not use information on the aggregate traffic over each link. We compare this algorithm theoretically and via computer experiments to Nesterov's fast gradient descent method as applied to the NUM problem in [1].

REFERENCES

[1] A. Beck, A. Nedić, and A. Ozdaglar, *An $O(1/k)$ gradient method for network resource allocation problems*, IEEE Trans. Control Netw. Syst., 1 (2014), pp. 64–73.

[2] F. P. Kelly, A. K. Maulloo, and D. K. H. Tan, *Rate control for communication networks: Shadow prices, proportional fairness and stability*, J. Oper. Res. Soc., 49 (1998), pp. 237–252.

K. S. Ryadovkin (St. Petersburg Department of Steklov Mathematical Institute of Russian Academy of Sciences, Russia)**. On branching random walks on periodic lattices.**[24]

Let $g_1, \dots, g_d \in \mathbb{Z}^d$ be a family of linearly independent vectors with integer coordinates. We designate a set $\Gamma = \{g \in \mathbf{Z}^d \colon g = \sum_{j=1}^d n_j g_j,\ n_j \in \mathbf{Z},\ j = 1, \dots, d\}$ as a lattice. Let us consider a branching random walk on $\mathbb{Z}^d$ with transition intensities $a(v,u)$ satisfying the conditions

$$a(v,u) = a(u,v) = a(v+g, u+g) \quad \text{and} \quad \sum_{w \in \mathbf{Z}^d} \|w\|^2 |a(v,w)| < \infty$$

for all $u, v \in \mathbf{Z}^d$ and $g \in \Gamma$. We also assume that the random walk is irreducible. Assume that the branching sources are located at each vertex $v \in \mathbf{Z}^d$ and that the branching intensities $b_k(v)$ into k offsprings satisfy the conditions

$$\beta_n(v) = \sum_{k=1}^{+\infty} k^n b_k(v) < \infty \quad \text{and} \quad \beta_n(v+g) = \beta_n(v)$$

for all $v \in \mathbf{Z}^d$, $g \in \Gamma$, and $n = 1, 2$. Asymptotic behavior of the mean number of particles for such a random walk was studied in [1].

We denote the evolution operator of the mean number of particles by $\mathscr{A}$ and the right edge of the spectrum of this operator by λ. By $M_2(v,u,t)$ we denote the second moment of the number of particles at the vertex u at time t, provided that at the initial time $t = 0$ there is one particle at the vertex v.

Theorem 1. *Let $\lambda > 0$. Then, as $t \to +\infty$,*

$$M_2(v,u,t) = \frac{e^{2\lambda t} t^{-d} c(v,u)}{\lambda} (1 + O(t^{-1})),$$

where $c(v,u)$ can be explicitly evaluated.

REFERENCES

[1] M. V. Platonova and K. S. Ryadovkin, *Branching random walks on $\mathbf{Z}^d$ with periodic branching sources*, Theory Probab. Appl., 64 (2019), pp. 229–248.

[24]Supported by the Russian Science Foundation (grant 17-11-01136).

V. V. Rykov (RUDN University, Moscow, Russia)**, D. V. Kozyrev** (RUDN University and V. A. Trapeznikov Institute of Control Sciences of RAS, Moscow, Russia). **On sensitivity of stochastic models.** [25]

Stability of output characteristics of different systems under a variation of their input data and external influences is one of the key problems in natural science. For stochastic systems, stability often means insensitivity of their characteristics to the shape of their input information distributions. For some stochastic systems, we give certain classical and modern results on their strong and asymptotic insensitivity to input information distributions.

1. *Sevast'yanov's theorem* [1]. This result can be treated as the insensitivity of stationary probabilities of an Erlang system with Poisson input to the shape of service time distribution with fixed mean service time. Analogously, the BCMP-*theorem* [2] asserts the insensitivity of output characteristics for a wide class of stochastic networks to the shape of service time distribution in their nodes.

2. *Kovalenko's theorem* [3] provides necessary and sufficient conditions for insensitivity of steady state probabilities for a redundant renewable system to the shape of its component repair time distributions. In *Gnedenko's* and *Solov'ev's theorems* [4], [5] it is shown that under "quick" restoration the reliability function of a redundant system tends to have an exponential form for any distribution of its life and repair times. This result can be looked upon also as asymptotic insensitivity of the system output characteristics to the shape of its input information distributions. In ICSM-3 (see [6]), an extension of this result was proposed for different classes of systems under wider assumptions about component failures.

3. *Sparre Andersen's theorem* on approximation of the ruin probability under Cramér–Lundberg conditions can be treated as the insensitivity of the ruin probability to the form of distributions of claim interarrival times. Moreover, the ruin probability is known to be quite sensitive to the shape of claim size distributions.

4. The membership of an optimal control strategy for a Markov process in the class of simple Markov strategies can be treated also as its insensitivity to observations about the process up to the moment of decision. An extension of the result to the class of discrete time controllable semiregenerative processes (which are capable of modeling many applied stochastic systems) was found in [7].

5. We propose sensitivity investigation for storage and warranty models as problems for future studies.

REFERENCES

[1] B. A. Sevast'yanov, *An ergodic theorem for Markov processes and its application to telephone systems with refusals*, Theory Probab. Appl., 2 (1957), pp. 104–112.
[2] I. N. Kovalenko, *Studies in the Reliability Analysis of Complex Systems*, Naukova Dumka, Kiev, 1975 (in Russian).
[3] F. Baskett, K. M. Chandy, R. R. Muntz, and F. D. Palacios, *Open, closed and mixed networks of queues with different classes of customers*, J. Assoc. Comput. Mach., 22 (1975), pp. 248–260.
[4] B. V. Gnedenko, *Doubling with renewal*, Izv. Akad. Nauk SSSR Tehn. Kibernet., no. 5, 1964, pp. 111–118 (in Russian).
[5] A. D. Solov'yev, *Standby with rapid renewal*, Izv. Akad. Nauk SSSR Tehn. Kibernet., no. 1, 1970, pp. 56–71.

[25] Supported by the Russian Foundation for Basic Research (grant 17-01-00633).

[6] V. V. Rykov and D. V. Kozyrev, *On problems of sensitivity of stochastic models*, abstract, Third International Conference on Stochastic Methods, Theory Probab. Appl., 64 (2019), p. 157.
[7] M. Yu. Kitaev and V. V. Rykov, *Controlled Queueing Systems*, CRC Press, Boca Raton, FL, 1995.

A. I. Rytova (Moscow, Russia)**. Branching walk with an infinite number of initial particles and heavy tails.** [26]

We consider a random field $n(t,\cdot)$ of particles on $\mathbb{Z}^d$, $d \geqslant 1$, such that their evolution includes walking of particles on the lattice and branching (i.e., death of particles and birth of an arbitrary number of descendants) at the origin. At the initial moment of time, at each point $x \in \mathbf{Z}^d$ there is a single particle, which determines the subpopulation $n_x(t,\cdot)$ of particles originating from it. Using [1], [2], [3], we obtain the following result for a branching random walk (BRW) with heavy tails (see the definitions in [1]).

