

**Afanasyev V.I.** (Moscow, Russia). **Boundary problems for a random walk in a random environment.**

Let  $\{X_k, k \geq 0\}$  be a random walk in a random environment and the random environment is a sequence of independent and identically distributed random vectors  $(p_i, q_i), i \in \mathbf{Z}$ , where  $p_0 + q_0 = 1, p_0 > 0, q_0 > 0$ . By definition it means that for a fixed random environment the sequence  $\{X_k, k \geq 0\}$  is a discrete Markov chain starting from 0 with the set of states  $\mathbf{Z}$  and transition probabilities  $p_{ij}$  such that  $p_{i,i+1} = p_i, p_{i,i-1} = q_i, i \in \mathbf{Z}$ .

Suppose that

$$\mathbf{E} \ln \frac{q_0}{p_0} = 0, \quad \mathbf{E} \ln^2 \frac{q_0}{p_0} := \sigma^2, \quad \sigma^2 \in (0, +\infty). \quad (1)$$

Let  $T_n = \min \{k \geq 1 : X_k = n\}$ , where  $n \in \mathbf{Z}$ . The random variable  $\ln T_n$ , as  $n \rightarrow \infty$ , is well studied. In particular, limit theorems are established for it as in the case when the condition (1) is valid so and in the case when the value  $\mathbf{E} \ln^2 (q_0/p_0)$  is infinite (see, for example, [1]).

Consider a two-boundary problem about the first exit of the sequence  $\{X_k, k \geq 0\}$  from the interval  $(-\lfloor an \rfloor, \lfloor bn \rfloor)$ , where  $a, b > 0$ . The following results are valid (see [2]).

**Theorem 1.** *If the condition (1) is valid and  $a, b > 0$ , then*

$$\lim_{n \rightarrow \infty} \mathbf{P} (T_{\lfloor bn \rfloor} < T_{-\lfloor an \rfloor}) = \frac{2}{\pi} \arctan \sqrt{a/b}.$$

Consider for  $x, y > 0$  and  $k, l \in \mathbf{Z}$  the triangle

$$S_{k,l}(x, y) = \{(u, v) \in \mathbf{R}^2 : u/x + v/y \leq k + l + 1, u \geq kx, v \geq ly\}.$$

Let  $G(x, y) = \bigcup_{k,l \in \mathbf{Z}} S_{4k, 2l+1}(x, y)$ ,  $D(x, y) = \bigcup_{k,l \in \mathbf{Z}} S_{4k+2, 2l+1}(x, y)$ . Suppose that  $\xi_1, \xi_2$  are independent random variables with standard normal distribution. Let

$$F(x, y) = \mathbf{P} ((\xi_1, \xi_2) \in G(x, y)) - \mathbf{P} ((\xi_1, \xi_2) \in D(x, y)).$$

**Theorem 2.** *If the condition (1) is valid and  $a, b > 0$ , then for any  $x > 0$*

$$\lim_{n \rightarrow \infty} \mathbf{P} \left( \frac{\ln T_{\lfloor bn \rfloor}}{\sigma \sqrt{n}} < x \mid T_{\lfloor bn \rfloor} < T_{-\lfloor an \rfloor} \right) = 2\pi \frac{F(x/\sqrt{b}, x/\sqrt{a})}{\arctan(\sqrt{a/b})}.$$

## REFERENCES

1. *Afanasyev V.I.* About time of reaching a high level by a random walk in a random environment, *Theory Probab. Appl.*, 2013, vol. 57, N 4, pp. 547–567.
2. *Afanasyev V.I.* Two-boundary problem for a random walk in a random environment, *Teoriya veroyatnostei i ee primeneniye* (in Russian), 2018, vol. 63, N 2.

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