Gliklikh Yu.E. (Voronezh, Russia). **Investigation of completeness of stochastic flows, generated by equations with current velocities.**

This is a joint talk with T.A. Shchichko. The main purpose of the talk is to find conditions (sufficient and necessary and sufficient) for completeness of the stochastic flow generated by the equation in terms of the so-called current velocities (symmetric Nelson's mean derivatives).

The preliminaries can be found in [1 - 4].

Let on $\mathbb{R}^n$ a Borel vector field $v(t,x)$ and a field of symmentric positive semi-definite matrices $\alpha(t,x)$ be given. The equation with current velocities (symmetric mesn derivatives) is the system in $\mathbb{R}^n$m of the form

$$\begin{cases} D_S\xi(t) = a(t,\xi(t)) \\ D_2\xi(t) = \alpha(t,\xi(t) \end{cases} \quad (1)$$

where D_S is the symmetric mean derivative (current velocity) and D_2 is the quadratic mean drivetive. It is shown in [5] that if a and α are smooth, satisfy (together with the first derivatives of α) the estimates of Ito type, α is positive definite and the initial value is random veriable who's density is smooth and nowhere equal to zero, (1) has a solution well-posed for $t \in [0,\infty)$. Our purpose is to find conditions for existence of the solution for $t \in [0,\infty)$ (completeness of the flow) without the estimates of Ito type being satisfied.

Recall that the function $\varphi : \mathbb{R}^n \to \mathbb{R}$ is called proper if its preimage of any relatively compact set in $\mathbb{R}$ is relatively compact in $\mathbb{R}^n$.

Theorem 1 *Let there exist a smooth positive proper function φ on $\mathbb{R}^n$ such that $\mathcal{L}(t,x)\varphi < C$ for a certain $C > 0$ for all $t \in \mathbb{R}$, $x \in \mathbb{R}^n$, where $\mathcal{L}$ is the generator of flow $\xi(s)$. Then the flow $\xi(s)$ is complete.*

Theorem 2 *Let on $\mathbb{R}^n$ there exist smooth positive proper function u such that $\tilde{\mathcal{L}}u < C$ for a certain constant $C > 0$, where $\tilde{\mathcal{L}}$ is the generator of inverse flow $\tilde{\eta}(t)$. Then the forward flow $\eta(t)$ is continuous at infinity on $[0,T]$.*

Introduce the direct product $\mathbb{R}^n_+ = [0,T] \times \mathbb{R}^n$.

Theorem 3 *Both forward flow $\xi(s)$ and inverse flow $\tilde{\xi}(s)$, generated by equation (1), are both continuous at infinity and complete on $[0,T]$, if and only if there exist smooth positive proper functions $u(t,x)$ and $\tilde{u}(t,x)$ on $\mathbb{R}^n_+$ such that the inequalities $(\frac{\partial}{\partial t} + \mathcal{A})u < C$ and $(-\frac{\partial}{\partial t} + \tilde{\mathcal{A}})\tilde{u} < \tilde{C}$ hold for some positive constants C and $\tilde{C}$, where $\mathcal{A}$ and $\tilde{\mathcal{A}}$ are generators of forward and inverse flows, respectively.*

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