THEOREM 1. *For a symmetric BRW on $\mathbb{Z}^d$, $d \geqslant 1$, with parameter $\alpha \in (0,2)$ determining the heaviness of the random walk tails as $t \to \infty$,*

$$\mathbf{E}\, n(t,y) \sim C(y)\, v(t), \quad \mathbf{E}\, n_x(t,y) \sim C(x,y)\, u(t),$$

where the functions $v(t)$, $u(t)$ are equal to the functions $e^{\lambda t}$, $e^{\lambda t}$ for a supercritical BRW or to the functions $v_{cr}(t)$, $u_{cr}(t)$ and $v_{sub}(t)$, $u_{sub}(t)$ for a critical and a subcritical BRW, respectively, and have the forms

(a) $v_{\mathrm{cr}}(t) = 1$, $u_{\mathrm{cr}}(t) = t^{-1/\alpha}$, $v_{\mathrm{sub}}(t) = t^{1/\alpha - 1}$, $u_{\mathrm{sub}}(t) = t^{1/\alpha - 2}$;
(b) $v_{\mathrm{cr}}(t) = 1$, $u_{\mathrm{cr}}(t) = t^{-1}$, $v_{\mathrm{sub}}(t) = \ln^{-1} t$, $u_{\mathrm{sub}}(t) = t^{-1} \ln^{-2} t$;
(c) $v_{\mathrm{cr}}(t) = t^{d/\alpha - 1}$, $u_{\mathrm{cr}}(t) = t^{d/\alpha - 2}$, $v_{\mathrm{sub}}(t) = 1$, $u_{\mathrm{sub}}(t) = t^{-d/\alpha}$;
(d) $v_{\mathrm{cr}}(t) = t \ln^{-1} t$, $u_{\mathrm{cr}}(t) = \ln^{-1} t$, $v_{\mathrm{sub}}(t) = 1$, $u_{\mathrm{sub}}(t) = t^{-d/\alpha}$;
(e) $v_{\mathrm{cr}}(t) = t$, $u_{\mathrm{cr}}(t) = 1$, $v_{\mathrm{sub}}(t) = 1$, $u_{\mathrm{sub}}(t) = t^{-d/\alpha}$;

where λ, $C(x,y)$, $C(y)$ are positive constants, and the following cases are identified for all possible combinations of the dimension d of the space and of the branching parameter $\alpha \in (0,2)$: (a) $d/\alpha \in (1/2, 1)$; (b) $d/\alpha = 1$; (c) $d/\alpha \in (1,2)$; (d) $d/\alpha = 2$; (e) $d/\alpha \in (2,\infty)$.

REFERENCES

[1] A. Rytova and E. Yarovaya, *Survival analysis of particle populations in branching random walks*, Comm. Statist. Simulation Comput. (published online 30 May 2019).
[2] E. Ermakova, P. Makhmutova, and E. Yarovaya, *Branching random walks and their applications for epidemic modeling*, Stoch. Models, 35 (2019), pp. 300–317.
[3] I. I. Khristolyubov and E. B. Yarovaya, *A limit theorem for supercritical random branching walks with branching sources of varying intensity*, Theory Probab. Appl., 64 (2019), pp. 365–384.

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N. A. Saifutdinova, D. A. Butko, S. S. Saifutdinova (Rostov-on-Don, Russia)**. Algorithm for evaluating the reliability of a water supply network with consideration of the equipment wear.**

An important problem in the operation of water supply networks is their reliability. This problem was studied, for example, in [1]. We present a new approach to evaluating the reliability of a ring water supply network with the use of its block diagram.

Let a network S_1 contain N_1 nodes and M_1 pipelines. We set $I_1 = \{1, 2, \dots, N_1\}$, $J_1 = \{1, 2, \dots, M_1\}$. The structural diagram is a directed graph $G_1 = \{V^{(1)}, X^{(1)}\}$, where $V^{(1)} = \{v_i, i \in I_1\}$ are vertices of the graph and $X^{(1)} = \{x_j,\, j \in J_1\}$ are arcs of the graph. The node corresponding to the vertex v_1 is called the water feeder, and the node corresponding to the vertex v_{N_1} is the consumer. The reliability of a network is understood as the probability of supplying water from a water supply to the consumer (the event A). Let $\mathbf{P}(A_i)$, $i \in I_1$, be the probabilities of failure-free operation of the nodes, and let $\mathbf{P}(B_j)$, $j \in J_1$, be the probabilities of trouble-free work of the pipeline sections. Also let l_1 be the number of paths from the vertex v_1 to v_{N_1}. Each path s_k, $k = 1, 2, \dots, l_1$, is a set of nodes and pipelines that the water flows through from v_1 to v_{N_1}. In this regard, one can introduce the event $D_k = B^{(k)}_{j_1} A^{(k)}_{i_1} B^{(k)}_{j_2} \cdots A^{(k)}_{i_n} B^{(k)}_{j_m}$, where $i_1, i_2, \dots, i_n \in I_1$ and $j_1, j_2, \dots, j_m \in J_1$. The following result holds.

THEOREM 1. *The reliability of the network S_1 equals*

$$P^{(1)}(A) = \mathbf{P}(A_1)\biggl(1 - \prod_{k=1}^{l_1} \mathbf{P}(\overline{D_k})\biggr)\mathbf{P}(A_{N_1}).$$

Consider a different network S_2 containing N_2 nodes and M_2 pipeline sections. If we consider the case $\mathbf{P}(A_i) = p$, $\mathbf{P}(B_j) = q$ for all possible i and j, then we can compare the reliability of these networks. The main result is given in the following theorem.

THEOREM 2. *If $N_1 < N_2$, then $P^{(1)}(A) < P^{(2)}(A)$.*

REFERENCES

[1] E. M. GAL'PERIN, *About definition of functioning reliability of water supply and wastewater systems*, Vestn. SGASU. Grad. Arkh., no. 1(14), 2014, pp. 52–67 (in Russian).

L. K. Shiryaeva (Samara State University of Economics, Russia)**. On rotated versions of the three-parameter Grubbs copula.**

Consider the Grubbs statistics $T_{n,(1)} = (\overline{X} - \min\{X_i\})/S$ and $T_n^{(1)} = (\max\{X_i\} - \overline{X})/S$, as calculated for a normal sample of size n (see [1]). Assume that in the sample $\{X_i\}_{i=1}^n$ there is one abnormal observation (outlier) and its number is unknown. We believe that the outlier differs from the other observations by the shift α and the scale parameters $\nu > 0$. We denote $G_{n,(1)}(t; \alpha, \nu) = \mathbf{P}(T_{n,(1)} < t)$, $G_n^{(1)}(t; \alpha, \nu) = \mathbf{P}(T_n^{(1)} < t)$, $\Upsilon_n(x, y; \alpha, \nu) = \mathbf{P}(T_{n,(1)} < x,\, T_n^{(1)} < y)$. Recursive relations for the marginal distribution functions $G_{n,(1)}(\,\cdot\,)$, $G_n^{(1)}(\,\cdot\,)$ and $\Upsilon_{n)}(\,\cdot\,)$ were found in [2]. According to Sclar's theorem [3], the copula C^{Gr}, as extracted from the joint distribution Υ_n, reads as $C^{\mathrm{Gr}}(G_{n,(1)}(x; \alpha, \nu), G_n^{(1)}(y; \alpha, \nu); n, \alpha, \nu) = \Upsilon_n(x, y; \alpha, \nu)$.

Grubbs copula describes negative interdependencies between r.v.'s. Copula versions rotated by 90° and 270° can be used to model positive interdependencies; i.e., $C_{90}^{\mathrm{Gr}}(u,v;n,\alpha,\nu) = v - C^{\mathrm{Gr}}(1-u,v;n,\alpha,\nu)$ and $C_{270}^{\mathrm{Gr}}(u,v;n,\alpha,\nu) = u - C^{\mathrm{Gr}}(u,1-v;n,\alpha,\nu)$. The following theorem describes the properties of the rotated versions of the copula.

THEOREM. *Let* $\Xi_n^{(90)} = \{0 \leqslant u \leqslant 1;\ \delta_n(1-u;\alpha,\nu) \leqslant v \leqslant 1\}$, $\Xi_n^{(270)} = \{0 \leqslant u \leqslant 1;\ 0 \leqslant v \leqslant 1-\delta_n(u;\alpha,\nu)\}$, *and let* $M(u,v) = \min(u,v)$ *be a maximum copula. Then, for* $n \geqslant 3$, $C_{90}^{\mathrm{Gr}}(u,v;n,\alpha,\nu) = M(u,v)$ *for all* $(u,v) \in \Xi_n^{(90)}$; *and* $C_{270}^{\mathrm{Gr}}(u,v;n) = M(u,v)$ *for all* $(u,v) \in \Xi_n^{(270)}$, *where* $\delta_n(u;\alpha,\nu) = G_n^{(1)}(\theta_n(\phi_{n,(1)}(u,v;n,\alpha,\nu)))$ *and* $\phi_{n,(1)}(\,\cdot\,)$ *is the quasi-inverse of* $G_{n,(1)}(\,\cdot\,)$.

