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Path dependence is of paramount importance in finance as it can be present in the dynamics of the assets or in the definition of claim pay-offs. We first review the Functional Ito Calculus which is the framework needed for path dependence and allows for the extension of many results to the non-Markov case.

We show that it is possible to characterize which contingent claims can be replicated by either:

- 1) A dynamic trading of the underlying asset.
- 2) A static position in European options.
- 3) A combination of the former two.

We show that the answer lies in the properties of the Intrinsic Value functional, which attributes to each asset price path up to a current date before maturity the payoff obtained by freezing the asset price until maturity. More precisely it depends on the behavior of the second functional space derivative of the Intrinsic Value.

We illustrate the power of the result by applying the associated algorithm to a variety of path dependent claims.