COROLLARY. *If* $n = 3$, *then, for all* α *and* $\nu > 0$, $C_{90}^{\mathrm{Gr}}(u,v;3,\alpha,\nu) = C_{270}^{\mathrm{Gr}}(u,v;3,\alpha,\nu) = M(u,v)$ *for all* $(u,v) \in [0,1]^2$.

We also prove the following lemma.

LEMMA. *Let statistics* $T_{3,(1)}$ *and* $T_3^{(1)}$ *be calculated from a set of continuous r.v.'s* X_1, X_2, X_3 *with arbitrary distributions. Then the copula, which describes the joined distribution of* $T_{3,(1)}$ *and* $T_3^{(1)}$, *is a minimum one. Moreover, the versions of this copula rotated by* 90° *and* 270° *coincide with the maximum copula.*

REFERENCES

[1] F. E. GRUBBS, *Sample criteria for testing outlying observations*, Ann. Math. Statist., 21 (1950), pp. 27–58.
[2] L. K. SHIRYAEVA, *On distribution of Grubbs' statistics in case of normal sample with outlier*, Russian Math. (Iz. VUZ), 61 (2017), pp. 72–88.
[3] R. B. NELSEN, *An Introduction to Copulas*, 2nd ed., Springer Ser. Statist., Springer, New York, 2006.

M. M. Shumafov (Maikop, Russia)**, V. B. Tlyachev** (Maikop, Russia)**. Stochastic stability of the second-order differential equations and systems.**

We consider the problem of stability of second-order nonlinear stochastic differential equations and two-dimensional linear stationary stochastic systems. We use Lyapunov's direct method (the method of Lyapunov functions) developed originally by Kushner [1] and Khas'minskii [2] and later by other authors for problems of stability of stochastic systems. Sufficient conditions for stability in probability and exponential stability in the mean square are given for second-order nonlinear differential equations perturbed by a Gaussian white noise. For linear stationary second-order systems of stochastic differential equations, we obtain necessary and sufficient conditions of the mean-square exponential stability. For our analysis, we construct special Lyapunov functions for the stochastic equations and systems under consideration. As an example, we consider a harmonic oscillator with one of its parameters perturbed by white noise.

The main result is formulated for a stochastic system of the form

$$(1)\qquad dx = y\,dt, \qquad dy = [-yf(x) - g(x)]\,dt - \sigma_1 y\,d^*\xi_1(t) - \sigma_2(x)\,d^*\xi_2(t),$$

where $f(x)$, $g(x)$, $\sigma_2(x)$ satisfy the Lipschitz condition for all $x \in \mathbf{R}$, and $d^*\xi_1$, $d^*\xi_2$ are stochastic differentials in the sense of Stratonovich.

THEOREM. *Suppose that there exist positive constants* b_1, $\bar{b}_1$, b_2, $\bar{b}_2$, *and* s_2 *such that*

(1) $0 < b_1 + \sigma_1^2/2 < f(x) < \bar{b}_1 + \sigma_1^2/2$ *for all* $x \in \mathbf{R}$;

(2) $0 < b_2 < g(x)/x < \bar{b}_2$ *for all* $x \in \mathbf{R}$ $(x \neq 0)$, $g(0) = 0$;

(3) $0 < \sigma_2(x)/x < s_2$ *for all* $x \in \mathbf{R}$ $(x \neq 0)$, $\sigma_2(0) = 0$;

(4) $s_2^2 < 2b_1 b_2$.

Then the trivial solution $(x(t) \equiv 0,\ y(t) \equiv 0)$ *of system* (1) *is exponentially stable in the mean square.*

REFERENCES

[1] H. J. KUSHNER, *Stochastic Stability and Control*, Math. Sci. Eng. 33, Academic Press, New York–London, 1967.

[2] R. Z. HAS'MINSKIĬ, *Stochastic Stability of Differential Equations*, Translated from the Russian by D. Louvish, Monographs Textbooks Mech. Solids Fluids: Mech. Anal. 7, Sijthoff & Noordhoff, Alphen aan den Rijn; Germantown, MD, 1980.

N. A. Slepov (Moscow, Russia). **Refinement of upper bounds in limit theorems for random sums of random variables.** [27]

A modified Stein's method and auxiliary machinery based on centered equilibrium transformation (see [1]) are employed to obtain upper bounds for several types of distances between distributions of random sums of r.v.'s to the Laplace distribution. In particular, for a geometrically distributed number of independent terms, we find an optimal estimate in the limit theorem for a perfect metric of order three. Furthermore, new upper bounds for the Kantorovich distance d_L refine the recent estimates from [1] and [2].

THEOREM 1. *Let* $\{X_i\}_{i=1}^{\infty}$ *be, in a broad sense, a stationary sequence of* m*-dependent r.v.'s with* $\mathbf{E}X_1 = 0$, $\mathbf{E}X_1^2 = \sigma^2$, $\rho = \sup_i \mathbf{E}|X_i^3| < \infty$, *and let an r.v.* $N_p \sim \operatorname{Geom}(p)$ *be independent of this sequence. We set* $W_p = \sqrt{p}\sum_{k=1}^{N_p} X_k$. *Then*

$$d_L(W_p, Z) \leqslant \frac{2}{\widetilde{\sigma}^2}\left[\frac{\rho}{3} + (\rho + \sigma^3)(4m^2 + m)\right]\sqrt{p} + \frac{1}{\sqrt{2}\,\widetilde{\sigma}}\big[\widetilde{\sigma}^2 + m(m+1)\sigma^2\big]p,$$

where $Z \sim \operatorname{Laplace}(0, (1/\sqrt{2})\widetilde{\sigma})$, $\widetilde{\sigma}^2 = \mathbf{E}X_{m+1}^2 + 2\sum_{i=1}^{m}\operatorname{Cov}(X_{m+1}, X_i)$.

REFERENCES

[1] J. PIKE AND H. REN, *Stein's method and the Laplace distribution*, ALEA Lat. Am. J. Probab. Math. Stat., 11 (2014), pp. 571–587.

[2] R. E. GAUNT, *Wasserstein and Kolmogorov error bounds for variance-gamma approximation via Stein's method* I, J. Theoret. Probab., (2018), pp. 1–41.

[27] Supported by the Program "Modern Problems of Fundamental Mathematics and Mechanics" of Moscow State University.

N. V. Smorodina (St. Petersburg Department of Steklov Mathematical Institute of Russian Academy of Sciences, Russia). **Reflecting Lévy processes and corresponding families of linear operators.** [28]

We consider one-dimensional special Markov processes that are Lévy processes taking values on an interval (for definiteness, on the interval $[0,\pi]$) and reflecting from boundary points. Informally, the behavior of such a process can be described as follows. Given a process, we equip it with some additional reflection mechanism. Namely, the process starts from a point $x \in [0,\pi]$ and moves along its trajectory until it reaches the boundary of the interval. At this time, the process reflects elastically from the boundary, leaves on it a "jump of momentum," and continues to move on. The jumps of momentum are accumulated at each point of the boundary with time. It is obvious that we need a precise definition of the reflection because the Lévy processes trajectories are nondifferentiable or, moreover, can have purely jump-type trajectories. A standard way to define the reflection for a continuous trajectory is related to a solution of the so-called Skorokhod problem. Since trajectories of a Lévy process are not continuous in general, we use another approach based on ideas of the generalized function theory. Let us describe the main ideas of this approach. Let $\xi(t)$, $\xi(0) = 0$ be a Lévy process, and let $\xi_x(t) = x + \xi(t)$. An initial function f is considered as a test function, and the operation of reflection is transferred to it (the standard approach in the theory of distributions). This means that we consider a new "reflected" function $\widetilde{f}$. So, we define a reflected trajectory by the formula $\mathbf{E}f(\widetilde{\xi_x}(t)) = \mathbf{E}\widetilde{f}(\xi_x(t))$. Such a reflected process $\widetilde{\xi_x}(t)$ is a Markov process, and so the last formula automatically defines all finite-dimensional distributions.

We consider the family $\widetilde{\xi_x}(t)$, $x \in [0,\pi]$, of reflected processes (in the above sense). The evolution of a reflected process is described by two families of operators R^t, Q^t. The first family, which is a semigroup of operators from $L_2[0,\pi]$ into $L_2[0,\pi]$, describes the evolution of probability law inside the interval. The elements of the second family can also be considered as operators acting on functions defined on the interval boundary (that is, on the two-point set $\{0,\pi\}$) and which map them into functions from $L_2[0,\pi]$. This family of operators describes the evolution of the mean value of momentum accumulated at boundary points. For the Lévy processes under consideration, we define not only the average accumulated momentum, but also the random accumulated momentum $\mathcal{Q}^t(\xi(\cdot))$ that is an accumulated momentum of each "individual" trajectory. This momentum is also considered as the random operator $\mathscr{Q}^t(\xi(\,\cdot\,))$, which now depends on the trajectory of the original process, so that for every function g, we have $(Q^t g)(x) = \mathbf{E}[\mathscr{Q}^t(\xi(\,\cdot\,))g](x)$. The random accumulated momentum is a nonnegative additive functional of the process trajectory. For a reflected Wiener process, the growth of the accumulated momentum is localized on the interval boundary (that is, it increases only at the moments when the process reaches the boundary) and can be expressed in terms of the local time of the reflected Wiener process on the boundary (see [1], [2]). For jump Lévy processes, such a localization does not take place even if the local time exists (see [3]).

REFERENCES

[1] G. A. Brosamler, *A probabilistic solution of the Neumann problem*, Math. Scand., 38 (1976), pp. 137–147.

[2] K. Sato and H. Tanaka, *Local times on the boundary for multi-dimensional reflecting diffusion*, Proc. Japan Acad., 38 (1962), pp. 699–702.

[28] Supported by the Russian Science Foundation (grant 17-11-01136).

[3] I. A. Ibragimov, N. V. Smorodina, and M. M. Faddeev, *A construction of reflecting Lévy processes*, Dokl. Math., 99 (2019), pp. 71–74.

E. S. Sopin, K. A. Ageev, K. E. Samouylov (Moscow, Russia)**. Effective algorithm for stationary characteristic evaluation of queuing systems with limited resources.** [29]

Queuing systems with limited resources [1] (which generalize the Erlang model) have received increasing attention in recent years due to the capability of adequate modeling of radio resource allocations in modern wireless networks. In [2], analytic product-form expressions for the stationary probability distribution of the random process $X(t) = \{\xi(t), \theta(t)\}$ were obtained:

$$q_n(\mathbf{r}) = G(N, \mathbf{R})^{-1} \frac{\rho^n}{n!} p^{(n)}(\mathbf{r}), \tag{1}$$

where $p^{(n)}(\mathbf{r})$ is the n-fold convolution of the resource requirement distribution $p(\mathbf{r})$, and $G(N, \mathbf{R})$ is the normalizing constant. Evaluation of convolutions of probability distributions is an involved task. Therefore, as an extension of [3], we develop an efficient algorithm for stationary characteristic evaluation of a considered process $X(t)$.

Theorem. *The normalizing constant $G(N, \mathbf{R})$ can be found by the recurrent formula*

$$G(n, \mathbf{r}) = G(n-1, \mathbf{r}) + \frac{\rho}{n} \sum_{\mathbf{0} \leqslant \mathbf{i} \leqslant \mathbf{r}} p(\mathbf{i}) \big(G(n-1, \mathbf{r}-\mathbf{i}) - G(n-2, \mathbf{r}-\mathbf{i}) \big) \tag{2}$$

with the initial values $G(0, \mathbf{r}) = 1$, $G(1, \mathbf{r}) = 1 + \rho \sum_{\mathbf{0} \leqslant \mathbf{i} \leqslant \mathbf{r}} p(\mathbf{i})$, $\mathbf{0} \leqslant \mathbf{r} \leqslant \mathbf{R}$.

REFERENCES

[1] E. L. Romm and V. V. Skitovich, *On certain generalization of problem of Erlang*, Autom. Remote Control, 32 (1971), pp. 1000–1003.
[2] V. A. Naumov, K. E. Samuilov, and A. K. Samuilov, *On the total amount of resources occupied by serviced customers*, Autom. Remote Control, 77 (2016), pp. 1419–1427.
[3] E. S. Sopin, K. A. Ageev, E. V. Markova, O. G. Vikhrova, and Yu. V. Gaidamaka, *Performance analysis of M2M traffic in LTE network using queuing systems with random resource requirements*, Automat. Control Comput. Sci., 52 (2018), pp. 345–353.

M. A. Stepovich (Kaluga, Russia)**, E. V. Seregina** (Kaluga, Russia)**, D. V. Turtin** (Ivanovo, Russia)**. On some aspects of correctness and stochastic features of mathematical models of diffusion and cathodoluminescence in semiconductors.** [30]

We consider stochastic models of diffusion [1] and subsequent radiative recombination of nonequilibrium minority charge carriers (MCC) generated in homogeneous semiconductors by wide electron or light beams. The following result is proved.

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THEOREM. *For the rate of convergence of matrix series approximating the projection characteristics of MCC after their diffusion* (C^{m_p} *is the expectation and* C^{R_p} *is the autocorrelation function*), *the following estimates hold*:

$$\begin{aligned}\|C^{m_p} - C^{m_p}_{ik}\| &\leqslant C_1 \int_{r_1}^{r_2} \frac{\Xi^i \exp\{-(\mu+2)r^2/(2\mu)\}}{1-\Xi}\, dr \\ &\quad + C_2 \int_{r_1}^{r_2} \Xi^i \exp\biggl\{-\frac{(\mu+2)r^2}{2\mu}\biggr\}\, dr, \\ \|C^{R_p} - C^{R_p}_{ik}\| &\leqslant C_3(k) \int_{r_1}^{r_2} \frac{\Xi^i \exp\{-(\mu+4)r^2/(2\mu)\}}{(1-\Xi)^2}\, dr \\ &\quad + C_4(k) \int_{r_1}^{r_2} \Xi^i \exp\biggl\{-\frac{(\mu+4)r^2}{2\mu}\biggr\}\, dr.\end{aligned}$$

Here r *is a continuous r.v. that is distributed according to the normal law and has zero expectation and unit vairance*; k *is the degree of the Chebyshev polynomial of the first kind*; $\Xi \equiv \|\widetilde{W}^k_p(r)\| < 1$, $\mu > 0$; *and* C_1, C_2, $C_3(k)$, *and* $C_4(k)$ *are positive constants that do not depend on* i; *moreover*, $C_3(k)$ *and* $C_4(k)$ *decrease with increasing* k.

REFERENCES

[1] E. V. SEREGINA, *The Use of the Projection Method for Mathematical Modeling of Statistical Distribution of Minor Charge Carriers in Semiconductoring Materials*, abstract of Ph.D. thesis (05.13.18), Keldysh IPM, RAS, Moscow, 2014.

D. A. Suchkova (Ufa, Russia). **Construction of the solution of a new version of the stochastic long wave equation (BBM) with white noise dispersion.**

The deterministic BBM (Benjamin–Bona–Mahony) equation

$$u_t + u_x + uu_x - u_{xxt} = 0 \tag{1}$$

has several advantages in comparison with the well-known Korteweg–de Vries equation [1] as an approximation for the description of unidirectional propagation of waves with small wave-amplitude and large wavelength in nonlinear dispersive systems. In particular, the phase velocitiy and the group velocity corresponding to the linearized BBM equation (1) are bounded for all wave numbers; moreover, both velocities tend to zero for large wave numbers.

The stochastic BBM equation (the regularized long wave equation) with white noise dispersion,

$$d_t u - d_t u_{xx} + u_x\, dt + u_x * dW + uu_x\, dt = 0, \qquad u(s) = u_s, \tag{2}$$

and the stochastic BBM equation with white noise in the variance and in the nonlinear term,

$$d_t u - d_t u_{xx} + u_x\, dt + u_x * dW + uu_x\, dt + uu_x * dW = 0, \qquad u(s) = u_s, \tag{3}$$

represent a more adequate model for particular physical systems that are stochastic in nature. Equation (2), as well as the equation proposed in [2], has the advantage of being the deterministic equation (1) if the noise is insignificant. This is valid for equation (3) also.

THEOREM. *Solution of* (2) *is reduced* [3] *to that of the following equations:*

$$u_t + u_x + uu_x - u_{xxt} = 0, \quad u_v + u_x - u_{xxv} = 0.$$

Solution of (3) *is reduced to that of the equations*

$$u_t + u_x + uu_x - u_{xxt} = 0, \quad u_v + u_x + uu_x - u_{xxv} = 0.$$

We give particular solutions of the stochastic equations (2) and (3) and, in particular, of (3) in the form of a traveling wave.

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REFERENCES

[1] T. B. BENJAMIN, J. L. BONA, AND J. J. MAHONY, *Model equations for long waves in nonlinear dispersive systems*, Philos. Trans. Roy. Soc. London Ser. A, 272 (1972), pp. 47–78.
[2] M. CHEN, O. GOUBET, AND Y. MAMMERI, *Generalized regularized long wave equation with white noise dispersion*, Stoch. Partial Differ. Equ. Anal. Comput., 5 (2017), pp. 319–342.
[3] F. S. NASYROV, *Local Times, Symmetric Integrals and Stochastic Analysis*, Fizmatlit, Moscow, 2011 (in Russian).

A. I. Sukhinov (Rostov-on-Don, Russia)**, V. V. Sidoryakina** (Taganrog, Russia)**, S. V. Protsenko** (Rostov-on-Don, Russia)**. Numerical investigation of a nonlinear stochastic model of sediment transport in coastal systems.** [31]

We consider the problem of construction and numerical study of a 2D model of sediment transport [1] in coastal zones of water bodies and take into account the stochastic nature of suspended matter precipitation.

THEOREM. *Let*

$$(1-\varepsilon)\,\frac{\partial H^n}{\partial t} = \operatorname{div}\left(k^{n-1}\frac{\tau_{bc}}{\sin\varphi_0}\operatorname{grad} H^n\right) - \operatorname{div}(k^{n-1}\vec{\tau}_b) + f,$$
$$t_{n-1} < t \leqslant t_n, \qquad n = 1, 2, \dots, N,$$

be the sediment transport equation, where f is the function describing precipitation of suspended matter (*it has a stochastic character*), $H \in C^2(V_T) \cap C(\overline{V}_T)$, $\operatorname{grad}_{(x,y)} H^n \in C(\overline{V}_T)$, *and H is the solution of the nth equation of the above system in the domain $V_T = D \times (0,T)$ with the initial conditions* $H^1(x,y,0) = H_0(x,y)$, $H_0(x,y) \in C^2(D) \cap C(\overline{D})$, $\operatorname{grad}_{(x,y)} H_0 \in C(\overline{D})$, $(x,y) \in \overline{D}$, $\overline{D}/D = G$, $H^n(x,y,t_{n-1}) = H^{n-1}(x,y,t_{n-1})$, $(x,y) \in \overline{D}$, $n = 2, \dots, N$, *and the Dirichlet boundary conditions. Now if* $k^{n-1} \geqslant k_0 > 0$, $k_0 = \text{const}$, $k^{n-1} \in C^1(\overline{D})$, $n = 1, 2, \dots, N$, *and* $\sin\varphi_0 \leqslant 2\tau_{bc}$, *then, under the condition* $H \geqslant c_0 \equiv \text{const}$,

$$\iint_D H(T)\,dx\,dy \leqslant \iint_D H(0)\,dx\,dy + \frac{M_1}{c_0(1-\varepsilon)}\int_0^T\left(\iint_D(\tau_{bx}^2 + \tau_{by}^2)\,dx\,dy\right)dt$$
$$+ M_2\int_0^T\left(\int_G H_G\,dx\right)dt + \frac{1}{1-\varepsilon}\int_0^T\left(\iint_D f\,dx\,dy\right)dt,$$

where M_1, M_2 are some constant functions, and G is the boundary of the computational domain.

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REFERENCES

[1] A. I. SUKHINOV AND V. V. SIDORYAKINA, *Development and correctness analysis of the mathematical model of transport and suspension sedimentation depending on bottom relief variation*, Vest. Don. Gos. Tekhn. Univ., 18 (2018), pp. 350–361.

M. S. Tikhov, V. A. Grishin (NNSU, Nizhni Novgorod, Russia)**. A simple bias reduction method in nonparametric distribution function estimation.**

This work continues the research started in [1], [2] on estimating the d.f. in a dose-response relationship. Namely, let $\mathscr{U}^{(n)} = \{(U_i, W_i)\}_{i=1}^n$ be a sample of a pair (U, W), where $W_i = I(X_i < U_i)$ is an indicator of the event $(X_i < U_i)$, and the sequences $(U_i)_{1\leqslant i\leqslant n}$ and $(X_i)_{1\leqslant i\leqslant n}$ are independent. The pair (X, U) has a joint distribution function $F(x)G(u)$ and a joint density $f(x)g(u) > 0$. We are interested in estimating the d.f. $F(x)$ of the r.v. X from the sample $\mathscr{U}^{(n)}$. One often considers the statistic

$$\widehat{F}_n(x) = \frac{S_{2n}(x)}{S_{1n}(x)}$$

as such an estimate of $F(x)$. Here

$$S_{2n}(x) = n^{-1}\sum_{i=1}^n W_i K_h(x - U_i), \qquad S_{1n}(x) = n^{-1}\sum_{i=1}^n K_h(x - U_i),$$

$\mathbf{W} = (W_1, \dots, W_n)^\top$, $K_h(x) = h^{-1}K(h^{-1}x)$, and $h = h(n) = n^{-1/5}$ is a smoothing parameter. It is known that the estimator $\widehat{F}_n(x)$ is not $\sqrt{nh}$-consistent. Hence we define the diagonal matrix $\mathbf{A} = \operatorname{diag}\{K_h(x - U_1), \dots, K_h(x - U_n)\}$ and the norm $\|\mathbf{x}\| = \sum_{j=1}^n |x_j|$ of a vector $\mathbf{x} = (x_1, \dots, x_n)^\top$. We also consider the operator norm $\|\mathbf{A}\| = \sup_{\|\mathbf{x}\|=1} \|\mathbf{A}\mathbf{x}\|$. Let $\mathbf{A}_k = \mathbf{I} - (\mathbf{I} - \mathbf{A})^k$, $\widehat{S}_{2n}^{(k)}(x) = \mathbf{A}_k \mathbf{W}$, $\|\mathbf{I} - \mathbf{A}\| = \lambda$. Making $n \to \infty$, we have $0 < \lambda < 1$; i.e., the operator $\mathbf{I} - \mathbf{A}$ is a contraction. Hence, as $k \to \infty$ the estimator $\widehat{S}_{2n}^{(\infty)}(x)$ is an unbiased estimator of the function $F(x)g(x)$. In a similar manner, one gets rid of the bias of the estimator for $g(x)$ based on the statistic $S_{1n}(x)$. Let $\widetilde{F}_n(x) = \widehat{S}_{2n}^{(\infty)}(x)/\widehat{S}_{1n}^{(\infty)}(x)$, $\sigma^2(x) = F(x)(1 - F(x))\|K\|^2/g(x)$.

THEOREM. *Under certain regularity conditions, $\tilde{F}_n(x)$ is an unbiased estimator of $F(x)$ and $n^{2/5}(\widetilde{F}_n(x) - F(x)) \xrightarrow{d} N(0, \sigma^2(x))$, $n \to \infty$.*

In this talk, we also consider the method of stochastic approximation for delivering estimates for the d.f. $F(x)$. The consistency and asymptotic normality of these estimates are proved.

REFERENCES

[1] M. S. TIKHOV, *Fourier method for recursive estimation of distribution function in dose-effect relationship*, abstract, Third International Conference on Stochastic Methods, Theory Probab. Appl., 64 (2019), p. 165.

[2] M. S. TIKHOV, *Fourier methods for recursive estimating of distribution function in dose-effect relationship*, Vestnik TVGU. Ser. Prikl. Matem., no. 4, 2018, pp. 31–49.

V. V. Ulyanov (Moscow, Russia)**, G. Christoph** (Magdeburg, Germany)**, M. M. Monakhov** (Moscow, Russia)**. Asymptotic expansions for distributions of statistics with random sample size.** [32]

In practice, one often encounters situations where the sample size is not defined in advance and can be random itself. In [1] it was demonstrated that the asymptotic properties of the statistics can be radically changed when the nonrandom sample size is replaced by a random value. In this talk, the second-order Chebyshev–Edgeworth and Cornish–Fisher type expansions (see [2]) based on Student's t- and Laplace distributions and their quantiles are derived for sample mean and sample median with random sample size of special kind. We use a general transfer theorem (see [3]), which enables us to construct asymptotic expansions for the distributions of the randomly normalized statistics applying the asymptotic expansions for the distributions of the nonrandomly normalized statistics being considered and for the distributions of the random size of the underlying sample.

Let r.v.'s $X, X_1, X_2, \ldots \in \mathbf{R}$ and $N_1, N_2, \ldots \in \mathbf{N}$ be defined on a probability space $(\Omega, \mathbf{A}, \mathbf{P})$ and be independent. In mathematical statistics, X_n is an observation, and N_n is a random size of the sample. Let $T_m := T_m(X_1, \ldots, X_m)$ be a statistic based on the sample of fixed size $m \in \mathbf{N}$. We set $T_{N_n}(\omega) := T_{N_n(\omega)}\big(X_1(\omega), \ldots, X_{N_n}(\omega)\big)$, $\omega \in \Omega$; i.e., T_{N_n} is the statistic based on the statistic T_m with random sample size N_n. For example, we can take the sample median as T_m. Let the distribution function of N_n for integer k be $(k/(1+k))^n$. This corresponds to the fact that N_n is the maximum over n independent r.v.'s with some discrete Pareto distribution. Then one can get nonasymptotic second-order approximations for the distribution of N_n (see [4]) and for T_m (see [5]) under some regularity conditions on the density function of X_1. Moreover, using the refined transfer theorem (see [5]), we get the following.

THEOREM. *The following inequality holds*: $\sup_x |F(T_{N_n}, x) - L(n, x)| \leqslant c/n^{-3/2}$. *Here* $F(T_{N_n}, x)$ *is the d.f. of the normalized random sample median* T_{N_n}, *and* $L(n, x)$ *is the second-order approximation in powers* $n^{-1/2}$ *with Laplace distribution as the limit function with density* $\exp\{-\sqrt{2}\,|x|\}/\sqrt{2}$.

For other options for T_m and their approximations, see [6, Chaps. 13–16], including the case of high-dimensional observations. The proofs and formulations of the results of this talk can be found in [4], [5], [7].

REFERENCES

[1] B. V. GNEDENKO, *Estimating the unknown parameters of a distribution with a random number of independent observations*, Trudy Tbiliss. Mat. Inst. Razmadze Akad. Nauk Gruzin. SSR, 92 (1989), pp. 146–150 (in Russian).

[2] V. V. ULYANOV, M. AOSHIMA, AND Y. FUJIKOSHI, *Non-asymptotic results for Cornish–Fisher expansions*, J. Math. Sci. (N.Y.), 218 (2016), pp. 363–368.

[3] V. E. BENING, N. K. GALIEVA, AND V. YU. KOROLEV, *Asymptotic expansions for the distribution functions of statistics constructed from samples with random sizes*, Inform. Primen., 7 (2013), pp. 75–83 (in Russian).

[4] G. CHRISTOPH, M. M. MONAKHOV, AND V. V. ULYANOV, *Second order Chebyshev–Edgeworth and Cornish–Fisher expansions for distributions of statistics constructed from samples with random sizes*, J. Math. Sci., 244(5) (2020), pp. 811–839.

[5] G. CHRISTOPH, V. V. ULYANOV, AND V. E. BENING, *Second Order Expansions for Sample Median with Random Sample Size*, preprint, arXiv:1905.07765, 2019.

[32] This work was supported by the Russian Science Foundation (grant 18-11-00132) and by the Russian Academic Excellence Project "5–100."

[6] Y. Fujikoshi, V. V. Ulyanov, and R. Shimizu, *Multivariate Statistics. High-Dimensional and Large-Sample Approximations*, Wiley Ser. Probab. Stat., John Wiley & Sons, Hoboken, NJ, 2010.

[7] A. S. Markov, M. M. Monakhov, and V. V. Ulyanov, *Generalized Cornish–Fisher expansions for distributions of statistics based on samples of random size*, Inform. Primen., 10 (2016), pp. 84–91 (in Russian).

T. A. Volosatova, I. V. Pavlov, S. I. Uglich (Rostov-on-Don, Russia)**. Minimax problem in a task with priorities.** [33]

Consider the function $\mathbf{F} = \prod_{j=1}^{k} \mathbf{E}^{\mathbf{P}}(u_j^{\alpha_j})$, where all $u_j > 0$ and $\alpha_j = \alpha_j(\omega)$ are r.v.'s defined on $(\Omega, \mathscr{F}, \mathbf{P})$ and satisfying the conditions $\mathbf{P}(0 < \alpha_j < 1) > 0$. We investigate the case when $u_k = -\sum_{i=1}^{k-1} c_i u_i + \sum_{i=1}^{k-1} c_i b_i + b_k > 0$ and all constants b_j are positive. So, for any fixed vector of parameters $(c_1, \dots, c_{k-1})$, the function $\mathbf{F}$ depends on $(u_1, \dots, u_{k-1})$ (it is denoted below by $F = F(u_1, \dots, u_{k-1})$) and is defined on the set $\bigl\{u_1 > 0, \dots, u_{k-1} > 0, \sum_{i=1}^{k-1} c_i u_i < \sum_{i=1}^{k-1} c_i b_i + b_k\bigr\}$. From [1], [2] it follows that if all c_j are positive and fixed, then the function F has a single point $(u_1^*, \dots, u_{k-1}^*)$ of local maximum (which is also a global maximum point of F) in its domain of definition. We set $F_{\max}(c_1, \dots, c_{k-1}) = F(u_1^*, \dots, u_{k-1}^*)$.

Theorem. *The function F_{max} has a unique stationary point in its domain of definition $\{c_1 > 0, \dots, c_{k-1} > 0\}$. This point is a minimum point.*

This minimax theorem can be applied to the problem of optimization of interactions within a unified system consisting of a number of institutions and an "optimizer" that is interested in successful performance of the system and acts on the basis of expert evaluations implemented in the form of independent random priorities α_j.

REFERENCES

[1] I. V. Pavlov and S. I. Uglich, *Optimization of complex systems of quasilinear type with several independent priorities*, Vestn. RGUPS, no. 3(67), 2017, pp. 140–145.

[2] N. P. Krasii, *Existence and uniqueness of the maximum point in problems of optimization of quasiline models with independent priorities*, Vestn. RGUPS, no. 4(72), 2018, pp. 144–151 (in Russian).

L. Vostrikova (LAREMA, Université d'Angers, France)**. On the ruin problem with investment when the risky asset is a semimartingale.**

In this talk, we give new results on the ruin probability of an insurance company having initial capital $y > 0$ with investment, where the business part of the insurance company X is modeled by a Lévy process with parameters (a_X, σ_X, ν_X), and the return of an investment process R is a semimartingale. We obtain upper and lower bounds on the finite and infinite time ruin probabilities $\mathbf{P}(\tau(y) \leqslant T)$, $\mathbf{P}(\tau(y) < \infty)$ and find a logarithmic asymptotic formula for them. More precisely, let

$$I_T = \int_0^T e^{-\widehat{R}_s}\, ds \quad \text{and} \quad J_T(\alpha) = \int_0^T e^{-\alpha \widehat{R}_s}\, ds,$$

with $\alpha > 0$ and $\widehat{R}_t = \ln \mathscr{E}(R)_t$, where $\mathscr{E}(R)$ is the Doléan–Dade exponential of R. In addition, we put $\beta_T = \sup\{\beta \geqslant 0\colon \mathbf{E}(J_T^{\beta/2}) < \infty,\, \mathbf{E}(J_T(\beta)) < \infty\}$.

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THEOREM 1. *Let $T > 0$. Assume that $\beta_T > 0$ and that, for some $0 < \alpha < \beta_T$,*

$$\int_{|x|>1} |x|^\alpha \,\nu_X(dx) < \infty. \tag{1}$$

Then, for all $y > 0$,

$$\mathbf{P}(\tau(y) \leqslant T) \leqslant \frac{C_1\mathbf{E}(I_T^\alpha) + C_2\mathbf{E}(J_T^{\alpha/2}) + C_3\mathbf{E}(J_T(\alpha))}{y^\alpha},$$

where $C_1 \geqslant 0$, $C_2 \geqslant 0$, and $C_3 \geqslant 0$ are constants depending only on α. Moreover, if (1) holds for all $0 < \alpha < \beta_T$, then

$$\limsup_{y\to\infty} \frac{\ln \mathbf{P}(\tau(y) \leqslant T)}{\ln y} \leqslant -\beta_T.$$

In Theorem 2 we obtain a lower bound for the ruin probability, which gives the logarithmic asymptotic for it. From Theorems 1 and 2, we can also obtain similar results for $\mathbf{P}(\tau(y) < \infty)$. In addition, we prove the following result for the ruin with probability 1.

THEOREM 3. *Assume that $a_X < 0$ or $\sigma_X > 0$ or $\nu_X([-a, a]) > 0$ for some $a > 0$. We also assume that ($\mathbf{P}$-a.s.), $I_\infty = +\infty$, $J_\infty(2) = +\infty$ and that there exists a strictly positive finite limit $\lim_{t\to\infty}(I_t/\sqrt{J_t(2)}) = L$. Then, for all $y > 0$,*

$$\mathbf{P}(\tau(y) < \infty) = 1.$$

In the case when R is a Lévy process, we retrieve from this theorem the known results for the ruin with probability 1.

This talk is a joint work with J. Spielmann.

A. L. Yakymiv (Steklov Mathematical Institute of Russian Academy of Sciences, Moscow, Russia)**. Moment asymptotics of a cycle number in a random *A*-permutation.**[34]

We fix an arbitrary set A of natural numbers. Let $T_n(A)$ be the set of all permutations of n elements with cycle lengths belonging to the set A (the so-called A-permutations). A random permutation τ_n uniformly distributed at the set $T_n(A)$ is considered. By ζ_n we denote the number of its cycles. We also put $A(n) = A \cap [1, n]$, $l(n) = \sum_{i\in A(n)} 1/i$.

THEOREM 1. *Assume that*

$$\frac{|T_n(A)|}{n!} = Cn^{\varrho-1}\bigl(1 + O(n^{-\lambda}L(n))\bigr) \qquad (n \to \infty)$$

for some constants $C > 0$, $\lambda \geqslant 0$, and $\varrho \in (0, 1]$, where $\{L(n),\, n \in \mathbf{N}\}$ is slowly varying at infinity. Then

$$\mathbf{E}\zeta_n = l(n) + \sigma(n) + O(n^{-\varrho}) + O(n^{-\lambda}L(n)) + \bigl(O(n^{-\varrho}) + O(n^{-\lambda})\bigr)\int_1^n \frac{L(x)}{x}\,dx$$

as $n \to \infty$, where, for arbitrary $x \geqslant 1$, we set $L(x) = L([x])$ and

$$\sigma(n) = \sum_{m\in A(n-1)} \frac{1}{m}\left(\left(1 - \frac{m}{n}\right)^{\varrho-1} - 1\right) \to \varrho\int_0^1 \frac{1}{x}\bigl((1-x)^{\varrho-1} - 1\bigr)\,dx.$$

[34]This work was completed within the framework of the MIAN state assignment.

We also obtain asymptotic formulas for the kth moments of ζ_n with fixed $k > 1$. Corresponding examples of sets A are given. Note that asymptotic properties of $\mathbf{E}\zeta_n$ and $\mathbf{D}\zeta_n$ were studied with $A = N$ in Gončarov's fundamental paper [1].

REFERENCES

[1] V. Gončarov, *On the field of combinatory analysis*, Amer. Math. Soc. Transl. (2), 19 (1962), pp. 1–46.

E. B. Yarovaya (Moscow, Russia). **Large deviations and asymptotic behavior of stochastic evolutionary systems.** [35] We consider continuous-time processes with generation and walking of particles on $\mathbf{Z}^d$, $d \geqslant 1$. Points of $\mathbb{Z}^d$ where the particle generation (that is, birth and death of particles) can occur are called *sources of branching*, and the process itself is called a *branching random walk* (BRW). The principal attention is paid to the asymptotic analysis of the behavior of the particle numbers and/or their integer moments, as $t \to \infty$, for the following models: (1) a symmetric BRW with one source of branching and a finite or infinite number of the initial particles (see [1]); (2) a symmetric BRW with a finite number of sources of various positive intensities and one initial particle (see [2]); (3) a BRW with pseudosources, admitting possible violation of symmetry at sources of branching, and with one initial particle (see [3]).

Let $p(t,x,y)$ be the transition probability of the underlying random walk. As shown in [4], [5], for a homogeneous symmetric random walk, the analysis of large deviations significantly depends on the behavior of $p(t,x,y)$ for $|y-x|+t \to \infty$ (under various assumptions about the relationship between $|y-x|$ and t with their joint growth) and on the behavior of the Green function $G_\lambda(y-x) = \int_0^\infty e^{\lambda t} p(t,x,y)\,dt$ as $|y-x| \to \infty$ under different assumptions on the parameter λ. The description of the models and the related proofs can be found, e.g., in [1], [2], [3], [4], [5].

We present a new limit theorem on the behavior of numbers of particles for a BRW with one initial particle and pseudosources u_i, $i = k+1,\dots,m$, admitting possible violation of symmetry at some of the sources of branching v_j, $j = 1,\dots,k$. Let $\mathscr{A}$ be a self-adjoint operator generating a symmetric random walk over $\mathbb{Z}^d$, and let Δ_x be the orthogonal projector on an element x. Then the operator $\mathscr{H} = \mathscr{A} + \sum_{i=1}^{k+m} \zeta_i \Delta_{u_i} \mathscr{A} + \sum_{j=1}^{k+l} \beta_j \Delta_{v_j}$ (which is non-self-adjoint in the case when $\zeta_i \neq 0$ for some i) defines the evolution of the mean number of particles $\mathbf{E}\mu_t(y)$ at every point $y \in \mathbf{Z}^d$ and also of the mean number $\mathbf{E}\mu_t$ of the total number of particles $\mu_t = \sum_{y\in\mathbf{Z}^d} \mu_t(y)$ on the whole lattice; see [3].

Theorem 1. *Let the operator $\mathscr{H}$ have an isolated eigenvalue $\lambda_0 > 0$, and let the rest of its spectrum belong to the semiaxis $\{\lambda \in \mathbf{R}\colon \lambda \leqslant \lambda_0 - \varepsilon\}$, where $\varepsilon > 0$. If $\zeta_i > -1$, $\beta_j^{(1)} > 0$, and $\beta_j^{(r)} = \mathrm{O}(r!\, r^{r-1})$ for all $j = 1,\dots,k+l$ and $r \in \mathbf{N}$, then, in the sense of convergence in distribution,*

$$\lim_{t\to\infty} \mu_t(y) e^{-\lambda_0 t} = \psi(y)\xi, \quad \lim_{t\to\infty} \mu_t e^{-\lambda_0 t} = \xi,$$

where $\phi(y)$ is a nonnegative nonrandom function and ξ is a nondegenerate r.v.

The proof of Theorem 1 is based on the asymptotic analysis of the moments of the r.v.'s $\mu_t(y)$ and $\mu_t(y)$. The convergence in distribution is established with the help of the Carleman condition in a way similar to that suggested in [2].

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REFERENCES

[1] E. Ermakova, P. Makhmutova, and E. Yarovaya, *Branching random walks and their applications for epidemic modeling*, Stoch. Models, 35 (2019), pp. 300–317.
[2] I. I. Khristolyubov and E. B. Yarovaya, *A limit theorem for supercritical random branching walks with branching sources of varying intensity*, Theory Probab. Appl., 64 (2019), pp. 365–384.
[3] E. B. Yarovaya, *Branching random walks with several sources*, Math. Popul. Stud., 20 (2013), pp. 14–26.
[4] S. A. Molchanov and E. B. Yarovaya, *Limit theorems for the Green function of the lattice Laplacian under large deviations of the random walk*, Izv. Math., 76 (2012), pp. 1190–1217.
[5] S. A. Molchanov and E. B. Yarovaya, *Large deviations for a symmetric branching random walk on a multidimensional lattice*, Proc. Steklov Inst. Math., 282 (2013), pp. 186–201.

V. G. Zadorozhniy (Voronezh, Russia)**. On the expectation of solution of a linear system of differential equations with three random coefficients.**

We consider the Cauchy problem

$$\frac{dx}{dt} = \varepsilon_1(t)Ax + \varepsilon_2(t)x + \varepsilon_3(t)f(t), \tag{1}$$

$$x(t_0) = x_0, \tag{2}$$

where $t \in \mathbf{R}$, $T = [t_0, t_1]$; $x\colon \mathbf{R} \to \mathbf{R}^n$ is the desired vector function; $\varepsilon_1, \varepsilon_2$ are the Laplace random processes defined by the characteristic functionals (see [1, p. 30]),

$$\varphi_j(u) = \frac{\exp\bigl(i\int_T \xi_j(s)u(s)\,ds\bigr)}{1 + \int_T\int_T b_j(s_1,s_2)u(s_1)u(s_2)\,ds_1\,ds_2}, \qquad j = 1,2;$$

ε_3 is a random process; A is an $n \times n$-matrix; $f\colon T \to \mathbf{R}^n$ is a given vector function; and x_0 is a random vector. Here $\xi_j(s) = \mathbf{E}(\varepsilon_j(s))$ is the expectation of the random process ε_j; $b_j(s_1,s_2) = \mathbf{E}(\varepsilon_j(s_1)\varepsilon_j(s_2)) - \mathbf{E}(\varepsilon_j(s_1))\mathbf{E}(\varepsilon_j(s_2))$ is the covariance function of the random process ε_j, $i = (-1)^{0.5}$; and $u\colon T \to \mathbf{R}$ is a summable function on T.

Theorem. *Suppose that*

$$\|A\|^2 \int_T\int_T b_1(s_1,s_2)\,ds_1\,ds_2 < 1, \qquad \int_T\int_T b_2(s_1,s_2)\,ds_1\,ds_2 < 1,$$

ε_1, ε_2, ε_3, x_0 *are independent, and the functions* ξ_j, b_j, f *are continuous on* T. *Then*

$$\begin{aligned}\mathbf{E}(x(t)) = {}& \exp\biggl(A\int_{t_0}^t \xi_1(s)\,ds\biggr)\sum_{k=0}^{\infty} A^{2k}\biggl(\int_{t_0}^t\int_{t_0}^t b_1(s_1,s_2)\,ds_1\,ds_2\biggr)^k \\ &\times \frac{\exp\bigl(\int_{t_0}^t \xi_2(s)\,ds\,\mathbf{E}(x_0)\bigr)}{1 - \int_{t_0}^t\int_{t_0}^t b_2(s_1,s_2)\,ds_1\,ds_2} \\ &+ \int_{t_0}^t \exp\biggl(A\int_s^t \xi_1(\tau)\,d\tau\biggr)\sum_{k=0}^{\infty} A^{2k}\biggl(\int_s^t\int_s^t b_1(s_1,s_2)\,ds_1\,ds_2\biggr)^k \\ &\times \frac{\exp\bigl(\int_s^t \xi_2(\tau)\,d\tau\bigr)}{1 - \int_s^t\int_s^t b_2(s_1,s_2)\,ds_1\,ds_2}\mathbf{E}(\varepsilon_3(s))f(s)\,ds\end{aligned}$$

is the expectation of the solution to problem (1), (2).

Of special interest is the case $T = [0, \infty)$. In this case, system (1) is asymptotically mean-stable [2] if

$$\left\| \exp\left(A \int_{t_0}^{t} \xi_1(s)\, ds \right) \sum_{k=0}^{\infty} A^{2k} \left(\int_{t_0}^{t} \int_{t_0}^{t} b_1(s_1, s_2)\, ds_1\, ds_2 \right)^k \right\| \times \frac{\exp\left(\int_{t_0}^{t} \xi_2(s)\, ds \right)}{1 - \int_{t_0}^{t} \int_{t_0}^{t} b_2(s_1, s_2)\, ds_1\, ds_2} \to 0$$

as $t \to \infty$.

REFERENCES

[1] V. G. Zadorozhniy, *Methods of Variational Analysis*, RkHD, Moscow–Izhevsk, 2006 (in Russian).

[2] V. G. Zadorozhniy, *Stabilization of linear systems by a multiplicative random noise*, Differ. Equ., 54 (2018), pp. 728–747.

M. V. Zhitlukhin (Steklov Mathematical Institute of Russian Academy of Sciences, Moscow, Russia). **Asymptotically optimal strategies in a market model with competition** [4]. [36]

We consider a multistep game model where several players (investors) compete for distribution of payments made by several assets. Our goal is to study strategies which are optimal on the infinite time horizon in the sense that they do not lose to any other strategies of the competitors. Problems of this type were considered for the first time in [1] for a particular model with discrete time; later they were studied (also w.r.t. discrete time), for example, in [2], [3]. Here we consider a more general model in both discrete and continuous time.

For simplicity, in this abstract, the model is described only for the case of discrete time. Suppose that on a filtered probability space $(\Omega, \mathscr{F}, (\mathscr{F}_t)_{t=1}^{\infty}, \mathbf{P})$ we are given N adapted nonnegative sequences A_t^n, which represent payments of the assets n at time t. By a strategy of the player m (where $m = 1, \dots, M$) we call a sequence of $\mathscr{F}_{t-1} \otimes \mathscr{B}(\mathbf{R}_+^M)$-measurable functions $l_t^{m,n}(\omega, y)\colon \Omega \times \mathbf{R}_+^M \to \mathbf{R}_+$, which express the amount of wealth invested by this player into the asset n at time t; the argument $y \in \mathbb{R}_+^M$ corresponds to the vector of wealth of all players at time $t-1$. It is assumed that the players select the values $l_t^{m,n}$ simultaneously and independently of one other.

Let the vector $\overline{Y}_0 = (Y_0^1, \dots, Y_0^M)$, with $Y_0^m > 0$, denote a given initial amount of the wealth of the players at the initial moment of time. Then, by definition, the amounts of wealth at the subsequent moments of time are given by

$$Y_t^m = Y_{t-1}^m - \sum_n l_t^{m,n}(\overline{Y}_t) + \sum_n \left(\frac{l_t^{m,n}(\overline{Y}_t)}{\sum_k l_t^{k,n}(\overline{Y}_t)} A_t^n \right). \tag{1}$$

The first sum on the right is equal to the investment expenses of the player m, and the second sum is the profit received by the player m from the assets (the payment from each asset is split between the players proportionally to their investments in it).

Let $r_t^m = Y_t^m / \sum_k Y_t^k$ be the fraction of wealth of player m in the total wealth. The main result reads as follows.

[36] Supported by the Russian Science Foundation (grant 18-71-10097).

THEOREM 1. *There exists a strategy* $\widehat{l}$ *such that if the player* m *uses it, then* $\inf_t r_t^m > 0$ *a.s. for any strategies of the other players.*

Thus the fraction of wealth of a player who uses the strategy $\widehat{l}$ remains bounded away from zero at all time. Here, this strategy is found in explicit form in a general model with continuous time. Moreover, it is shown that it is essentially unique: all strategies that have this property are asymptotically close to it in some sense.

REFERENCES

[1] R. Amir, I. V. Evstigneev, and K. R. Schenk-Hoppé, *Asset market games of survival: A synthesis of evolutionary and dynamic games*, Ann. Finance, 9 (2013), pp. 121–144.

[2] L. Blume and D. Easley, *Evolution and market behavior*, J. Econom. Theory, 58 (1992), pp. 9–40.

[3] I. Evstigneev, T. Hens, and K. R. Schenk-Hoppé, *Evolutionary behavioral finance*, in The Handbook of Post Crisis Financial Modeling, Palgrave Macmillan, London, 2016, pp. 214–234.

[4] M. V. Zhitlukhin, *Asymptotic Distribution of Capital in a Model of an Investment Market with Competition*, preprint, arXiv:1811.12491v1, 2018.